# Constructions on Anti Q-Fuzzy Soft Gamma Ideals of Rings

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***ABSTRACT:*** In this paper, we introduce and Study the concept of anti fuzzy soft Gamma ring and investigated some of the properties related to them. Further we study anti Fuzzy soft ideals on gamma rings and established a one-one correspondence between anti fuzzy soft left ideal of a gamma ring R and Level set $\delta_t, t \in [0, 1]$ left ideal of R.

**Keywords:** Soft set, Q-fuzzy set, Q-fuzzy soft set, Anti Q-fuzzy soft gamma ring, Anti Q-fuzzy soft gamma ideals

**INTRODUCATION:** Researchers studying to solve complicated problems in Economics, Engineering, environmental Science, sociology, medical science and many other fields deal with the complex problems of modeling uncertain data. While some Mathematical theories such as probability theory, fuzzy set theory [24, 25], rough set theory [18, 19], vague set theory [9] and the interval mathematics [10] are useful approaches to describing uncertainty, each of these theories has its inherent difficulties as mentioned by Molodtsov [17]. Consequently, Molodtsov [17] prosed a completely new approach for modeling vagueness and uncertainty. This approach called soft set theory is free from the difficulties affecting existing methods. Soft set theory has potential application in many fields, including the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory and measurement theory. Most of these applications have already been demonstrated in Molodtsov's paper [17]. At present, works on soft theory are progressing rapidly. Maji et al [15] investigated the applications of soft set theory to a decision making problem. Maji [16] defined and studied several operations on soft sets. Jun and Park [13] discussed the application of soft sets in ideal theory of BCK/BCI algebras. Aktas and Cagman [2] compared the soft sets to the related concepts of fuzzy and rough sets. They also defined and studied soft groups, soft subgroups, normal soft subgroups and soft homomorphism. Feng et al.[8] introduced and investigated soft semi ring, soft sub semirings, soft ideals, idealistic soft semi ring and soft semiring homomorphism. The algebraic structure of set theories dealing with uncertainties has also been studied by some anthers. Rosenfeld [21] proposed the concept of fuzzy groups in order to establish the algebraic structures of fuzzy sets. Abou-Zaid [1] introduced the notion of a fuzzy subnear-ring and studied fuzzy ideals of a near-ring. This concept is also discussed by many authors [6, 7, 14, 22]. Rough groups were defined by Biswas et al. [4] and some other authors (e.g., [5, 11]) have studied the algebraic properties of rough sets as well. In [23], Sezgin et al. introduced union soft subnear-rings (ideals) of a near-ring and union soft (N -ideals) of an N -group by using Molodtsov's definition of soft sets and investigated their related properties with respect to soft set operations, soft anti-image and lower a-inclusion of soft sets. Throughout this study, applying to soft set theory, we define the notion of soft subnear-rings, soft ideals and soft N -subgroups of near-rings and give several illustrating examples. We also establish the bi-intersection and product operation of soft subnear rings, soft ideals and soft N -subgroup of near-rings.

PRELIMINARIES:

**Definition 1:** Let R and Γ be two additive Commutative semi groups. Then R is called gamma ring if there exists a mapping $R \times \Gamma \times R \to R$ satisfying the follow conditions:

$$R1: a\alpha(b + c) = a\alpha b + a\alpha c;$$
$$R2: (a + b)\alpha c = a\alpha c + b\alpha c;$$
$$R3: a(\alpha + \beta)c = a\alpha c + a\beta c;$$
$$R4: a\alpha(b\beta c) = (a\alpha b)\beta c, \forall a, b, c \in R, \alpha, \beta \in \Gamma.$$

**Example:** Let N be the Set of natural numbers and Γ = {1, 2, 3}. Then {N, max} and {Γ, max} are Commutative semi groups. Define a mapping by $N \times \Gamma \times N \to N$ by $a\alpha b = \min\{a, \alpha, b\}$, $\forall a, b \in N$ and $\alpha \in \Gamma$. Then N is a gamma ring.

**Definition 2:** A non-empty subset S of a gamma ring R is called a sub gamma ring of R if (s, +) is a sub-semi group of (R, +) and $a\alpha b \in S$, for all $a, b \in S, \gamma \in \Gamma$.

**Definition 3:** A non-empty subset S of a gamma ring R is called a gamma ideal of R if (s, +) is a sub-semi group of (R, +) and $x\gamma a \in S$ and $\alpha\gamma x \in S$, for all $a \in S, x \in R$ and $\gamma \in \Gamma$.

**Definition 4:** Let R be the any non-empty set. A mapping $\delta : R \times Q \to [0, 1]$ is called a Q-fuzzy soft subset R.

**Definition 5:** Let δ be any Q-fuzzy soft subset of a set R and let $t \in [0, 1]$ .The set $\delta t = \{x \in R / \delta(x, q) \leq t\}$ is called a lower level subset of δ. The set of all lower level subsets of δ is denoted by $F\delta$. (i.e) $F\delta = \{\delta t / t \in lm(\delta)\}$.

**Definition 6:** Let δ and Δ be any two Q-fuzzy soft subsets of a set R. Then δ is said to be contained in Δ, denoted by $\delta \subseteq \Delta$, if $\delta(x, q) \leq \Delta(x, q)$, for all $x \in R$ and $q \in Q$. If $\delta(x, q) = \Delta(x, q)$, for all $x \in R$, then δ and Δ are said to be equal.

**Definition 7:** Let δ and Δ be any two Q-fuzzy soft subsets of R, then

Union : $(\delta \cup \Delta)(x, q) = \max\{\delta(x, q), \Delta(x, q)\}$

Intersection : $(\delta \cap \Delta)(x, q) = \min\{\delta(x, q), \Delta(x, q)\}$ for all $x \in R$ and $q \in Q$.

**Definition 8:** The union and intersection of any family $\{\delta i / i \in I\}$ of Q-fuzzy soft subsets of a set of R are defined by

$$\left(\bigcup_{i \in I} \delta_i\right)(x, q) = \sup_{i \in I} \delta i(x, q)$$

$$\left(\bigcap_{i \in I} \delta_i\right)(x, q) = \inf_{i \in I} \delta i(x, q) \; \forall x \in R.$$

**III-ANTI Q-FUZZY SOFT GAMMA RING**

**Definition 9:** Let δ be a Q-fuzzy soft subset of a gamma ring R. Then δ is called a Q-fuzzy soft gamma ring if

$(COFSttR - 1) : \delta(x + y, q) \leq \max\{\delta(x, q), \delta(x, q)\}$

$(COFSttR - 2) : \delta(x \gamma y, q) \leq \max\{\delta(x, q), \delta(x, q)\}$ for all $x, y \in R$ and $\gamma \in \Gamma$

**Example 1:** Let R be the set of natural numbers with zero and let $\Gamma = \{0, 1\}$. Define the mapping $R \times \Gamma \times R \to R$ by $a, \alpha, b$ usual product of a, α, b, for all $a, b \in R, \alpha \in \Gamma$. Then R is a gamma ring. Define $\delta : R \times Q \to [0, 1]$ by

$$\delta(x,q) = \begin{cases} 0.7 & \text{if} \quad x = 0 \\ 0.6 & \text{if} \quad x \text{ is odd} \\ 0.5 & \text{if} \quad x \text{ is even} \end{cases}$$ Then δ is a anti Q-fuzzy soft gamma ring.

**Example 2:** Let R be the set of negative integers and Γ be the set of negative odd integers. Then R, Γ are additive commutative semi groups. Define the mapping $R \times \Gamma \times R \to R$ by $a, \alpha, b$ usual product of $a, \alpha, b, \forall\, a, b \in R; \alpha \in \Gamma$. Then R is a Gamma ring. Define $\delta: R \times Q \to [0,1]$ by

$$\delta(x,q) = \begin{cases} 0.3 & \text{if} \quad x = -1 \\ 0.6 & \text{if} \quad x = -2 \\ 0.5 & \text{if} \quad x < -2 \end{cases}$$ Then δ is a anti Q-fuzzy soft gamma ring.

**Example 3:** Consider the additive abelian groups $Z_8 = \{0,1,2,3,4,5,6,7\}$ and $\Gamma = 0,1,3,5$.

Define $Z_8 \times \Gamma \times Z_8 \to Z_8$ by $a, \alpha, b$ usual product of $a, \alpha, b, \forall a, b \in R; \alpha \in \Gamma$.Then $Z_8$ is a Gamma ring. Define $\delta : Z_8 \times Q \to [0,1]$ by

$$\delta(x,q) = \begin{cases} 0.3 & \text{if} \quad x = 0 \\ 0.6 & \text{if} \quad x \neq 0 \end{cases}$$. Then Γ is anti Q-fuzzy soft gamma ring.

**Theorem 1:** Let R be a gamma ring. A Q-fuzzy soft subset δ of R is a anti Q-fuzzy soft gamma ring iff its lower level set $\delta_t, t \in [0,1]$ is a sub gamma ring of R.

**Proof:** Let R be a gamma ring and let δ be a Q-fuzzy soft subset of R. Suppose δ is anti Q-fuzzy soft gamma ring.

**Claim:** $\delta_t = \{x \in R / \delta(x,q) \leq t\}$ is a sub gamma ring of R. Let $x, y \in \delta_t$ and $\gamma \in \Gamma$.

$$\Rightarrow \delta(x,q) \leq t, \delta(y,q) \leq t$$

$$\Rightarrow \max\{\delta(x,q), \delta(y,q) \leq t\}$$

$$\Rightarrow \delta(x+y,q) \leq t \text{ and } \delta(x\gamma y) \leq t$$

$$\Rightarrow x + y \in \delta, x\gamma y \in \delta_t.$$

Then $\delta_t$ is sub gamma ring of R. Conversely, suppose $\delta_t$ is a sub gamma ring of R.

**Claim:** δ is anti Q-fuzzy soft gamma sub ring of R. Let $x, y \in R$ and $\gamma \in \Gamma$.

Let $\delta(x,y) = l$; $\delta(y,q) = m$, Let $t = \max\{l, m\}$.

Then $\delta(x,q) = l \leq t$; $\delta(y,q) = m \leq t$

$$\Rightarrow x, y \in \delta_t$$

$$\Rightarrow x + y \in \delta_t \text{ and } x\gamma y \in \delta_t$$

$$\Rightarrow \delta(x+y,q) \leq t \text{ and } \delta(x\gamma y,q) \leq t$$

$$\Rightarrow \delta(x+y,q) \leq \max\{\delta(x,q), \delta(y,q)\} \text{ and}$$

$\delta(x\gamma y,q) \leq \max\{\delta(x,q), \delta(y,q)\}$. Then δ is anti Q-fuzzy soft gamma ring of R.

**Theorem 2:** Let R be a gamma ring. Let $\delta_1$ and $\delta_2$ be anti Q-fuzzy soft gamma rings of R. The $\delta_1 \cap \delta_2$ is anti Q-fuzzy soft gamma ring of R.

**Proof:** Let R be gamma ring and $\delta_1$ and $\delta_2$ be two anti Q-fuzzy soft gamma ring of R.

Let $x, y \in R$; $\gamma \in \Gamma$

(CFSGR− 1): $(\delta_1 \cap \delta_2)(x+y,q) = \min\{\delta_1(x+y,q), \delta_2(x+y,q)\}$

$$\leq \text{Min}\{\max\{\delta_1(x,q), \delta_1(y,q)\}, \max\{\delta_2(x,q), \delta_2(y,q)\}\}$$

$$= \max\{\min\{\delta_1(x,q), \delta_1(y,q)\}, \min\{\delta_2(x,q), \delta_2(y,q)\}\}$$

$$= \max\{\min\{\delta_1(x,q), \delta_2(x,q)\}, \min\{\delta_1(y,q), \delta_2(y,q)\}\}$$

$= \max\{(\delta_1 \cap \delta_2)(x, q), (\delta_1 \cap \delta_2)(y, q)\}$

(CFSGR − 2): $(\delta_1 \cap \delta_2)(x\gamma y, q) = \min\{\delta_1(x\gamma y, q), \delta_2(x\gamma y, q)\}$

$\leq \min\{\max\{\delta_1(x, q), \delta_1(y, q)\}, \max\{\delta_2(x, q), \delta_2(x, q)\}\}$

$= \min\{\min\{\delta_1(x,q), \delta_1(y,q)\}, \min\{\delta_2(x,q), \delta_2(x,q)\}\}$

$= \max\{\min\{\delta_1(x,q), \delta_2(x,q)\}, \min\{\delta_1(y,q), \delta_2(y,q)\}\}$

$= \max\{(\delta_1 \cap \delta_2)(x, q), (\delta_1 \cap \delta_2)(p, q)\}$

Thus $\delta_1 \cap \delta_2$ is the Q-fuzzy soft gamma ring of R.

In general union of two anti Q-fuzzy soft gamma rings may not be anti Q-fuzzy soft gamma ring.

**Example 4:** Consider the additive groups $Z_6 = \{0, 1, 2, 3, 4, 5\}$ and $\Gamma = \{0, 2\}$.

Define $Z_6 \times \Gamma \times Z_6 \to Z_6$ by $a\alpha b$ usual product of $a, \alpha, b, \forall a, b \in R;\ \alpha \in \Gamma$.

Then $Z_6$ is a gamma ring

Define $\delta_1 : Z_6 \times Q \to [0, 1]$ and $\delta_2 : Z_6 \times Q \to [0, 1]$ by

$$\delta 1(x, q) = \begin{cases} 0.6 & \text{if } x = 0 \\ 0.8 & \text{if } x = 1 \\ 0.3 & \text{otherwise} \end{cases} \quad \text{And} \quad \delta 2(x, q) = \begin{cases} 0.3 & \text{if } x = 2 \\ 0.8 & \text{if } x = 0 \\ 0.4 & \text{otherwise} \end{cases}$$

Then $\delta_1$ and $\delta_2$ are anti Q-fuzzy soft gamma rings of $Z_6$

But $\delta_1 \cup \delta_2$ is not anti Q-fuzzy soft gamma ring, In particular, we have the following theorem.

**Theorem 3:** Let R be a gamma ring. Let $\delta_1$ and $\delta_2$ be two anti Q-fuzzy soft gamma rings of R. Then $\delta_1$ and $\delta_2$ is anti Q-fuzzy soft gamma rings of R if $\delta_1 \subseteq \delta_2$ or $\delta_2 \subseteq \delta_1$.

**Proof:** Let R be a gamma ring.

Let $\delta_1, \delta_2$ be two anti Q-fuzzy soft gamma rings on R. Suppose $\delta_1 \subseteq \delta_2$

Let $x, y \in R; \gamma \in \Gamma$

(CFSGR − 1): $(\delta_1 \cup \delta_2)(x + y, q) = \max\{\delta_1(x + y, q), \delta_2(x + y, q)\}$

$= \delta_2(x + y, q)$

$= \max\{\delta_2(x, q), \delta_2(y, q)\}$

$\leq \max\{\max\{\delta_1(x, q), \delta_2(x, q)\}\max\{\delta_1(y, q), \delta_2(y, q)\}\}$

$= \max\{(\delta_1 \cup \delta_2)(x, q), (\delta_1 \cup \delta_2)(y, q)\}$

(CFSGR − 2): $(\delta_1 \cup \delta_2)(x\gamma y) = \max\{\delta_1(x\gamma y, q), \delta_2(x\gamma y, q)\}$

$= \delta_2(x\gamma y, q)$

$\leq \max\{\delta_2(x, q), \delta 2(y, q)\}$

$= \max\{\max\{\delta_1(x, q), \delta_2(y, q), \max\{\delta_1(y, q), \delta_2(y, q)\}\}$

$= \max\{(\delta_1 \cup \delta_2)(x, q), (\delta_1 \cup \delta_2)(y, q)\}$

Thus $(\delta_1 \cup \delta_2)$ is anti Q-fuzzy soft gamma ring.

Similarly $(\delta_1 \subseteq \delta_2)$, we get $(\delta_1 \cup \delta_2)$ is anti Q-fuzzy soft gamma ring.

**Theorem 4:** Two anti Q-fuzzy soft gamma rings δ and Q of a gamma ring R such that cord Im $\delta < \infty$ and cord ImQ $< \infty$ are equal iff $\mathrm{Im}\delta = \mathrm{Im}Q$ and $F_\delta = F_Q$.

**Proof:** Let R be a gamma ring. Let δ and Q are two anti Q-fuzzy soft gamma rings of R such that cord Im$\delta < \infty$ and cord ImQ $< \infty$, suppose δ, Q are equal

**Claim:** $\mathrm{Im}\delta = \mathrm{Im}Q$ and $F_\delta = F_Q$

Let $t \in I$

$$\Leftrightarrow \delta(x,q) = t$$
$$\Leftrightarrow Q(x,q) = t \ \text{(because } \delta = Q)$$
$$\Leftrightarrow t \in \mathrm{Im}Q$$
$$\therefore \mathrm{Im}\,\delta = \mathrm{Im}Q$$
$$F_\delta = \{\delta/t \in \mathrm{Im}\delta\} = \{Q_t/\mathrm{Im}Q\} = F_Q$$
$$\therefore F_\delta = F_Q$$

Conversely, Suppose $\mathrm{Im}\delta = \mathrm{Im}Q$ and $F_\delta = F_Q$

**Claim:** $\delta = Q$. Let $t \in \mathrm{Im}\delta \Rightarrow t \in \mathrm{Im}Q$.

Then $\delta(x,q) = t$ and $Q(x,t) = t$, (i.e) $\delta(x,q) = t = Q(x,q)$, for all $x \in R$ and $q \in Q$

$$\Rightarrow \delta = Q.$$

**Theorem 5:** Let R be the gamma ring. Let $\delta$ be a anti Q-fuzzy soft gamma ring on R. Define $\delta^*$ on $R \cup \{0\}$ by $\delta^* = \{x \in R/\delta(x,q) = \delta(0,q)\}$. Then $\delta^*$is a sub gamma ring on R.

**Proof:** Let $\delta$ be anti Q-fuzzy soft gamma ring of R.

Let $x, y \in \delta^* \Rightarrow \delta(x,q) = \delta(0,q)$ and $\delta(y,p) = \delta(0,q)$

$$\text{(CFSGR} - 1): \delta(x+y,q) \le \max\{\delta(x,q),\delta(y,q)\}$$
$$= \max\{\delta(0,q),\delta(0,q)\}$$
$$= \delta(0,q)$$
$$\Rightarrow x+y \in \delta^*.$$
$$\text{(CFSGR} - 2): \delta(x\gamma y,q) \le \max\{\delta(x,q),\delta(y,q)\}$$
$$= \max\{\delta(0,q),\delta(0,q)\}$$
$$= \delta(0,q)$$
$$\Rightarrow x\gamma y \in \delta^*.$$

Thus $\delta^*$ is a sub gamma ring on R.

**ANTI Q-FUZZY SOFT GAMMA IDEALS**

**Definition 10:** Let $\delta$ be a Q-fuzzy soft subset of a gamma ring R. Then $\delta$ is called anti Q-fuzzy left (right) ideal of R if

$$\text{(CFSGI} - 1): \delta(x+y,q) \le \max\{\delta(x,q),\delta(y,q)\}$$
$$\text{(CFSGI} - 2): \delta(x\gamma y,q) \le \delta(y,q)\big(\delta(x,q)\big), \ \forall\, x,y \in R, \gamma \in \Gamma, q \in Q.$$

$\delta$ is anti Q-fuzzy soft gamma ideal if it is both Q-fuzzy soft left and right ideal.

Definition **11:** Let S is a subset of a gamma ring R. The characteristic function of S taking values in [0, 1] is a Q-fuzzy soft set given by

$$\chi(x,q) = \begin{cases} 1 & \text{if } x \notin S \\ 0 & \text{if } x \in S \end{cases}$$

Then $\chi_s$ is anti fuzzy characteristic function of S in [0, 1].

**Theorem 6:** Let R be a gamma ring. A Q-fuzzy soft subset $\delta$ of R is a anti Q-fuzzy soft left (right) ideal of a gamma ring R iff the lower level subset $\delta_t, t \in Im\delta$ is a left (right) ideal of R.

**Proof:** Let R be a gamma ring. Let δ be a anti Q-fuzzy soft left ideal of R.

**Claim:** $\delta_t, t \in Im\delta$ is a left ideal of R. Let $x, y \in \delta_t$ and $\gamma \in \Gamma$.

$$\Rightarrow \delta(x,q) \le t \text{ and } \delta(y,t) \le t.$$

$(CFSGI - 1): \delta(x+y,q) \le \max\{\delta(x,q), \delta(y,q)\} \le t$

$$\Rightarrow x + y \in \delta_t.$$

Let $x \in \delta t;\ a \in R, \gamma \in \Gamma \ \Rightarrow \delta(x,q) \le t.$

$(CFSGI - 2): \delta(\alpha\gamma x, q) \le \delta(\gamma, q) \le t \Rightarrow \alpha\gamma x \in \delta_t.$

Thus $\delta_t, t \in Im\delta$ is left (right) ideal of R. Conversely, suppose that $\delta_t$is left (right) ideal of R.

**Claim:** δ is anti Q-fuzzy soft left ideal. Let x, y ∈ R; γ ∈ Γ.

Suppose $\delta(x,q) = a, \delta(y,q) = b$, where $a, b \in [0,1]$. Let $t = \max\{a, b\}$

Therefore $\delta(x,q) \le t$ and $\delta(y,q) \le t$

$$\Rightarrow x, y \in \delta_t$$

$$\Rightarrow x + y \in \delta_t$$

$$\Rightarrow \delta(x+y,q) \le t = \max\{\delta(x,q), \delta(y,q)\}$$

Now $x\gamma y \in \delta_t$ and $y \gamma x \in \delta_t$. Suppose $a < b \Rightarrow t = a$.

Therefore $\delta(y\gamma x, q) \le t = a = \delta(x,q)$.

Suppose $b < a$ , we get $\delta(x\gamma y, q) \le \delta(y,q)$. Thus δ is anti Q-fuzzy soft left (right) ideal of R.

**Theorem 7:** Let S be a non empty subset of a gamma ring R. Then $\chi_s$ is anti Q-fuzzy soft left (right) ideal of R iff S is a left (right) ideal of R.

**Proof:** Suppose $\chi_s$ is a anti Q-fuzzy soft left (right) ideal of R. Let $x, y \in R, a \in R, \gamma \in \Gamma$.

Now $\chi_s(x+y,q) \le \max\{\chi_s(x,q), \chi_s(y,q)\}$

$\Rightarrow x + y \in S$ Also

$\chi_s(\alpha\gamma x, q) \le \chi_s(x,q) = 1 \Rightarrow \alpha\gamma x \in S$. Then S is a left ideal of R.

Conversely, suppose that S is a left (right) ideal of R. Let $x, y \in S$, then $x + y \in S$ and $x\gamma y \in S$

$\Rightarrow \chi_s(x+y,q) = 1 = \max\{\chi_s(x,q), \chi_s(y,q)\} = 1 = \chi_s(y,q)$ of $x, y \notin S$.

Then $x + y \notin S$ and $x\gamma y \notin S$,

$\chi_s(x+y,q) = 0 = \max\{\chi_s(x,q), \chi_s(y,q)\}$ and $\chi_s(x\gamma y, q) = 0 = \chi_s(y,q)$.

If one of the x and y not in S, then $x + y \notin S$ and $x\gamma y \notin S$

$\Rightarrow \chi_s(x+y,q) = 0 = \min\{\chi_s(x,q), \chi s(y,q)\}$ and $\chi_s(x\gamma y,q) = 0 = \chi_s(y,q)$

Then $\chi_s$ is anti Q-fuzzy soft left (right) ideal of R,

**Theorem 8:** Let R be a gamma ring. Let $\delta_1$ and $\delta_2$ be anti Q-fuzzy soft left (right) ideal of R. The $\delta_1 \cap \delta_2$ is a anti Q-fuzzy soft left (right) ideal of R.

**Proof:** Let R be a gamma ring and let $\delta_1, \delta_2$ be two fuzzy soft left (right) ideal of R.

Let x, y ∈ R; γ ∈ Γ. Now,

$(CFSGI - 1): (\delta_1 \cap \delta_2)(x+y,q) = \min\{\delta_1(x+y,q), \delta_2(x+y,q)\}$

$$\leq \min\{\max\{\delta_1(x,q),\delta_1(y,q)\},\max\{\delta_2(x,q),\delta_2(y,q)\}\}$$
$$= \min\{\min\{\delta_1(x,q),\delta_1(y,q)\},\min\{\delta_2(x,q),\delta_2(y,q)\}\}$$
$$= \max\{\min\{\delta_1(x,q),\delta_2(x,q)\},\min\{\delta_1(y,q),\delta_2(y,q)\}\}$$
$$= \max\{(\delta_1\cap\delta_2)(x,q),(\delta_1\cap\delta_2)(y,q)\}$$

$$(\text{CFSGI}-2): (\delta_1\cap\delta_2)(x\gamma y,q) = \min\{\delta(x\gamma y,q),\delta_2(x\gamma y,q)\}$$
$$\leq \max\{\delta_1(y,q),\delta_2(y,q)\}$$
$$= (\delta_1\cap\delta_2)(y,q)$$

This $(\delta_1\cap\delta_2)$ is a anti Q-fuzzy soft left (right) ideal of R.

**Theorem 9:** Let $\{\delta_i/i\in I\}$ be a family of anti Q-fuzzy soft left (right) ideals of gamma ring R. Then $\underset{i\in I}{\cup}\delta_1$ is anti Q-fuzzy soft left (right) ideals of R

**Proof:** Let $\{\delta_i/i\in I\}$ be a family of anti Q-fuzzy soft left ideal of a gamma ring R.

**Claim:** $\delta=\underset{i\in I}{\cup}\delta_1$ anti Q-fuzzy soft left (right) ideals of R. Let $x,y\in R,\gamma\in\Gamma$

$$(\text{CFSGI}-1):\ \delta(x+y,q) = \underset{i\in I}{\cup}\delta_i(x+y,q)$$
$$= \sup_{i\in I}\delta_i(x+y,q)$$
$$\leq \sup_{i\in I}\{\max\{\delta_i(x,q),\delta_i(y,q)\}\}$$
$$= \max\{\sup_{i\in I}\delta_i(x,q),\sup_{i\in I}\delta_i(y,q)\}$$
$$= \max\{\sup\{\delta_i(x,q)/i\in I\},\sup\{\delta_i(y,q)/i\in I\}\}$$
$$= \max\{\underset{i\in I}{\cup}\delta_i(x,q),\underset{i\in I}{\cup}\delta_i(y,q)\}$$
$$= \max\{\delta(x,q),\delta(y,q)$$

$$(\text{CFSGI}-2):\ \delta(x\gamma y,q) = \underset{i\in I}{\cup}\delta_i(x\gamma y,q)$$
$$= \sup_{i\in I}\delta_i(x\gamma y,q)$$
$$= \sup_{i\in I}\delta_i(y,q)$$
$$= \underset{i\in I}{\cup}\delta_i(y,q)$$
$$= \delta(y,q)$$

Hence $\underset{i\in I}{\cup}\delta_i$ is anti Q-fuzzy left (right) ideal of R

**CONCLUSION:** In this paper, the concept of anti Q-fuzzy soft gamma rings is introduced and we established a one-one correspondence between anti Q-fuzzy soft gamma ring and its lower level set. Further we investigated a one-one correspondence between anti Q-fuzzy left (right) ideal of a gamma ring and its lower level set left (right) ideal of a gamma ring.

**APPLICATION:** These structures are useful in developing fuzzy prime ideals, fuzzy maximal ideals and fuzzy semi prime ideals of a gamma ring.

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