International Journal of Mathematics Trends and Technology Volume 66 Issue 10, 24-32, October 2020
ISSN: 2231 - 5373 /doi:10.14445/22315373/IJMTT-V66I10P505 © 2020 Seventh Sense Research Group®

SOME CURVATURE PROPERTIES OF LP-SASAKIAN MANIFOLD WITH RESPECT TO QUARTER SYMMETRIC NON METRIC $\xi-$CONNECTION

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Abstract

The purpose of the present paper is to study some properties of LP-Sasakian manifold with respect to quarter symmetric non metric $\xi-$ connection. Also, we study Conharmonically flat, $\xi-$Conharmonically flat and quasi conharmonically flat LP-Sasakian manifolds with respect to quarter symmetric non metric $\xi-$ connection. Moreover, we study Ricci soliton on LP-Sasakian manifold with respect to Quarter symmetric non metric $\xi-$connection.

Key words and phrases : *LP-Sasakian manifolds, Quarter Symmetric non metric $\xi-$ connection, Conharmonic Curvature tensor, Ricci soliton*

Mathematics Subject Classification 2010: 53C15, 53C25, 53C50.

1 INTRODUCTION

In 1989, K. Matsumoto[9] first introduced the notion of Lorentzian Para-Sasakian manifolds. Also in 1992, I. Mihai and R. Rosca[10] introduced independently the notion of Lorentzian Para-Sasakian manifolds (briefly, LP-Sasakian Manifolds) in classical analysis.

Levi-Civita was the first to define a linear connection for a Riemannian space generalizing the concept of parallelism in Euclidean space. A linear connection $\widetilde{\nabla}$ on a pseudo Riemannian manifold M is said to be symmetric or torsion free if the torsion tensor $\widetilde{T}$ is zero i.e., $\widetilde{T}(X,Y) = 0$, for all $X, Y \in \chi(M)$. On contrast, the linear connection $\widetilde{\nabla}$ is said to be non-symmetric if its torsion tensor does not vanish. Again, if the torsion tensor $\widetilde{T}$ of the connection $\widetilde{\nabla}$ have the form $\widetilde{T}(X,Y) = \eta(Y)X - \eta(X)Y$, for all $X, Y \in \chi(M)$, then the connection $\widetilde{\nabla}$ is called semi-symmetric linear connection. Also if, $\widetilde{T}(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y$, for all $X, Y \in \chi(M)$, then the linear connection is called a quarter symmetric connection[6]. A quarter symmetric connection is said to be metric compatible if $\widetilde{\nabla} g = 0$. And for a non metric compatible quarter

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symmetric connection, $\widetilde{\nabla} g \neq 0$. The quarter symmetric non metric connections on different structures have been studied by many researcher, we cite ([1],[4],[15]) and their references.

For an n-dimensional LP-Sasakian manifold M with a Lorentzian metric g, a $(1,1)$ tensor field ϕ, a vector field ξ and a 1-form η, a new type of quarter symmetric non metric connection $\widetilde{\nabla}$, called a quarter symmetric non metric $\xi-$connection have been recently introduced by S.K. Chaubey and U.C. De[2] and the connection $\widetilde{\nabla}$ is defined as

$$\widetilde{\nabla}_X Y = \nabla_X Y + \eta(Y)\phi X - g(\phi X, \phi Y)\xi \tag{1.1}$$

satisfying

$$\left(\widetilde{\nabla}_X g\right)(Y,Z) = \eta(Y)\{g(\phi X, \phi Z) - g(\phi X, Z)\} + \eta(Z)\{g(\phi X, \phi Y) - g(\phi X, Y)\} \tag{1.2}$$

for all $X, Y, Z \in \chi(M)$, where ∇ denotes the Levi-Civita connection. From (1.1), it follows that $\widetilde{\nabla}_X \xi = 0$, i.e., M is $\xi-$ parallel with respect to $\widetilde{\nabla}$.

In 1957, Y. Ishii[8] introduced and defined Conharmonic curvature tensor of type $(0,3)$ on Riemannian manifold of dimenssion n in terms of Riemannian curvature tensor, Ricci curvature tensor, scalar curvature and metric tensor. The Conharmonic curvature tensor was further studied by many authors. For instance, see ([3],[14],[5]). A Conharmonic curvature tensor K of rank three for an n-dimensional Riemannian Manifold M is given by

$$\begin{aligned} K(X,Y)Z &= R(X,Y)Z - \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y] \\ &\quad - \frac{1}{n-1}[g(Y,Z)QX - g(X,Z)QY] \end{aligned} \tag{1.3}$$

for all $X, Y, Z \in \chi(M)$, where R is the Riemannian tensor of type $(0,3)$, K is the Conharmonic curvature tensor of type $(0,3)$ and S denotes the Ricci tensor of type $(0,2)$, Q is the Ricci operator.

The concept of Ricci flow and its existence was introduced by R.S. Hamilton[7] in the year 1982. R.S. Hamilton observed that the Ricci flow is an excellent tool for symplifying the structure of a manifold. This concept was developed to answer Thurston's geometric conjecture which says that each closed three manifolds admits a geometric decomposition. The Ricci flow equation is given by

$$\frac{\partial g}{\partial t} = -2S \tag{1.4}$$

where g is Riemannian metric, S is Ricci curvature tensor and t is the time. A Ricci soliton is a self similar solution of the Ricci flow equation, where the metrices at different times differ by a diffeomorphism of the manifold. A Ricci soliton is represented by a tripple (g, V, λ), where g is Riemannian metric, V is a vector field and λ is a scalar, which satisfies the equation:

$$L_V g + 2S + 2\lambda g = 0 \tag{1.5}$$

where, S is Ricci curvature tensor, $L_V g$ denotes the Lie derivative of g along the vector field V. The Ricci soliton is said to be shrinking, steady or expanding according as $\lambda < 0, \lambda = 0$ or $\lambda > 0$ respectively. If the vector field V is gradient of a smooth function h, then the Ricci soliton (g, V, λ) is called a gradient Ricci soliton and the function h is called the potential function. Ricci soliton was further studied by many researchers. For nore details, we refer ([11],[12],[13],[16]) and their references there in.

Definition 1.1 *An n-dimensional LP -Sasakian manifold M is said to be generalized $\eta-$Einstein manifold if the Ricci tensor of type(0,2) is of the form*

$$S(Y,Z) = k_1 g(Y,Z) + k_2 \eta(Y)\eta(Z) + k_3 \omega(X,Y) \tag{1.6}$$

for all $X, Y \in \chi(M)$, set of all vector fields of the manifold M and k_1, k_2 and k_3 are constants on M and $\omega(X,Y)$ is a 2-form given by $\omega(X,Y) = g(X, \phi Y)$.

Definition 1.2 *An n-dimensional LP-Sasakian manifold is said to be Conharmonically if* $K(X,Y)Z = 0$ *for all* $X,Y,Z \in \chi(M)$.

Definition 1.3 *An n-dimensional LP-Sasakian manifold is said to be $\xi-$ Conharmonically flat if* $K(X,Y)\xi = 0$ *for all* $X,Y \in \chi(M)$.

Definition 1.4 *An n-dimensional LP-Sasakian manifold is said to be quasi $\phi-$ Conharmonically flat if* $g(K(\phi X,Y)Z,\phi W) = 0$ *for all* $X,Y,Z,W \in \chi(M)$.

This paper is structured as follows:

After introduction, a short description of LP-Sasakian manifold is given in section (2). In section (3) ,we have discussed LP-Sasakian manifold admitting quarter symmetric non metric $\xi-$ connection and obtained curvature tensor $\widetilde{R}$, Ricci tensor $\widetilde{S}$, Scalar curvature tensor $\widetilde{r}$. Section (4) contains Conharmonically flat, $\xi-$Conharmonically flat LP-Sasakian manifolds with respect to the quarter symmetric non metric $\xi-$connection. Section (5) concerns with quasi Conharmonically flat LP-Sasakian manifold with respect to the quarter symmetric non metric $\xi-$connection. In section (6), we have disscused LP-Sasakian manifold satisfying $\widetilde{K}(\xi,U) \circ \widetilde{R}(X,Y)Z = 0$, where $\widetilde{K}$ is the Conharmonic curvature tensor with respect to $\widetilde{\nabla}$. In section (7), we have discussed Ricci soliton on LP-Sasakian manifold with respect to the connection $\widetilde{\nabla}$.

2 PRELIMINARIES

An n-dimensional differentiable manifold is called an Lorentzian Para-Sasakian manifold if it admits a $(1,1)$ tensor field ϕ, a vector field ξ, a 1-form η and a Lorentzian metric g which satisfies

$$
\begin{aligned}
\phi^2 Y &= Y + \eta(Y)\xi, \eta(\xi) = -1, \eta(\phi X) = 0,\ \phi\xi = 0, && (2.1)\\
g(\phi X, \phi Y) &= g(X,Y) + \eta(X)\eta(Y), && (2.2)\\
g(X, \phi Y) &= g(\phi X, Y), \eta(Y) = g(Y,\xi), && (2.3)\\
\nabla_X \xi &= \phi X, \quad g(X,\xi) = \eta(X) && (2.4)\\
(\nabla_X \phi) Y &= g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi && (2.5)\\
\forall X, Y &\in \chi(M.)
\end{aligned}
$$

where ∇ denotes the operator of covarient differentiation with respect to the Lorentzian metric g.

Let us introduced a symmetric $(0,2)$ tensor field such that

$$\omega(X,Y) = g(X,\phi Y) \tag{2.6}$$

Also since the vector field η is closed in LP- Sasakian manifold we have

$$(\nabla_X \eta) Y = \omega(X,Y), \omega(X,\xi) = 0, \forall X, Y \in \chi(M) \tag{2.7}$$

In LP- Sasakian manifold, the following relations also hold:

$$
\begin{aligned}
\eta(R(X,Y)Z) &= g(Y,Z)\eta(X) - g(X,Z)\eta(Y) && (2.8)\\
R(X,Y)\xi &= \eta(Y)X - \eta(X)Y && (2.9)\\
R(\xi,Y)Z &= g(Y,Z)\xi - \eta(Z)Y && (2.10)\\
R(\xi,Y)\xi &= \eta(Y)\xi + Y && (2.11)\\
S(X,\xi) &= (n-1)\eta(X) && (2.12)\\
S(\phi X, \phi Y) &= S(X,Y) + (n-1)\eta(X)\eta(Y) && (2.13)
\end{aligned}
$$

$$Q\xi = (n-1)\xi, Q\phi = \phi Q, S(X,Y) = g(QX,Y), S^2(X,Y) = S(QX,Y) \tag{2.14}$$

3 SOME PROPERTIES OF LP-SASAKIAN MANIFOLD ADMITTING QUARTER SYMMETRIC NON METRIC $\xi-$ CONNECTION

Due to [2], the Riemannian curvature tensor with respect to the quarter symmetric non metric $\xi-$ connection is given by

$$\begin{aligned} \widetilde{R}(X,Y)Z &= R(X,Y)Z + g(\phi X,Z)\phi Y + \eta(Y)\eta(Z)X \\ &\quad - g(X,\phi Z)\eta(Y)\xi - g(Y,\phi Z)\phi X \\ &\quad - \eta(X)\eta(Z)Y + g(Y,\phi Z)\eta(X)\xi \end{aligned} \tag{3.1}$$

Considering a frame field and then contracting the equation (3.1) along the vector field X, we find that

$$\widetilde{S}(Y,Z) = S(Y,Z) + g(Y,Z) + n\eta(Y)\eta(Z) - (1+\psi)g(Y,\phi Z) \tag{3.2}$$

for all $X, Y \in \chi(M)$, where $\psi = tr(\phi)$.

Consequently one can easily bring out the following results

$$\widetilde{S}(Y,\xi) = \widetilde{S}(\xi,Z) = 0 \tag{3.3}$$

$$\widetilde{Q}Y = QY + Y + n\eta(Y)\xi - (1+\psi)\phi Y \tag{3.4}$$

$$\widetilde{Q}\xi = 0, \tag{3.5}$$

$$\widetilde{r} = r - \psi(1+\psi) \tag{3.6}$$

$$\widetilde{R}(X,Y)\xi = 0 \tag{3.7}$$

$$\begin{aligned} \widetilde{R}(X,\xi)Z &= \eta(Z)X - g(X,Z)\xi - \eta(Z)X \\ &\quad + g(X,\phi Z)\xi - \eta(X)\eta(Z)\xi \end{aligned} \tag{3.8}$$

$$\widetilde{R}(\xi,Y)Z = g(Y,Z)\xi - g(Y,\phi Z)\xi + \eta(Y)\eta(Z)\xi \tag{3.9}$$

Thus we can state the followings:

Proposition 3.1: Let M be an n-dimensional LP-Sasakian manifold admitting Quarter symmetric non metric $\xi-$connection $\widetilde{\nabla}$, Then

(i) The curvature tensor $\widetilde{R}$ of $\widetilde{\nabla}$ is given by (3.1),

(ii) The Ricci tensor $\widetilde{S}$ of $\widetilde{\nabla}$ is given by (3.2),

(iii) The scalar curvature $\widetilde{r}$ of $\widetilde{\nabla}$ is given by (3.6)

(iv) The Ricci tensor $\widetilde{S}$ of $\widetilde{\nabla}$ is symmetric.

Now if we suppose that the LP-Sasakian manifold is Ricci flat with respect to the Quarter symmetric non metric $\xi-$ connection. Then from (3.2) we get

$$S(Y,Z) = -g(Y,Z) - n\eta(Y)\eta(Z) + (1+\psi)\omega(Y,Z)$$

for all $Y, Z \in \chi(M)$ where $\omega(Y,Z) = g(Y,\phi Z)$

This leads to the following theorem:

Theorem 3.1 *If the LP-Sasakian manifold M is Ricci flat with respect to the Quarter symmetric non metric $\xi-$ connection, then M is a generalized η-Einstein manifold.*

4 CONHARMONICALLY FLAT AND $\xi-$CONHARMONICALLY FLAT LP-SASAKIAN MANIFOLD WITH RESPECT TO $\widetilde{\nabla}$

The Conharmonic curvature tensor with respect to Quarter symmetric non metric $\xi-$ connection is given by

$$\begin{aligned} \widetilde{K}(X,Y)Z &= \widetilde{R}(X,Y)Z - \frac{1}{n-1}\left[\widetilde{S}(Y,Z)X - \widetilde{S}(X,Z)Y\right] \\ &\quad -\frac{1}{n-1}\left[g(Y,Z)\widetilde{Q}X - g(X,Z)\widetilde{Q}Y\right] \end{aligned} \tag{4.1}$$

Let us assume the LP-sasakian manifold M be Conharmonically flat with respect to $\widetilde{\nabla}$, then from (4.1)

$$\begin{aligned} \widetilde{R}(X,Y)Z &= \frac{1}{n-1}\left[\widetilde{S}(Y,Z)X - \widetilde{S}(X,Z)Y\right] \\ &\quad \frac{1}{n-1}\left[g(Y,Z)\widetilde{Q}X - g(X,Z)\widetilde{Q}Y\right] \end{aligned} \tag{4.2}$$

Taking inner product of (4.2) with a vector field W, we get

$$\begin{aligned} \widetilde{R}(X,Y,Z,W) &= \frac{1}{n-1}\left[\widetilde{S}(Y,Z)g(X,W) - \widetilde{S}(X,Z)g(Y,W)\right] \\ &\quad \frac{1}{n-1}\left[g(Y,Z)\widetilde{S}(X,W) - g(X,Z)\widetilde{S}(X,W)\right] \end{aligned} \tag{4.3}$$

Let $\{e_i\}\,(1 \le i \le n)$ be an orthonormal basis of the tangent space at any point of the manifold M. Setting $X = W = e_i$ and taking summation over $i(1 \le i \le n)$ and using (3.3) we get

$$\widetilde{S}(Y,Z) = \frac{1}{n-1}(n-2)\widetilde{S}(Y,Z) + \frac{\widetilde{r}}{n-1}g(Y,Z) \tag{4.4}$$

Using (3.2), (3.6) in (4.4), we get

$$S(Y,Z) = \{r - \psi(1+\psi) - 1\}g(Y,Z) - n\eta(Y)\eta(Z) + (1+\psi)\omega(Y,Z)$$

for all $Y, Z \in \chi(M)$ where $\omega(Y,Z) = g(Y,\phi Z)$
Thus we have the following theorem:

Theorem 4.1 *If an $n-$dimensional LP-sasakian manifold is Conharmonically flat, then it is a generalized $\eta-$ Einstein manifold.*

In reference to (3.1) and (4.1), we get

$$\begin{aligned} \widetilde{K}(X,Y)Z &= R(X,Y)Z + g(\phi X,Z)\phi Y + \eta(Y)\eta(Z)X \\ &\quad -g(X,\phi Z)\eta(Y)\xi - g(Y,\phi Z)\phi X - \eta(X)\eta(Z)Y \\ &\quad +g(Y,\phi Z)\eta(X)\xi - \frac{1}{n-1}\left[\widetilde{S}(Y,Z)X - \widetilde{S}(X,Z)Y\right] \\ &\quad -\frac{1}{n-1}\left[g(Y,Z)\widetilde{Q}X - g(X,Z)\widetilde{Q}Y\right] \end{aligned} \tag{4.5}$$

Using (1.3), (3.2) in (4.5), we get

$$\begin{aligned}\widetilde{K}(X,Y)Z &= K(X,Y)Z + g(\phi X,Z)\phi Y + \eta(Y)\eta(Z)X \\ &-g(X,\phi Z)\eta(Y)\xi - g(Y,\phi Z)\phi X - \eta(X)\eta(Z)Y \\ &+g(Y,\phi Z)\eta(X)\xi - \frac{1}{n-1}[g(Y,Z)X - g(X,Z)Y] \\ &-\frac{n}{n-1}[\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] \\ &-\frac{n}{n-1}[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi] \\ &+\frac{1}{n-1}[(1+\psi)g(Y,\phi Z)X - (1+\psi)g(X,\phi Z)Y] \\ &+\frac{1}{n-1}[(1+\psi)g(Y,Z)QX - (1+\psi)g(X,Z)QY] \end{aligned} \tag{4.6}$$

Setting $Z=\xi$ in (4.6)

$$\begin{aligned}\widetilde{K}(X,Y)\xi &= K(X,Y)\xi - 2\eta(Y)X + 2\eta(X)Y \\ &+\frac{1}{n-1}[(1+\psi)\eta(Y)QX - (1+\psi)\eta(X)QY] \end{aligned} \tag{4.7}$$

Thus we have the following theorem:

Theorem 4.2 *An $n-$dimensional LP-Sasakian manifold is $\xi-$Conharmonically flat with respect to the quarter symmetric non-metric $\xi-$connection if and only if it is so with respect to the Levi- Civita connection, provided the vector fields are horizontal vector fields.*

5 QUASI-CONHARMONICALLY FLAT LP-SASAKIAN MANIFOLD WITH RESPECT TO $\widetilde{\nabla}$

We consider an $n-$dimensional quasi-Conharmonically flat LP-Sasakian manifold with respect to quarter symmetric non metric $\xi-$connection, i.e.,

$$g\left(\widetilde{K}(\phi X,Y)Z,\phi W\right) = 0 \tag{5.1}$$

for all $X,Y,Z,W \in \chi(M)$.
In view of (4.1), we have

$$\begin{aligned} & g\left(\widetilde{R}(\phi X,Y)Z,\phi W\right) \\ = & \frac{1}{n-1}\left[\widetilde{S}(Y,Z)g(\phi X,\phi W) - \widetilde{S}(\phi X,Z)g(Y,\phi W)\right] \\ & +\frac{1}{n-1}\left[g(Y,Z)\widetilde{S}(\phi X,\phi W) - g(\phi X,Z)\widetilde{S}(Y,\phi W)\right] \end{aligned} \tag{5.2}$$

Let $\{e_i\}$ $(1\le i\le n)$ be an orthonormal basis of the tangent space at any point of the manifold M. Setting $Y=Z=e_i$ and taking summation over $i(1\le i\le n)$ and using (3.3) in (5.2), we get

$$\widetilde{S}(\phi X,\phi W) = \widetilde{r}g(\phi X,\phi W) \tag{5.3}$$

Using (3.2), (3.6) in (5.3), we get

$$\begin{aligned} S(X,W) &= \{r-\psi(1+\psi)-1\}g(X,W) \\ &+\{r-\psi(1+\psi)-n\}\eta(X)\eta(W) + (1+\psi)\omega(X,W) \end{aligned} \tag{5.4}$$

for all $X,W \in \chi(M)$ where $\omega(X,W) = g(X,\phi W)$ and $\psi = trace(\phi)$.
Thus we have the following theorem:

Theorem 5.1 *An $n-$dimensional quasi-Conharmonically flat LP-Sasakian manifold is a generalized $\eta-$Einstein manifold.*

6 LP-SASAKIAN MANFOLD SATISFYING $\widetilde{K}(\xi, U) \circ \widetilde{R} = 0$

Let us consider a LP- Sasakian manifold M satisfying the condition

$$\widetilde{K}(\xi, U) \circ \widetilde{R}(X, Y) Z = 0. \tag{6.1}$$

where $\widetilde{K}$ and $\widetilde{R}$ denote the Conharmonic curvature tensor and Riemmanian curvature tensor with respect to Quarter symmetric non metric $\xi-$connection respectively.

Equation (6.1) gives

$$\begin{aligned} 0 &= \widetilde{K}(\xi, U) \widetilde{R}(X, Y) Z - \widetilde{R}\left(\widetilde{K}(\xi, U) X, Y\right) Z \\ &\quad -\widetilde{R}\left(X, \widetilde{K}(\xi, U) Y\right) Z - \widetilde{R}(X, Y) \widetilde{K}(\xi, U) Z \end{aligned} \tag{6.2}$$

Replacing Z by ξ in (6.2), we get

$$\begin{aligned} 0 &= \widetilde{K}(\xi, U) \widetilde{R}(X, Y) \xi - \widetilde{R}\left(\widetilde{K}(\xi, U) X, Y\right) \xi \\ &\quad -\widetilde{R}\left(X, \widetilde{K}(\xi, U) Y\right) \xi - \widetilde{R}(X, Y) \widetilde{K}(\xi, U) \xi \end{aligned} \tag{6.3}$$

using (3.7) in (6.3), we get

$$0 = \psi \widetilde{R}(X, Y) QU - \widetilde{R}(X, Y) U \tag{6.4}$$

taking inner product of (6.4) with a vector field W we get

$$0 = \psi g\left(\widetilde{R}(X, Y) QU, W\right) - g\left(\widetilde{R}(X, Y) U, W\right) \tag{6.5}$$

Let $\{e_i\}$ $(1 \leq i \leq n)$ be an orthonormal basis of the tangent space at any point of the manifold M. Setting $X = W = e_i$ and taking summation over $i(1 \leq i \leq n)$ and using (3.3) in (6.5), we get

$$\begin{aligned} \psi S^2(Y, U) &= (1+\psi) \psi S(Y, \phi U) - (\psi - 1) S(Y, U) \\ &\quad + g(Y, U) - n(n-2) \eta(Y) \eta(U) - (1+\psi) \omega(Y, U) \end{aligned} \tag{6.6}$$

where $\omega(Y, U) = g(Y, \phi U)$.

Hence we have the following theorem:

Theorem 6.1 *If the condition $\widetilde{K}(\xi, U) \circ \widetilde{R}(X, Y) Z = 0$ holds in an $n-$dimensional LP-Sasakian manifold M, then equation (6.6) is satisfied in M.*

7 RICCI SOLITON ON LP-SASAKIAN MANIFOLD WITH RESPECT TO QUARTER SYMMETRIC NON METRIC $\xi-$CONNECTION $\widetilde{\nabla}$

Let (g, W, λ) be a Ricci soliton on an $n-$dimensional LP-Sasakian manifold M with respect to quarter symmetric non metric $\xi-$connection satisfying

$$\widetilde{L}_W g(Y, Z) + 2\widetilde{S}(Y, Z) + 2\lambda g(Y, Z) = 0 \tag{7.1}$$

for all $Y, Z, W \in \chi(M)$, where $\widetilde{L}_W$ denotes the Lie derivative operator with respect to $\widetilde{\nabla}$ along the vector field W

Using (3.2) in (6.1) we get,

$$\begin{aligned} & \widetilde{L}_W g(Y,Z) + 2\widetilde{S}(Y,Z) + 2\lambda g(Y,Z) \\ = & \; g\left(\widetilde{\nabla}_Y W, Z\right) + g\left(\widetilde{\nabla}_Z W, Y\right) + 2\widetilde{S}(Y,Z) + 2\lambda g(Y,Z) \\ = & \; L_W g(Y,Z) + 2S(Y,Z) + 2\lambda g(Y,Z) \\ & +2g(Y,\phi Z)\eta(W) - g(\phi Y,\phi W)\eta(Z) - g(\phi Z,\phi W)\eta(Y) \\ & +2g(Y,Z) + 2n\eta(Y)\eta(Z) - 2(1+\psi)g(Y,\phi Z) \end{aligned} \tag{7.2}$$

Thus we have the following theorem:

Theorem 7.1 *A Ricci soliton (g, W, λ) on an $n-$dimensional LP-Sasakian manifold M, with respect to Quarter symmetric non metric $\xi-$connection is invariant if and only if*

$$\begin{aligned} 2g(Y,\phi Z)\eta(W) = & \; g(\phi Y,\phi W)\eta(Z) + g(\phi Z,\phi W)\eta(Y) - 2g(Y,Z) \\ & -2n\eta(Y)\eta(Z) + 2(1+\psi)g(Y,\phi Z) \end{aligned} \tag{7.3}$$

for arbitrary vector fields Y, Z, W of M

Now considering the Ricci soliton (g, ξ, λ), (7.1) gives

$$\begin{aligned} 0 = & \; \widetilde{L}_\xi g(Y,Z) + 2\widetilde{S}(Y,Z) + 2\lambda g(Y,Z) \\ = & \; g\left(\widetilde{\nabla}_Y \xi, Z\right) + g\left(\widetilde{\nabla}_Z \xi, Y\right) + 2\widetilde{S}(Y,Z) + 2\lambda g(Y,Z) \\ = & \; \widetilde{S}(Y,Z) + \lambda g(Y,Z) \end{aligned} \tag{7.4}$$

Using (1.1),(3.2) in (7.4)

$$S(Y,Z) = -(1+2\lambda)g(Y,Z) - n\eta(Y)\eta(Z) + (1+\psi)\omega(Y,Z)$$

for all $Y, Z \in \chi(M)$, where $\omega(Y,Z) = g(Y,\phi Z)$

Thus we have the following theorem:

Theorem 7.2 *An $n-$dimensional LP-Sasakian manifold M with quarter symmetric non metric $\xi-$connection is a generalized $\eta-$Einstein manifold under the Ricci soliton (g, ξ, λ)*

Now putting $Z = \xi$ in (7.4), we get $\lambda = 0$ for $\eta(Y) \neq 0$

Corollary 7.1: *A Ricci soliton (g, ξ, λ) on an $n-$dimensional LP-Sasakian manifold M, with respect to quarter symmetric non metric $\xi-$connection is always steady for non horizontal vector fields.*

CONCLUSION

In this paper we have investigated that a Conharmonically flat,quasi Conharmonically flat LP-Sasakian manifolds with respect to quarter symmetric non metric $\xi-$connection are generalized $\eta-$ Einstein manifolds. Also, we find that a LP-Sasakian manifold is $\xi-$Conharmonically flat with respect to quarter symmetric non metric $\xi-$connection iff it is so with respect to Levi-Civita connection. Moreover, we investigated that a Ricci soliton (g, ξ, λ) *on LP-Sasakian manifold with respect to quarter symmetric non metric $\xi-$connection is always steady for non horizontal vector fields.*

ACKNOWLEDGEMENT

The authors would like to thank the referee for their valuable suggestions to improve the paper.

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