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Nano regular generalized continuous functions and Nano generalized regular continuous functions in Nano Topological spaces

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Abstract - *The Purpose of this paper is to introduce and study a new class of Nano regular generalized continuous function and Nano generalized regular continuous functions in Nano Topological spaces. Also examine some of their properties of such functions.*

Keywords: *Nano- continuous, Nr- continuous, Ng- continuous, Nrg- continuous, Ngr- continuous*.

I. Introduction

One of the main concepts of topology is continuous functions. The concept of Regular continuous functions was first introduced by Arya. S.P. and Gupta.R [1]. Also the concept of Generalized Regular continuous functions in topological spaces was introduced by Mahmood, S.I [5]. The concept of Nano continuous functions was first introduced by Lellis Thivagar, M. A. and Carmel Richard [4]. He has also defined a Nano Open mappings, Nano closed mappings and Nano homeomorphisms and their representations in terms of Nano closure and Nano interior.The concept of Nrg-closed set was introduced by Sulohana Devi [8]. Bhuvaneswari et al [2] introduced and investigate some properties of Nsg-continuous, Npg-continuous, Nαg-continuous and also Nrg-closed sets in Nano topological spaces. In this paper we introduce Nano regular generalized continuous functions and Nano generalized regular continuous function.

II. Preliminaries

Definition 2.1: Let U be a non-empty finite set of objects called the Universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$. 1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$.

That is $L_R(X) = U \{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by $x \in U$.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = U\{R(x): R(x) \cap X \neq \Phi\}$.

3. The boundary region of X with respect to R is the set of all objects , which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X) = U_R(X) – L_R(X)$.

Definition 2.2: Let U be a non-empty finite universe of objects and R be an equivalence relation on U. Let X $\subseteq$ U. Let $\tau_R(X)$ = {U,Φ, $L_R(X)$, $U_R(X)$, $B_R(X)$}. Then $\tau_R(X)$ is a topology on U, called as the Nano topology with respect to X. Elements of the Nano topology are known as the Nano open sets in U and (U, $\tau_R(X)$) is called the Nano topological space. $[\tau_R(X)]^c$ is called as the dual Nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called as Nano closed sets.

Definition 2.3: If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set β = {U, $L_R(X)$, $B_R(X)$} is the basis for $\tau_R(X)$.

Definition 2.4 : If (U, $\tau_R(X)$) is a Nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then the Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by NInt(A). That is, NInt(A) is the largest Nano open subsets of A. The Nano closure of A is defined as the intersection of all Nano closed sets containing A and is denoted by NCl(A). That is, NCl(A) is the smallest Nano closed set containing A.

Definition 2.5: Let (U, $\tau_R(X)$) be a Nano topological space and A $\subseteq$ U. Then A is said to be

(i) Nano semi - open if A $\subseteq$ NCl(NInt(A))

(ii) Nano pre-open if A $\subseteq$ NInt(NCl(A))

(iii) Nano α-open if A $\subseteq$ NInt(NCl(NInt(A)))

(iv) Nano regular open if A = NInt(NCl(A))

NSO(U,X), NPO(U,X),NαO(U,X) and NRO(U,X) respectively denote the families of all Nano semi - open, Nano pre -open, Nano α - open and Nano regular open subsets of U.

Let (U, $\tau_R(X)$) be a Nano topological space and A $\subseteq$ U. A is said to be Nano semi - closed, Nano pre closed, Nano α - closed and Nano regular closed if its complement is respectively Nano semi - open, Nano pre- open, Nano α - open and Nano regular open.

Definition 2.6: A subset A of a Nano topological space (U, $\tau_R(X)$) is called

(i) Nano generalized closed (briefly Nano g-closed), if NCl(A) $\subseteq$ G whenever A $\subseteq$ G and G is Nano open in U.

(ii) Nano semi generalized closed (briefly Nano sg-closed), if NsCl(A) $\subseteq$ G whenever A $\subseteq$ G and G is Nano semi open in U.

(iii) Nano α - generalized closed (briefly Nano αg-closed), if NαCl(A) $\subseteq$ G whenever A $\subseteq$ G and G is Nano open in U.

(iv) Nano regular generalized closed(briefly Nano rg-closed), if NrCl(A) $\subseteq$ G whenever A $\subseteq$ G and G is Nano regular open in U.

(v) Nano generalized regular closed (briefly Nano gr-closed), if NrCl(A) $\subseteq$ G whenever A $\subseteq$ G and G is Nano open in U.

Definition 2.7: Let (U, τ_R(X)) and (V, τ_R'(Y)) be Nano topological spaces. Then a mapping f: (U, τ_R(X)) $\rightarrow$ (V, τ_R'(Y)) is said to be

(i)Nano continuous if f^{-1}(B) is Nano open in U for every Nano open set B in V.

(ii)Nano generalized continuous if f^{-1}(B) is Nano g-open in U for every Nano open set B in V.

(iii)Nano regular continuous if f^{-1}(B) is Nano regular open in U for every Nano open set B in V.

(iv) Nano α- continuous if f^{-1}(B) is Nano α-open in U for every Nano open set B in V.

(v)Nano semi continuous if f^{-1}(B) is Nano semi open in U for every Nano open set B in V.

(vi) Nano pre continuous if f^{-1}(B) is Nano pre-open in U for every Nano open set B in V.

III. Nano Regular Generalized Continuous Function in Nano Topological space

In this section we introduce Nano regular generalized continuous function and investigate some their properties.

Definition 3.1: Let (U, τ_R(X)) $\rightarrow$ (V, τ_R'(Y) are any two Nano topological spaces. Let f: (U, τ_R(X)) $\rightarrow$ (V, τ_R'(Y)) be Nano regular generalized continuous function (shortly Nrg-continuous). If the inverse image of every Nano open (resp. Nano closed) set in (V, τ_R'(Y)) is Nrg – open (resp. Nrg – closed) in (U, τ_R(X)).

Theorem 3.2: Let (U, τ_R(X)) and (V, τ_R'(Y)) are any two Nano topological spaces. Let f: (U, τ_R(X)) $\rightarrow$ (V, τ_R'(Y)) be Nano continuous function. If f is Nano continuous function, then f is Nrg-continuous but not conversely.

Proof: Let B be any Nano closed set in V. Then f^{-1}(B) is Nrg-closed in U. Since every Nano closed set is Nrg-closed. Hence f^{-1}(B) is Nrg- closed in U. Therefore, f is f^{-1}(B) is Nrg-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example3.3: Let U = {a, b, c, d} with U|R = {{a}, {c}, {b, d}} and X = {a, b}.Then τ_R(X) = {U, φ, {a}, {b, d}, {a, b, d}}. Let V={x, y, z, w} with V|R' = {{y}, {w}, {x, z}}. Then τ_R'(Y) = {V, φ, {y}, {x, y, z}, {x, z}}. Define a mapping f: (U, τ_R(X)) → (V, τ_R'(Y)) as, f (a) = x, f (b) = w, f (c) = z, f (d) = y. Then f is Nrg-continuous but not Nano continuous. Thus the inverse image of a Nano closed set f^{-1}({x, y, z}) = {a, c, d}, {a, c, d} in V is {x, y, z} in which is not Nano closed in U.

Theorem 3.4: Let f: (U, τ_R(X)) $\rightarrow$ (V, τ_R'(Y)) be Nano continuous function and (U, τ_R(X)) and (V, τ_R'(Y)) are any two Nano topological spaces. If f is Nano regular continuous (briefly Nr-continuous) function, then f is Nrg- continuous but not conversely.

Proof: Let B be any Nano closed set in (V, τ_R'(Y)). Then f^{-1}(B) is Nr-closed in (U, τ_R(X)) as f is Nano regular continuous. Since every Nano regular closed set is Nrg-closed. Hence f^{-1}(B) is Nrg - closed in (U, τ_R(X)). Therefore f is Nrg-continuous.

The converse of the above theorem need not be true as seen from the following example;

Example3.5 : Let U={a ,b, c, d} with U|R={{b, c}, {a}, {d}} and X={b, d} ⊆ U with τ_R(X) = {U, φ, {d}, {b, c}, {b, c, d}} and let V={x, y, z, w} with V|R'= {{x},{y, z},{w}} and Y={x, z} ⊆ V. Then τ_R'(Y) = {V, φ, {x}, {y, z}, {x, y, w}}. Let (U, τ_R(X)) → (V, τ_R'(Y)) be a function defined by f(a) = x, f(b) = y, f(c) = w and f(d) = z. It is clear that f is not Nano regular continuous but f is Nrg-continuous. Since f^{-1}(φ) = φ, f^{-1}(V) = U and f^{-1}({x, y, w}) = {a, b, d} are Nrg-closed in U.

Theorem 3.6: Let f: (U, τ_R(X)) → (V, τ_R'(Y)) be Nano continuous function and (U, τ_R(X)) and (V, τ_R'(Y)) are any two Nano topological spaces. If f is Nano g- continuous function, then f is Nrg- continuous.

Proof: let f be Ng-continuous function and let B be a Nano closed set in (V, τ_R'(Y)). Then f^{-1}(B) is Ng-closed in (U, τ_R(X)). Since every Ng-closed set is Nrg-closed set. Thus f^{-1}(B) is Nr- closed. Hence f is Nrg-continuous.

Theorem 3.7: Let f: (U, τ_R(X)) → (V, τ_R'(Y)) be Nano continuous function and (U, τ_R(X)) and (V, τ_R'(Y)) are any two Nano topological spaces. If f is Nano continuous (briefly N-continuous) function, then f is Nr- continuous but not conversely.

Proof: Let B be any Nano closed set in (V, τ_R'(Y)). Since f is Nano continuous, then f^{-1}(B) is Nano closed in (U, τ_R(X)). Since every Nano closed is Nr-closed, then f^{-1}(B) is Nr-closed set in (U, τ_R(X)). Hence f is Nr-continuous.

The converse of the above theorem need not be true as seen from the following example;

Example 3.8: Let U = {a, b, c, d} with τ_R(X) = {U, φ, {d}, {a, c}, {a, c, d}} and let V = {x, y, z, u, v} with τ_R'(Y) = {V, φ, {u, x}, {y, z}, {u, x, y, z}}. Let f: (U, τ_R(X)) → (V, τ_R'(Y)) be a function defined by f (a) = u, f (b) = v, f(c) = x, f (d) = y. It is clear that f is Nano continuous but f is not Nr-continuous.

Theorem 3.9: A function f: (U, τ_R(X)) → (V, τ_R'(Y)) is Nrg – continuous if and only if f(NrgCl(A)) ⊆ NCl(f(A)) for every subset A of (U, τ_R(X)).

Proof: Let f be a Nrg-continuous and A ⊆ U. Then f (A) ⊆ V. Since f is Nrg-continuous and NCl(f(A)) is Nano closed in V, f^{-1}(NCl(f(A))) is Nrg-closed in U. Since f (A) ⊆ NCl(f(A)), f^{-1}(f(A)) ⊆ f^{-1}(NCl(f(A))). Then NrgCl(A) ⊆ NrgCl[f^{-1}(NCl(f(A)))] = f^{-1}[NCl(f(A))]. Thus, NrgCl(A) ⊆ f^{-1}(NCl(f(A))). Therefore, f(NrgCl(A)) ⊆ NCl(f(A)) for every subset A of U.

Conversely, let f (NrgCl(A)) $\subseteq$ NCl(f(A)) for every subset A of U. If F is Nano - closed in V. Since $f^{-1}(F) \subseteq$ U. $f(NrgCl(f^{-1}(F))) \subseteq NCl(f(f^{-1}(F))) = NCl(F) = F$. That is, $f(NrgCl(f^{-1}(F))) \subseteq F$. Thus, $NrgCl(f^{-1}(F)) \subseteq f^{-1}(F)$. But $f^{-1}(F) \subseteq NrgCl(f^{-1}(F))$. Hence $NrgCl(f^{-1}(F)) = f^{-1}(F)$. Therefore $f^{-1}(F)$ is Nrg-closed in U for every Nano closed set F in V. That is f is Nrg – continuous.

IV. Nano Generalized Regular Continuous in Nano topological spaces

Definition 4.1: A function f: (U, $\tau_R(X)$) $\to$ (V, $\tau_R'(Y)$) is called Nano Generalized Regular continuous (briefly Ngr-continuous) if $f^{-1}(B)$ is Ngr-open (resp. Ngr-closed) in (U, $\tau_R(X)$) for every Nano open set (resp. Nano closed set) B in (V, $\tau_R'(Y)$). That is the inverse image of every Nano open (resp. Nano closed) set in (V, $\tau_R'(Y)$) is Ngr – open (resp. Ngr-closed) set in (U, $\tau_R(X)$).

Theorem 4.2: Let (U, $\tau_R(X)$) and (V, $\tau_R'(Y)$) are any two Nano Topological spaces. Let

f: (U, $\tau_R(X)$) $\to$ (V, $\tau_R'(Y)$) be Nano continuous function. If f is Nano continuous function, then f is Ngr – continuous, but not conversely.

Proof: Let B be any closed set in (V, $\tau_R'(Y)$). Since every Nano - closed set is Ngr - closed. Then $f^{-1}(B)$ is Ngr – closed in (U, $\tau_R(X)$). Therefore, $f^{-1}(B)$ is Ngr – continuous.

The converse of the theorem need not be true as seen from the following example;

Example 4.3: Let U = {a, b, c, d} with $\tau_R(X)$ = {U, φ, {a}, {a, b, d}, {b, d}} and let V = {x, y, z, w} with $\tau_R'(Y)$ = {V, φ, {w}, {y, z, w}, {x, w}}. Define a mapping f: (U, $\tau_R(X)$) $\to$ (V, $\tau_R'(Y)$) as f (a) = x, f (b) = y, f(c) = z, f (d) = w. Then f is Ngr – continuous as the inverse image of a Nano closed set {x, y, z} in V is {a, b, c} which is not Nano closed in U.

Theorem 4.4: Let f: (U, $\tau_R(X)$) $\to$ (V, $\tau_R'(Y)$) be a Nano continuous functions. Then every Nano regular continuous, Nano α – continuous, Nano semi-continuous, Nano pre -continuous functions are Ngr – continuous.

Proof: It follows from the definition.

Remark 4.5: The converse of the above theorem need not be true which can be seen from the following examples;

Example 4.6: Let U = {a, b, c, d} with U|R = {{a, c}, {b}, {d}} and X = {a, d}. Then the Nano Topology is defined as $\tau_R(X)$ = {U, φ, {d}, {a, c}, {a, c, d}}. Let V = {x, y, z, w} with V|R'= {{x}, {y, z}, {w}}. Then $\tau_R'(Y)$ = {V, φ, {x}, {x, y, z}, {{x, z}}. Let f: (U, τ_R(X)) → (V, τ_R'(Y)) is defined as f (a) = x, f(b) = w, f(c)=y and f(d) = z. Then f is Ngr –continuous but f is not Nano regular continuous and Nano semi continuous. Since f^{-1}({y, w}) = {b, c} and f^{-1}({y, z, w}) = {b, c, d} which are not Nano regular closed and Nano semi closed in U. Where {y, w} and {y, z, w} are Nano closed in V. Thus Ngr – continuous function is not Nano regular continuous and Nano semi continuous.

Example 4.7: Let U = {a, b, c, d} with U|R = {{a, c}, {b}, {d}} and X = {a, d} ⊆ U. Then the Nano Topology is defined as τ_R(X) = {U, φ, {d}, {a, c}, {a, c, d}}. Let V = {x, y, z, w} with V|R'= {{y}, {z}, {v}, {u, x}} and Y = {x, y, z} ⊆ V. Then τ_R'(Y) = {V, φ, {x}, {x, y, z}, {{x, z}}. Let f: (U, τ_R(X)) → (V, τ_R'(Y)) is defined as f (a) = y, f(b) = x, f(c) = z and f(d) = w. Then f^{-1}({y, z, w} = {a, c, d}. Since {a, c, d} is not Nano α – closed but it is Nano pre closed in U. Thus Ngr – continuous function is not Nano α – continuous and Nano pre continuous function.

Theorem 4.8: A function f: (U, τ_R(X)) → (V, τ_R'(Y)) is Ngr - continuous if and only if the inverse image of every Nano closed set in (V, τ_R'(Y)) is Ngr – closed in (U, τ_R(X)).

Proof: Let F be a Nano closed set in V and f be Ngr – continuous. That is V - F is Nano closed in V. Since f is Ngr – continuous, f^{-1}(V – F) is Ngr – closed in U. But f^{-1}(V – F) = U – f^{-1}(F). Hence f^{-1}(F) is Ngr – closed in U.

Conversely, let the inverse image of every Nano closed set in V is Ngr – closed in U. Assume B is an Nano closed set in V, then B^c is Nano closed in V. By the assumption $f^{-1}(B^c)$ = U – f^{-1}(B) is Ngr – closed in U. Hence f^{-1}(B) is Ngr – open in U. Hence f is Ngr – continuous.

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