Radiation Pressure and its effect on the Halo orbits of Venus-Sun and Satellite system

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Venus is our sister planet which is less explored because of its atmosphere. It is unique in the sense of its rotation. It rotates in opposite direction. Its brightness as natural objects comes as second after Moon in the night sky. Indian space research organization (ISRO) has planned to send Shukrayan-I in 2023 to explore the surface and atmosphere of Venus. In this paper Venus-satellite and sun are considered in the mathematical model of "circular restricted three body problem".Since Venus is very near to the Sun so it is important to study the radiation pressure and its effect on the halo orbit through more accurate and modern technique. Here the continuation method up to fourth order approximation (recently introduced) have been used to analyze halo orbits around L-points i.e. L_1 and L_2 .

Keywords:- Restricted three body problem; "Photogravitational"; "Halo orbits". 2010 Mathematics Subject Classification:- 70F07

I. Introduction

Indian Space Research Organization (ISRO) has proposed to send an orbiter named Shukrayann-1 in year 2023. The aim of this interplanetary satellite is to study the atmosphere and surface of Venus. Slingsloting is a very good method which is used in designing the trajectories of an interplanetary mission, basically for outer planets viz Saturn, Jupiter, Uranus and beyond. This method requires very high energy which makes the spacecraft too fast and increases the probability of collision. That's the reason this method is used for outer planets and its applications are limited [1]. Lo [2] introduced a new design which needs very low energy. In his paper halo orbits were used as transfer station and according to him it is possible because it is weakly stable. Koon et.al[3, 4] have proposed a very good method of transferring the satellite from halo orbit to halo orbit. In his work L_1 and L_2 equilibrium points were used for the said purposes. Dynamics and geometry of the neighbourhood of collinear points of the centre manifold was explained by Gomez [5] and Jorba [6]. Romagnoli[7] tested the performance and verified the applicability of LGPS (Lunar Global positioning system) around all sides of Moon.Also they investigated the Lissajous trajectories around L_1 and L_2 . Eapen

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and Sharma [8] investigated the effect of photo gravitation on the halo orbits around L_1 of Mars in CRTBP ("Circular Restricted Three body Problem"). Findings of their work are (1) Radiation pressure is directly proportional to the trajectories of halo orbits and (2) when time period increases radiation pressure sends the trajectories of the halo orbits towards the Sun. Here it is aimed is to study the effect of photo gravitation on the halo orbit around L_1 and L_2 through fourth order approximation of continuation method.

This paper is organized as follows : In Section II, we have described the configuration of the Venus-Sun and satellite in the "restricted three body problem" with zero eccentricity. In perturbation theory, we need to approximate the periodic solutions uniformly which can remove secular terms. For this, we have used a continuation method of fourth order for the computation of halo orbits which are described in Section III. In Section IV we describe the results and discussion followed with conclusion of the paper in Section V.

II. Configuration of Venus-Sun and satellite model and Equations of Motion

It is assumed that the interplanetary satellite with infinitesimal mass is moving around the centre of mass of Sun and Venus on circular trajectories. This model is known as "spatial circular restricted three body problem" (SCR3BP). This model consist five Lagrangian points in the "synodic reference frame". At L_1 and L_2 points the orbital period of spacecraft is same as the orbital period of venus. There exist one abstract equilateral triangle having vertices the Sun, the Venus and L_4 and similarly another equilateral triangle with L_5 , Sun and venus as a vertices. L_1 , L_2 and L_3 lies on the same line which passes through The Sun and Venus.Here these three collinear points are unstable. Since these collinear points are weakly unstable so we can use it in "low energy space mission". The three dimensional periodic trajectories around these collinear points is called Halo orbits. Motion near collinear Lagrangian points are very complex but it is possible to send our spacecraft from the halo orbits of Earth-Sun system to the halo orbits of Venus-Sun system because of its positional and dynamical importance. Let $\mu = \frac{m_2}{m_1+m_2}$ is the mass parameter and $q = 1 - \frac{F_p}{F_q}$ [9] as the mass



Fig. 1 Geometrical view of the model.

radiation factor accounts the effect of radiation force in the system. Here the assumption of unit of length is considered

in such a way that the constant separation of two masses is unity. The model of the problem under consideration is given in Fig.(1). The equation of motion for the considered model is written as [8, 10, 11]

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x},\tag{1}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y},\tag{2}$$

$$\ddot{z} = \frac{\partial U}{\partial z},\tag{3}$$

$$U = \frac{(x^2 + y^2)}{2} + \frac{(1 - \mu)q}{r_1} + \frac{\mu}{r_2},$$
(4)

where,

$$r_{1} = \sqrt{(x + \mu)^{2} + y^{2} + z^{2}}$$

$$r_{2} = \sqrt{(x + \mu - 1)^{2} + y^{2} + z^{2}}.$$
(5)

III. Continuation method for the halo orbits

Continuation method is an important technique for "uniformly approximating periodic solutions" which remove secular terms comes in the use of perturbation theory to weakly nonlinear problems with finite oscillatory solutions [12].Periodic solution and its convergent series approximation can be obtained through periodicity of the results and expansion theorem . Here nonlinear terms alter the frequency of the linearized system [4]. By the help of successive approximations of continuation method periodic solutions upto third order was obtained by Richardson [13]. Motion about the halo orbit of L_1 , L_2 can be studied by the above method but for better insight we need to find the periodic solution up to fourth order. Tiwary and Kushwah [14] computed the halo orbit by translating the origin at the equilibrium point along with the combined effect of photogravitational impact of the Sun and the Earth's oblateness . We have considered only the radiation effect of the Sun excluding the effect of oblateness, as it is known that in the case of the Sun-Mercury system Mishra and Jha [15] the radiation factors is more dominant.

$$X = x + \mu \pm \gamma - 1, \quad Y = y, \quad Z = z,$$
 (6)

Using Equation (6) in (1 - 3) we have

$$\gamma(\ddot{X} - 2\dot{Y}) = \frac{\partial\Omega}{\partial X}, \tag{7}$$

$$\gamma(\ddot{Y} + 2\dot{X}) = \frac{\partial\Omega}{\partial Y},$$
(8)

$$\gamma\ddot{Z} = \frac{\partial\Omega}{\partial Z},$$
(9)

$$= \frac{\partial\Omega}{\partial Z},\tag{9}$$

$$\Omega = \frac{X^2 + Y^2}{2} + \frac{(1 - \mu)q}{R_1} + \frac{\mu}{R_2}$$
(10)

and R_1 , R_2 can be written as

$$R_1 = \sqrt{(X\gamma + 1 \mp \gamma)^2 + (Y\gamma)^2 + (Z\gamma)^2},$$

$$R_2 = \sqrt{(X\gamma \mp \gamma)^2 + (Y\gamma)^2 + (Z\gamma)^2}.$$

By the expansion of non-linear terms of equation 10, as in [16] it reduces to

$$\Omega = \gamma \frac{(X^2 + Y^2)}{2} + \frac{1}{\gamma^2} \left\{ \frac{(1 - \mu)q\gamma}{1 - \gamma} + \mu \right\} + \frac{X}{\gamma^2} \left\{ -\frac{(1 - \mu)q\gamma^2}{(1 - \gamma)^2} \right\} + \frac{(2X^2 - Y^2 - Z^2)}{2\gamma^2} \left\{ \frac{(1 - \mu)q\gamma^3}{(1 - \gamma)^3} \right\} + \frac{1}{\gamma^2} \left\{ \sum_{m \ge 3}^{\infty} C_m \rho^m P_m \left(\frac{X}{\rho} \right) \right\}.$$
(11)

Thus, through algebraic manipulation, equations of motion reduces in the following form

$$\ddot{X} - 2\dot{Y} - (1 + 2C_2)X = \frac{\partial}{\partial X} \sum_{m \ge 3}^{\infty} C_m \rho^m P_m\left(\frac{X}{\rho}\right),\tag{12}$$

$$\ddot{Y} + 2\dot{X} + (C_2 - 1)Y = \frac{\partial}{\partial Y} \sum_{m \ge 3}^{\infty} C_m \rho^m P_m\left(\frac{X}{\rho}\right),\tag{13}$$

$$\ddot{Z} + C_2 Z = \frac{\partial}{\partial Z} \sum_{m \ge 3}^{\infty} C_m \rho^m P_m\left(\frac{X}{\rho}\right).$$
(14)

where,

$$C_m = \frac{1}{\gamma^3} \left\{ \frac{(-1)^m q (1-\mu) \gamma^{m+1}}{(1\mp\gamma)^{m+1}} + (\pm 1)^m (\mu) \right\},\tag{15}$$

where $m = 0, 1, 2, 3, \dots$ Neglecting all portions except linear terms of the solution of equations 12-14, we have

$$X(t) = A_{11}e^{\alpha t} + A_{22}e^{-\alpha t} + A_{33}\cos\lambda t + A_{44}\sin\lambda t,$$
 (16)

$$Y(t) = -\kappa_1 A_{11} e^{\alpha t} + \kappa_1 A_{22} e^{-\alpha t}$$

$$-\kappa_2 A_{33} \sin \lambda t + \kappa_2 A_{44} \cos \lambda t, \tag{17}$$

$$Z(t) = A_{55} \cos \sqrt{C_2} t + A_{66} \sin \sqrt{C_2} t,$$
(18)

where A_{11} , A_{22} , A_{33} , A_{44} , A_{55} , and A_{66} are arbitrary constants whereas

$$\alpha = \sqrt{\frac{-(2-C_2) + \sqrt{9C_2^2 - 8C_2}}{2}},$$

$$\lambda = \sqrt{\frac{(2 - C_2) + \sqrt{9C_2^2 - 8C_2}}{2}},$$

$$\kappa_1 = \frac{(2C_2 + 1) - \alpha^2}{2\alpha},$$

$$\kappa_2 = \frac{(2C_2 + 1) + \lambda^2}{2\lambda}.$$

Here roots are real equal and of opposite signs it means the critical points will be a saddle points. By taking $A_{11} = A_{22} = 0$ and

$$A_{33} = -A_X \cos \phi, A_{44} = A_X \sin \phi,$$
$$A_{55} = A_Z \sin \psi, \text{ and } A_{66} = A_Z \cos \psi,$$

we will have the solutions as [4]

$$X(t) = -A_X \cos(\lambda t + \phi),$$

$$Y(t) = \kappa A_X \sin(\lambda t + \phi),$$

$$Z(t) = A_Z \sin(\sqrt{C_2}t + \psi),$$

(19)

Where A_X , A_Z are the amplitudes λ and C_2 are the frequencies ψ λ and C_2 are the out plane phase and In plane phase is ϕ . These out and in plane phases are responsible for halo orbits. To obtain a halo orbits we have to equate these two frequencies by introducing a frequency correct relation $\Delta = \lambda^2 - C_2$. However, we compute the orbits up to fourth order approximation with radiation pressure. For removing the secular terms in the successive approximation we use two variables ω and τ where $\tau = \omega t$.Now equations of motion can be written in terms of new independent variable τ as given in [14]

$$\omega^{2}X'' - 2n\omega Y' - (n^{2} + 2C_{2})X = \frac{3}{2}C_{3}(2X^{2} - Y^{2} - Z^{2}) + 2C_{4}(2X^{2} - 3Y^{2} - 3Z^{2})X + \frac{5}{8}C_{5}$$

$$[8X^{2}\{X^{2} - 3(Y^{2} + Z^{2})\} + 3(Y^{2} + Z^{2})^{2}], \qquad (20)$$

$$\omega^{2}Y'' + 2n\omega X' + (C_{2} - n^{2})Y = -3C_{3}XY - \frac{3}{2}C_{4}(4X^{2} - Y^{2} - Z^{2})Y - \frac{5}{2}C_{5}XY(4X^{2} - 3Y^{2} - 3Z^{2}), \qquad (21)$$

$$\omega^{2}Z'' + \lambda^{2}Z = -3C_{3}XZ - \frac{3}{2}C_{4}(4X^{2} - Y^{2} - Z^{2})Z - \frac{5}{2}C_{5}XZ(4X^{2} - 3Y^{2} - 3Z^{2}) + \Delta Z. \qquad (22)$$

here, $\Delta = \lambda^2 - C_2$, which is very small and is used for the frequency correction to get the halo orbit. Now we need to use the perturbation technique of Lindstedt-Poincaré. This technique will provide an approximate solution of the given problem in the neighbourhood of the critical point. Here the higher order terms of the equations of motion are considered to produce a series expansion of the solution of the equations of motion which have higher accuracy. Thus the solutions for equations (20), (21), and (22) in the form of perturbation, will be

$$X(\tau) = \epsilon X_1(\tau) + \epsilon^2 X_2(\tau) + \epsilon^3 X_3(\tau) + \epsilon^4 X_4(\tau) + \dots,$$
(23)

$$Y(\tau) = \epsilon Y_1(\tau) + \epsilon^2 Y_2(\tau) + \epsilon^3 Y_3(\tau) + \epsilon^4 Y_4(\tau) + \dots,$$
(24)

$$Z(\tau) = \epsilon Z_1(\tau) + \epsilon^2 Z_2(\tau) + \epsilon^3 Z_3(\tau) + \epsilon^4 Z_4(\tau) + \dots,$$
(25)

and

$$\omega = 1 + \epsilon \omega_1 + \epsilon^2 \omega_2 + \epsilon^3 \omega_3 + \epsilon^4 \omega_4 + \dots$$
(26)

Using this substitution in equation (20), (21), and (22) and equating the terms of $O(\epsilon)$, $O(\epsilon^2)$, $O(\epsilon^3)$, and $O(\epsilon^4)$ to obtain the 1st , 2nd, 3rd, and 4th order equations, respectively as [14, 16] with some modifications. We solve our model considering radiation pressure only whereas [14] considered radiation pressure and oblateness both.

Finally, we combine all the solutions considering the mapping as $A_X \mapsto \frac{A_X}{\epsilon}$ and $A_Z \mapsto \frac{A_Z}{\epsilon}$ to remove ϵ from all the solutions of equations upto fourth order approximations. Combining all solutions components wise in (23), (24), and (25), we get the final solution, for the expression of coefficients we refer [14, 16].

$$X(\tau) = (\rho_{20} + \rho_{31} + \rho_{40}) - A_X \cos \tau_1 + (\rho_{21} + \zeta \rho_{22} + \rho_{41}) \cos 2\tau_1 \cos 3\tau_1 + \rho_{42} \cos 4\tau_1,$$
(27)

$$Y(\tau) = (\kappa A_X + \sigma_{32}) \sin \tau_1 + (\sigma_{21} + \sigma_{41} + \zeta \sigma_{22}) \sin 2\tau_1 + \sigma_{31} \sin 3\tau_1 + \sigma_{42} \sin 4\tau_1,$$
(28)
$$Z(\tau) = (-1)^{\frac{p-1}{2}} A_Z \cos \tau_1 + (-1)^{\frac{p-1}{2}} (\kappa_{21} \cos \tau_1 + \kappa_{22} + \kappa_{32} \cos 3\tau_1) + \kappa_{40} + \kappa_{41} \cos 2\tau_1 + \kappa_{42} \cos 4\tau_1.$$
(29)

IV. Results and Discussion

In the Earth-Sun and satellite system halo orbits have been drawn by the third order approximation method. It can be seen in [4] and [16]. Here we investigate the halo orbits of the Venus-Sun and satellite system through fourth order continuation method using (27), (28), and (29) equations. Here amplitudes, $A_X = 200000$ km and $A_Z = 110000$ km are taken. Time period of halo orbits around L_1 at q = 1 is 47.94 days which can be seen in (Fig.2) in black color. The time period for halo orbit around L_2 is 42.80 days as shown in Fig.3. The other orbits with blue to red colors shows the 3D-halo orbits, when q = 0.99, 0.98, 0.97, 0.96 and 0.95 respectively. The red color orbit in both the Figs.2 and 3 shows when q = 0.95.

The variation of some parameters and time period with radiation factor q for the motion around L_1 and L_2 are given in Tables 1 and 2 respectively. It is clearly visible that radiation pressure is directly proportional to the time period of the orbits around L_1 but inversely proportional to the time period around L_2 . The effect on the halo orbit due to radiation pressure of the Sun are clearly visible in Fig.2-3, that the orbits are shifting towards the Sun position. Also Fig.(2-3) indicates that the eccentricity of orbit increases around L_1 with increase in radiation factor whereas in case of orbits around L_2 it decreases.

V. Conclusion

Here system of Venus-Sun and satellite have been studied analytically and numerically for getting the halo orbits of satellite. Further the radiation pressure and its effect on these orbits have been studied on the line shown by [14]. The

	$\tau(days)$	47.94	48.18	54.28	60.67	66.54	71.50
	$\tau(dimensionless)$	3.42480	3.01293	3.87684	4.33326	4.75252	5.10663
I	γ	1.83461	1.08541	1.62070	1.44999	1.32207	1.23040
	C_5	3.02829	2.06679	1.35456	0.86291	0.54476	0.34762
	C_4	3.02839	2.06691	1.35471	0.86312	0.54505	0.34801
	C_3	3.01862	2.05586	1.34202	0.84837	0.52784	0.32795
	C_2	4.05677	3.08889	2.37126	1.81530	1.55391	1.35442
	γ	0.00931	0.01058	0.01218	0.01415	0.01050	0.01916
	L_1	0.99068	0.98941	0.98781	0.98584	0.98549	0.98082
	q	1.00	0.99	0.98	0.97	0.96	0.95

Table 1 Radiation pressure effect around critical point L_1

L_2 γ	~		C_2	C_3	C_4	C_5	γ	$\tau(dimensionless)$	$\tau(days)$
1.00937 0.009	0.00	937	3.94461	-2.98125	2.97230	-2.97222	2.05802	3.05302	42.80
1.00839 0.009	0.00	937	3.93489	-2.98116	2.97230	-2.97222	2.05563	3.05658	42.79
1.00760 0.007	0.00	760	6.52587	-5.57513	5.56795	-5.56790	2.61435	2.40335	33.64
1.00697 0.006	0.00	697	8.17232	-7.22891	7.22238	-7.22223	2.91315	2.15683	30.19
1.00645 0.006	0.00	545	10.03840	-9.10283	9.09682	-9.09079	3.21812	1.95244	27.33
1.00603 0.00	0.00	502	12.11040	-11.18290	11.17740	-11.17730	3.53576	1.78208	24.95
	_			-					-

 Table 2
 Radiation pressure effect around critical point L2:

result shows that radiation pressure is directly proportional to eccentricity and the time period of the orbits around L_1 but it is inversely proportional in case of L_2 . Also due to the effect of radiation pressure the halo orbits shifting towards the Sun.

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Fig. 2 Halo orbit around L_1



Fig. 3 Halo orbit around *L*₂