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Causal relationship k

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Abstract* — *Aim: A detailed and sophisticated analysis of causal relationships and chains of causation in medicine, life and other sciences by logically consistent statistical methods is still not generally accepted.

Methods: In this publication, a hypothetico-deductive scientific method has been used to approach to the solution of view basic problems of causality.

Results: A method how to determine an exact probability of a single event has been derived. The causal relationship k has been established mathematically while relying on the axiom +1 = +1.

Conclusion: Experimental and non-experimental data can be analysed for causal relationships.

Keywords — ***Causality, Cause, Effect, Causal relationship, Law Of Nature relationship.***

I. Introduction

The history of the denialism of causality in Philosophy, Mathematics, Statistics, Physics et cetera is very long. We only recall David Hume's (1711-1776) account of causation and his inappropriate reduction of the cause-effect relationship to a simple habitual connection in human thinking or Immanuel Kant's (1724-1804) initiated trial to consider causality as nothing more but a '*a priori*'given category [70] in human reasoning and other similar attempts too. It is worth noting in this context that especially Karl Pearson (1857 - 1936) himself has been engaged in a long lasting and never-ending crusade against the principle of causality too. "Pearson categorically denies the need for an independent concept of causal relation beyond correlation ... he *exterminated* causation from statistics before it had a chance to take root "[see 78, p. 340] At the beginning of the 20^{th} century notable proponents of **conditionalism** like the German anatomist and pathologist David Paul von Hansemann [47] (1858 - 1920) and the biologist and physiologist Max Richard Constantin Verworn [101] (1863 - 1921) started a new attack [68] on the principle of causality. In his essay "Kausale und konditionale Weltanschauung"Verworn [101] presented "an exposition of 'conditionism'as contrasted with 'causalism,'[99] while ignoring cause and effect relationships completely. "**Das Ding ist also identisch mit der Gesamtheit seiner Bedingungen**."[101] However, Verworn's goal to exterminate causality completely out of science was hindered by the further development of research. The history of futile attempts to refute **the principle of causality** culminated in a publication by the German born physicist Werner Karl Heisenberg (1901 - 1976). Heisenberg put forward a logically inconsistent [6]–[8], completely unnecessary and confusing uncertainty principle [50] which opened the door to wishful thinking and logical fallacies in physics and in science as such. Heisenberg's unjustified reasoning ended in an act of a manifestly unfounded conclusion: "**Weil alle Experimente den Gesetzen der Quantenmechanik und damit der Gleichung (1) unterworfen sind, so wird durch die Quantenmechanik die Ungültigkeit des Kausalgesetzes definitiv festgestellt**."[50] while 'Gleichung (1)'denotes Heisenberg's uncertainty principle. Einstein's himself, a major contributor to quantum theory and in the same respect a major critic of quantum theory, disliked Heisenberg's uncertainty principle fundamentally while Einstein's opponents used Heisenberg's Uncertainty Principle against Einstein. After the End of the German Nazi initiated Second World War with unimaginable brutality and high human losses and a death toll due to an industrially organised mass killing of people by the German Nazis which did not exist in this way before, Werner Heisenberg visited Einstein in Princeton (New Jersey, USA) in October 1954 [75]. Einstein [23] agreed to meet Heisenberg only for a very short period of time but their encounter lasted longer. However, there where not only a number of differences between Einstein and Heisenberg, these two physicists did not really loved each other. "Einstein remarked that the inventor of the uncertainty principle was a 'big Nazi'... "[75] Albert Einstein (1879 - 1955) took again the opportunity to refuse to endorse **Heisenberg's uncertainty principle** as a fundamental law of nature and rightly too. Meanwhile, Heisenberg's uncertainty principle is refuted [6]–[8]

for several times but still not exterminated completely out of physics and out of science as such. In contrast to such extreme anti-causal positions as advocated by Heisenberg and the **Copenhagen interpretation of quantum mechancis**, the search for a (mathematical) solution of *the issue of causal inferences* is as old as human mankind itself ("*i. e. Aristotle's Doctrine of the Four Causes*") [53] even if there is still little to go on. It is appropriate to specify especially the position of D'Holbach [58]. D'Holbach (1723-1789) himself linked cause and effect or causality as such to changes. "**Une *cause*, est un être qui e met un autre en mouvement, ou qui produit quelque changement en lui. *L'effet* est le changement qu'un corps produit dans un autre ...**"[58] D'Holbach infers in the following: "**De l'action et de la réaction continuelle de tous les êtres que la nature renferme, il résulte une suite de causes et d'effets ..**"[58] With more or less meaningless or none progress on the matter in hand even in the best possible conditions, it is not surprising that authors are suggesting more and more different approaches and models for causal inference. Indeed, the hope is justified that logically consistent *statistical methods of causal inference* can help scientist to achieve so much with so little. One of the methods of causal inference in Bio-sciences are based on the known *Henle [52] (1809–1885) - Koch [64] (1843–1910) postulates* [33] which are applied especially for the identification of a causative agent of an (infectious) disease. However, the pathogenesis of most chronic diseases is more or less very complex and potentially involves the interaction of several factors. In practice, from the 'pure culture' requirement of the Henle-Koch postulates insurmountable difficulties may emerge. In light of subsequent developments (PCR methodology, immune antibodies et cetera) it is appropriate to review the full validity of the Henle-Koch postulates in our days. In 1965, Sir Austin Bradford Hill [57] published nine criteria (the '*Bradford Hill Criteria* ') in order to determine whether observed epidemiologic associations are causal. Somewhat worrying, is at least the fact that, Hill's "... fourth characteristic is *the temporal relationship of the association* " and so-to-speak just a reformulation of the '*post hoc ergo propter hoc'[3], [108]* logical fallacy through the back-door and much more then this. It is questionable whether association as such can be treated as being identical with causation. Unfortunately, due to several reasons, it seems therefore rather problematic to rely on Bradford Hill Criteria carelessly. Meanwhile, several other and competing mathematical or statistical approaches for causal inference have been discussed [3]–[5], [9]–[11], [27], [35], [39], [54]–[56], [67], [78], [91], [95], [111] or even established [3]–[5], [9]–[11]. Nevertheless, the question is still not answered, is it possible at all to establish a cause effect relationship between two factors while applying only certain statistical [93] methods?

II. Material and methods

A. Definitions

Reaching a generally valid consensus on the definition of the numbers +0 and +1 appears to be difficult. These numbers are of fundamental importance in classical logic, probability theory and so forth. The definition of the basic numbers +1 and +0 in terms of Euler's identity and physical 'constants 'offer us the possibility to test classical logic or mathematical theorems et cetera by reproduceable physical experiments too. In particular, it is very remarkable that Leibniz [71] himself published in 1703 the first self-consistent binary number system [20], [21] representing all numeric values while using typically +0 (zero) and +1 (one).

2.1.1 The number +0

Definition 1 (**The number +0**). *Let c denote the speed of light in vacuum [36], [97], [102], [103], let ε_0 denote the electric constant and let μ_0 the magnetic constant. Let i denote the imaginary number [28]. The number +0 is defined as the expression*

$$+0 \equiv +1-1 \equiv +1+i^2 \equiv +1+e^{i\pi} \equiv +\left(c^2 \times \varepsilon_0 \times \mu_0\right)+e^{i\pi} \tag{1}$$

while '= 'or $\equiv$ denotes the equals sign [86] or equality sign [87] used to indicate equality and '- '[77], [106] denotes minus signs used to represent the operations of subtraction and the notions of negative as well and '+ 'denotes the plus [86] signs used to represent the operations of addition and the notions of positive as well.

Remark 1. *Roger Cotes (1682 – 1716) [34] or Leonhard Euler's (1707 – 1783) identity [40] is regarded as one of the most beautiful equations [107]. In this context, it is provisionally presumed, that Euler's identity [40] is logically sound and correct.*

2.1.2 The number +1

Definition 2 (**The number +1**). *Again, let c denote the speed of light in vacuum [36], [97], [102], [103], let ε_0 denote the electric constant and let μ_0 the magnetic constant. Let i denote the imaginary number [28]. The number +1 is defined as the expression*

$$+1 \equiv +1+0 \equiv +1-0 \equiv -i^2 \equiv -e^{i\pi} \equiv +\left(c^2 \times \varepsilon_0 \times \mu_0\right) \tag{2}$$

while again '= 'or ≡ may denote the equals sign [86] or equality sign [87] used to indicate equality and '- '[77], [106] denotes minus signs used to represent the operations of subtraction and the notions of negative as well and '+ 'denotes the plus [86] signs used to represent the operations of addition and the notions of positive as well.

2.1.3 The n-th moment expectation value of U

Definition 3 (The n-th moment expectation value of U). *Let $_RU_t$ denote an event at a certain (period of) time or **Bernoulli trial t** [100]. Let p($_RU_t$) represent the probability of an event at a given Bernoulli trial t. Let E($_RU_t^n$) denote the n-th moment **expectation value** [61], [105] of $_RU_t$. Let E($_RU_t^1$) denote the first moment expectation value of $_RU_t$. Let E($_RU_t^2$) denote the second moment expectation value of $_RU_t$. In general, **the n-th moment expectation value of $_RU_t$** is defined as*

$$
\begin{aligned}
E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}}\right) &\equiv \left(\underbrace{{}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times \ldots}_{(n-times)}\right) \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \\
&\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}}\right) \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)
\end{aligned}
\tag{3}
$$

Furthermore, it is

$$
\begin{aligned}
E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}}\right)^{\mathrm{m}} &\equiv \left(\underbrace{{}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times \ldots}_{(n-times)}\right)^{\mathrm{m}} \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)^{\mathrm{m}} \\
&\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}}\right)^{\mathrm{m}} \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)^{\mathrm{m}}
\end{aligned}
\tag{4}
$$

The first moment expectation value of $_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ follows as

$$
\begin{aligned}
E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{1}\right) &\equiv \left(\underbrace{{}_{\mathrm{R}}U_{\mathrm{t}}{}^{1}}_{(one-times)}\right) \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \\
&\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{1}\right) \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \\
&\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)
\end{aligned}
\tag{5}
$$

The second moment expectation value of $_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ follows as

$$
\begin{aligned}
E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{2}\right) &\equiv \left(\underbrace{{}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{1}}_{(two-times)}\right) \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \\
&\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{2}\right) \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)
\end{aligned}
\tag{6}
$$

2.1.4 The n-th moment expectation value of anti U

Definition 4 (The n-th moment expectation value of anti U). *Let p($_RU_t$) represent the probability of a single event $_RU_t$ at a given Bernoulli trial t. Let (1-p($_RU_t$)) represent the probability that a single event $_RU_t$ will not occur, will not exist at a given Bernoulli trial t. Let E($_R\underline{U}_t^n$) denote the n-th moment **expectation value** [61], [105] of anti $_RU_t$. Let E($_R\underline{U}_t^1$) denote the first moment expectation value of anti $_RU_t$. Let E($_R\underline{U}_t^2$) denote the second moment expectation value of anti $_RU_t$. In general, **the n-th moment expectation value of anti $_RU_t$** is defined as*

$$
\begin{aligned}
E\left({}_{R}\underline{U}_{t}{}^{n}\right) &\equiv \left(\underbrace{{}_{R}U_{t}{}^{1} \times {}_{R}U_{t}{}^{1} \times {}_{R}U_{t}{}^{1} \times \ldots}_{(n-times)}\right) \times \left(1 - p\left({}_{R}U_{t}\right)\right) \\
&\equiv \left({}_{R}U_{t}{}^{n}\right) \times \left(1 - p\left({}_{R}U_{t}\right)\right)
\end{aligned}
\tag{7}
$$

The first moment expectation value of anti ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ follows as

$$
\begin{aligned}
E\left({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}{}^{1}\right) &\equiv \left(\underbrace{{}_{\mathrm{R}}U_{\mathrm{t}}{}^{1}}_{(one-times)}\right) \times (1-p({}_{\mathrm{R}}U_{\mathrm{t}})) \\
&\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{1}\right) \times (1-p({}_{\mathrm{R}}U_{\mathrm{t}})) \\
&\equiv ({}_{\mathrm{R}}U_{\mathrm{t}}) \times (1-p({}_{\mathrm{R}}U_{\mathrm{t}}))
\end{aligned} \tag{8}
$$

The second moment expectation value of anti ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ follows as

$$
\begin{aligned}
E\left({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}{}^{2}\right) &\equiv \left(\underbrace{{}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{1}}_{(two-times)}\right) \times (1-p({}_{\mathrm{R}}U_{\mathrm{t}})) \\
&\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{2}\right) \times (1-p({}_{\mathrm{R}}U_{\mathrm{t}}))
\end{aligned} \tag{9}
$$

2.1.5 The n-th moment expectation value of U and W

Definition 5 (**The n-th moment expectation value of U and W**). *Let p(${}_{R}U_{t}$, ${}_{R}W_{t}$) represent the joint probability of an occurring of the events ${}_{R}U_{t}$ and ${}_{R}W_{t}$ at the same (period of time or) Bernoulli trial t. Let E(${}_{R}U_{t}{}^{n}$, ${}_{R}W_{t}{}^{n}$) denote the n-th moment expectation value of ${}_{R}U_{t}$ and ${}_{R}W_{t}$. Let E(${}_{R}U_{t}{}^{1}$) denote the first moment expectation value of ${}_{R}U_{t}$. In general,* ***the n-th moment expectation value of ${}_{R}U_{t}$ and ${}_{R}W_{t}$*** *is defined as*

$$
\begin{aligned}
E\left({}_{R}U_{t}{}^{n}, {}_{R}W_{t}{}^{n}\right) &\equiv \left(\underbrace{\left({}_{R}U_{t}{}^{1} \times {}_{R}W_{t}{}^{1}\right) \times \left({}_{R}U_{t}{}^{1} \times {}_{R}W_{t}{}^{1}\right) \times \ldots}_{(n-times)}\right) \times p({}_{R}U_{t}, {}_{R}W_{t}) \\
&\equiv \left({}_{R}U_{t}{}^{n} \times {}_{R}W_{t}{}^{n}\right) \times p({}_{R}U_{t}, {}_{R}W_{t})
\end{aligned} \tag{10}
$$

The first moment expectation value of ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{t}}$ follows as

$$
\begin{aligned}
E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{1}, {}_{\mathrm{R}}W_{\mathrm{t}}{}^{1}\right) &\equiv \left(\underbrace{{}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}W_{\mathrm{t}}{}^{1}}_{(one-times)}\right) \times p({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}) \\
&\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}W_{\mathrm{t}}{}^{1}\right) \times p({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}) \\
&\equiv ({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}}) \times p({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}})
\end{aligned} \tag{11}
$$

2.1.6 The probability of a single event

Definition 6 (**The probability of a single event**). *In consideration of the definitions before, again let p(${}_{R}U_{t}$) represent the probability of a single event ${}_{R}U_{t}$ at Bernoulli trial t. Let $\Psi({}_{R}U_{t})$ represent the wavefunction, a probability amplitude [30] of an event or of finding an event inside a set at a given (period of) time / Bernoulli trial [100] t. Let $\Psi^{*}({}_{R}U_{t})$ denote the complex conjugate of the wave-function. In general, it is*

$$
\begin{aligned}
p({}_{R}U_{t}) &\equiv \frac{E({}_{R}U_{t})}{{}_{R}U_{t}} \\
&\equiv \frac{E({}_{R}U_{t})^{2}}{E({}_{R}U_{t}{}^{2})} \\
&\equiv \frac{p({}_{R}U_{t})^{2} \times ({}_{R}U_{t})^{2}}{p({}_{R}U_{t}) \times ({}_{R}U_{t})^{2}} \\
&\equiv \Psi({}_{R}U_{t}) \times \Psi^{*}({}_{R}U_{t})
\end{aligned} \tag{12}
$$

2.1.7 Wave function

Definition 7 (**Wave function**). *Taking into account especially the definitions before and the relationship that $p({}_{R}U_{t}) \equiv \Psi({}_{R}U_{t}) \times \Psi^{*}({}_{R}U_{t})$, some other relationships can be derived too. Recall again that p(${}_{R}U_{t}$) represent the* *probability of a certain single event at the Bernoulli trial t,* $\Psi(_RU_t)$ *represent the wave function, a probability amplitude [30] of an event at a given (period of) time / Bernoulli trial t and* $\Psi^*(_RU_t)$ *is the complex conjugate of the wave-function of* $_RU_t$*. Let* $f(_RU_t)$ *denote any kind of a mathematical function which describes the behaviour of* $_RU_t$ *while the same mathematical function satisfies the need that* $\frac{f(_RU_t)}{f(_RU_t)} \equiv +1$*. In general, it is*

$$\begin{aligned}
\Psi(_RU_t) &\equiv \frac{1}{\Psi^*(_RU_t)} \times p(_RU_t) \\
&\equiv \frac{p(_RU_t)}{\Psi^*(_RU_t \times f(_RU_t))} \times f(_RU_t) \\
&\equiv \underbrace{\frac{p(_RU_t)}{\Psi^*(_RU_t) \times f(_RU_t)}}_{Z} \times f(_RU_t) \\
&\equiv Z \times f(_RU_t) \\
&\equiv \frac{1}{\Psi^*(_RU_t)} \times \frac{E(_RU_t)}{_RU_t} \\
&\equiv \frac{1}{\Psi^*(_RU_t) \times {_RU_t}} \times E(_RU_t)
\end{aligned} \tag{13}$$

Under conditions where $_ou_t$ *and* $_o\underline{u}_t$ *are described by a wave-function,* ***the superposition principle****, first stated by Daniel Bernoulli (1700 – 1782) in 1753 ("****Later (1753), Daniel Bernoulli formulated the principle of superposition ...****"[see 72, p. 2]) demands that* $\Psi(_ou_t) + \Psi(_o\underline{u}_t) \equiv \Psi(_ou_t + {_o\underline{u}_t}) \equiv \Psi(_RU_t)$ *while the normalisation condition is necessary to be ensured.*

2.1.8 The complex conjugate

Definition 8 (The complex conjugate). *The conjugate of a complex number denoted as* $conjugate(a(_RU_t) + (i \times b(_RU_t)))$*, where* $i^2 \equiv -1$ *is the imaginary [28], is defined as*

$$conjugate(a(_RU_t) + (i \times b(_RU_t))) \equiv (a(_RU_t) - (i \times b(_RU_t))) \tag{14}$$

It is known that any complex number multiplied by its complex conjugate is a real number. It is

$$\begin{aligned}
(a(_RU_t) + (i \times b(_RU_t))) \times (a(_RU_t) - (i \times b(_RU_t))) &\equiv \left(a(_RU_t)^2\right) - \left(i^2 \times b(_RU_t)^2\right) \\
&\equiv \left(a(_RU_t)^2\right) + \left(b(_RU_t)^2\right)
\end{aligned} \tag{15}$$

2.1.9 The variance

Definition 9 (The variance). *Sir Ronald Aylmer Fisher (1890 – 1962), an English statistician, "the single most important figure in 20th century statistics"[37] coined the term variance as follows: "It is therefore desirable* ***in analysing the causes*** *of variability to deal with the square of the standard deviation as the measure of variability. We shall term this quantity the Variance ... "[see 41, p. 399] Again, let* $p(_RU_t)$ *represent the probability of a single event* $_RU_t$ *at a given point in space-time or Bernoulli trial t. Let* $E(_RU_t)$ *denote again the expectation value of* $_RU_t$*. The expectation value of* $_RU_t$ *is defined as*

$$E(_RU_t) \equiv p(_RU_t) \times (_RU_t) \equiv \Psi(_RU_t) \times {_RU_t} \times \Psi^*(_RU_t) \tag{16}$$

The expectation value of the other of $_RU_t$*, of the complementary of* $_RU_t$*, of the opposite of* $_RU_t$*, of* ***the anti*** $_RU_t$*, denoted by* $_R\underline{U}_t$*, is defined as*

$$E(_R\underline{U}_t) \equiv (1 - p(_RU_t)) \times (_RU_t) \tag{17}$$

In this context, $E(_RU_t{}^2)$ *is the expectation value of the second moment of* $_RU_t$*. The expectation value of* $_RU_t{}^2$ *is defined as*

$$E\left(_RU_t{}^2\right) \equiv p(_RU_t) \times \left(_RU_t{}^2\right) \equiv p(_RU_t) \times (_RU_t \times {_RU_t}) \tag{18}$$

Let $\sigma({}_{R}U_{t})$ denote the standard deviation of ${}_{R}U_{t}$. Let $\sigma({}_{R}U_{t})^2$ denote the variance of ${}_{R}U_{t}$. In general, the variance [see 66, p. 42] is defined as

$$
\begin{aligned}
\sigma({}_{R}U_{t})^2 &\equiv \sigma({}_{R}U_{t}) \times \sigma({}_{R}U_{t}) \\
&\equiv E\left({}_{R}U_{t} - E\left({}_{R}U_{t}\right)\right)^2 \\
&\equiv E\left({}_{R}U_{t}{}^2\right) - \left(E\left({}_{R}U_{t}\right)\right)^2 \\
&\equiv \left({}_{R}U_{t}{}^2 \times p\left({}_{R}U_{t}\right)\right) - \left(p\left({}_{R}U_{t}\right) \times {}_{R}U_{t}\right)^2 \\
&\equiv \left({}_{R}U_{t}{}^2\right) \times \left(p\left({}_{R}U_{t}\right) - p\left({}_{R}U_{t}\right)^2\right) \\
&\equiv \left({}_{R}U_{t}{}^2\right) \times \left(p\left({}_{R}U_{t}\right) \times \left(1 - p\left({}_{R}U_{t}\right)\right)\right) \\
&\equiv {}_{R}U_{t} \times \left(p\left({}_{R}U_{t}\right) \times {}_{R}U_{t} \times \left(1 - p\left({}_{R}U_{t}\right)\right)\right) \\
&\equiv E\left({}_{R}U_{t}\right) \times {}_{R}U_{t} \times \left(1 - p\left({}_{R}U_{t}\right)\right) \\
&\equiv E\left({}_{R}U_{t}\right) \times E\left({}_{R}\underline{U}_{t}\right)
\end{aligned}
\tag{19}
$$

From equation 19 follows that

$$
\begin{aligned}
p\left({}_{R}U_{t}\right) \times \left(1 - p\left({}_{R}U_{t}\right)\right) &\equiv \frac{\sigma({}_{R}U_{t})^2}{{}_{R}U_{t}{}^2} \\
&\equiv \frac{E\left({}_{R}U_{t}{}^2\right)}{{}_{R}U_{t}{}^2} - \frac{\left(E\left({}_{R}U_{t}\right)\right)^2}{{}_{R}U_{t}{}^2} \\
&\equiv \frac{E\left({}_{R}U_{t}\right) \times E\left({}_{R}\underline{U}_{t}\right)}{{}_{R}U_{t} \times {}_{R}U_{t}} \\
&\equiv p\left({}_{R}U_{t}\right) - p\left({}_{R}U_{t}\right)^2
\end{aligned}
\tag{20}
$$

and equally (Eq. 19) that

$$
{}_{R}U_{t} \equiv \frac{\sigma({}_{R}U_{t})}{\sqrt[2]{p\left({}_{R}U_{t}\right) \times \left(1 - p\left({}_{R}U_{t}\right)\right)}}
\tag{21}
$$

and that

$$
{}_{R}W_{t} \equiv \frac{\sigma({}_{R}W_{t})}{\sqrt[2]{p\left({}_{R}W_{t}\right) \times \left(1 - p\left({}_{R}W_{t}\right)\right)}}
\tag{22}
$$

2.1.10 The Chi square goodness of fit test of variance

Definition 10 (The $\tilde{\chi}^2$ goodness of fit test of variance).

In order to determine the population variance, denoted as $\sigma({}_{\mathrm{R}}U_{\mathrm{t}})_0{}^2$, it is appropriate to examine the entire population. However, often it is sufficient too, to obtain the population variance based on a representative data sample. Thus far, it is possible to determine whether the variance of a variable obtained from a data sample, denoted as $\sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2$, has the same size as the known population variance of the same variable. **The $\tilde{\chi}^2$ goodness of fit test of a variance** with degree of freedom (d. f.) of d. f. = 1 is defined as

$$
\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}\left(\sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 \mid \sigma({}_{\mathrm{R}}U_{\mathrm{t}})_0{}^2\right) &\equiv \frac{\left(\sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 - \sigma({}_{\mathrm{R}}U_{\mathrm{t}})_0{}^2\right)^2}{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})_0{}^2} + \\
&\quad \frac{\left(\sigma({}_{\mathrm{R}}U_{\mathrm{t}})_0{}^2 - \sigma({}_{\mathrm{R}}U_{\mathrm{t}})_0{}^2\right)^2}{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})_0{}^2} \\
&\equiv \frac{\left(\sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 - \sigma({}_{\mathrm{R}}U_{\mathrm{t}})_0{}^2\right)^2}{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})_0{}^2} + 0
\end{aligned}
\tag{23}
$$

and is not identical with **the chi-square test for variance** which is a non-parametric statistical procedure. In this context, Yate's [110] continuity correction has not been used.

2.1.11 The n-th moment co-variance

Definition 11 (**The n-th moment co-variance**). *Let $p({}_RU_t, {}_RW_t)$ represent the joint probability of ${}_RU_t$ and ${}_RW_t$ at the same (period of time) Bernoulli trial t. Let $E({}_RU_t{}^n, {}_RW_t{}^n)$ denote the n-th moment expectation value of ${}_RU_t$ and ${}_RW_t$. Let $E({}_RU_t{}^n)$ denote the n-th moment expectation value of ${}_RU_t$. Let $E({}_RW_t{}^n)$ denote the n-th moment expectation value of ${}_RW_t$. Let $\sigma({}_RU_t, {}_RW_t)$ denote the co-variance between ${}_RU_t$ and ${}_RW_t$. In general,* ***the n-th moment co-variance between ${}_RU_t$ and ${}_RW_t$*** *is defined as*

$$\begin{aligned}\sigma\left({}_RU_t{}^n, {}_RW_t{}^n\right) &\equiv \left({}_RU_t{}^n \times {}_RW_t{}^n\right) \times \left(p\left({}_RU_t, {}_RW_t\right) - \left(p\left({}_RU_t\right) \times p\left({}_RW_t\right)\right)\right) \\ &\equiv \left(\left({}_RU_t{}^n \times {}_RW_t{}^n \times p\left({}_RU_t, {}_RW_t\right)\right) - \left(\left({}_RU_t{}^n \times {}_RW_t{}^n\right) \times p\left({}_RU_t\right) \times p\left({}_RW_t\right)\right)\right) \\ &\equiv E\left({}_RU_t{}^n, {}_RW_t{}^n\right) - \left({}_RU_t{}^n \times p\left({}_RU_t\right)\right) \times \left({}_RW_t{}^n \times p\left({}_RW_t\right)\right) \\ &\equiv E\left({}_RU_t{}^n, {}_RW_t{}^n\right) - \left(E\left({}_RU_t{}^n\right) \times E\left({}_RW_t{}^n\right)\right)\end{aligned} \tag{24}$$

From equation 24 follows that

$$\begin{aligned}\sigma\left({}_RU_t, {}_RW_t\right) &\equiv \left({}_RU_t \times {}_RW_t\right) \times \left(p\left({}_RU_t, {}_RW_t\right) - \left(p\left({}_RU_t\right) \times p\left({}_RW_t\right)\right)\right) \\ &\equiv \left(\left({}_RU_t \times {}_RW_t \times p\left({}_RU_t, {}_RW_t\right)\right) - \left(\left({}_RU_t \times {}_RW_t\right) \times p\left({}_RU_t\right) \times p\left({}_RW_t\right)\right)\right) \\ &\equiv E\left({}_RU_t \times {}_RW_t\right) - \left({}_RU_t \times p\left({}_RU_t\right)\right) \times \left({}_RW_t \times p\left({}_RW_t\right)\right) \\ &\equiv E\left({}_RU_t \times {}_RW_t\right) - \left(E\left({}_RU_t\right) \times E\left({}_RW_t\right)\right)\end{aligned} \tag{25}$$

Equation 25 demands that

$${}_RU_t \times {}_RW_t \equiv \frac{\sigma\left({}_RU_t, {}_RW_t\right)}{\left(p\left({}_RU_t, {}_RW_t\right) - \left(p\left({}_RU_t\right) \times p\left({}_RW_t\right)\right)\right)} \tag{26}$$

2.1.12 Two by two table of Bernoulli random variables

Definition 12 (**Two by two table of Bernoulli random variables**).

Karl Pearson was the first to introduce the notion of a two by two or contingency[82] table in 1904. A contingency table is still an appropriate theoretical model too for studying the relationships between two *Bernoulli[24] (i. e. +0/+1) distributed random variables* existing or occurring at the same *Bernoulli trial* [100] (period of time) t. In this context, let a Bernoulli distributed random variable U_t denote a risk factor, a condition or a cause et cetera and occur or exist with the probability $p(U_t)$ at the *Bernoulli trial* [100] (period of time) t. Let $E(U_t)$ denote the expectation value of U_t. In the case of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned}E\left(U_t\right) &\equiv U_t \times p\left(U_t\right) \\ &\equiv p\left(a_t\right) + p\left(b_t\right) \\ &\equiv (+0+1) \times p\left(U_t\right) \\ &\equiv p\left(U_t\right)\end{aligned} \tag{27}$$

Let a Bernoulli distributed random variable W_t denote an outcome, a conditioned event or an effect and occur or exist et cetera with the probability $p(W_t)$ at the Bernoulli trial (period of time) t. Let $E(W_t)$ denote the expectation value of W_t. It is

$$\begin{aligned}E\left(W_t\right) &\equiv W_t \times p\left(W_t\right) \\ &\equiv p\left(a_t\right) + p\left(c_t\right) \\ &\equiv (+0+1) \times p\left(W_t\right) \\ &\equiv p\left(W_t\right)\end{aligned} \tag{28}$$

Let $p(a_t)= p(U_t \cap W_t)$ denote the joint probability distribution of U_t and W_t at the same Bernoulli trial (period of time) t. In general it is

$$\begin{aligned}E\left(a_t\right) &\equiv E\left(U_t \cap W_t\right) \\ &\equiv \left(U_t \times W_t\right) \times p(U_t \cap W_t) \\ &\equiv p(U_t \cap W_t) \\ &\equiv p\left(a_t\right)\end{aligned} \tag{29}$$

Let $p(b_t) = p(U_t \cap \neg W_t)$ denote the joint probability distribution of U_t and not W_t at the same Bernoulli trial (period of time) t. In general it is

$$\begin{aligned} E(b_t) &\equiv E(U_t \cap \neg W_t) \\ &\equiv (U_t \times \neg W_t) \times p(U_t \cap \neg W_t) \\ &\equiv p(U_t \cap \neg W_t) \\ &\equiv p(b_t) \end{aligned} \tag{30}$$

Let $p(c_t) = p(\neg U_t \cap W_t)$ denote the joint probability distribution of not U_t and W_t at the same Bernoulli trial (period of time) t. In general it is

$$\begin{aligned} E(c_t) &\equiv E(\neg U_t \cap W_t) \\ &\equiv (\neg U_t \times W_t) \times p(\neg U_t \cap W_t) \\ &\equiv p(\neg U_t \cap W_t) \\ &\equiv p(c_t) \end{aligned} \tag{31}$$

Let $p(d_t) = p(\neg U_t \cap \neg W_t)$ denote the joint probability distribution of not U_t and not W_t at the same Bernoulli trial (period of time) t. In general it is

$$\begin{aligned} E(d_t) &\equiv E(\neg U_t \cap \neg W_t) \\ &\equiv (\neg U_t \times \neg W_t) \times p(\neg U_t \cap \neg W_t) \\ &\equiv p(\neg U_t \cap \neg W_t) \\ &\equiv p(d_t) \end{aligned} \tag{32}$$

In general, it is

$$p(a_t) + p(b_t) + p(c_t) + p(d_t) \equiv +1 \tag{33}$$

Table 1 provides an overview of the definitions above.

		Conditioned W_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	$p(b_t)$	$p(U_t)$
U_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{U}_t)$
		$p(W_t)$	$p(\underline{W}_t)$	+1

Tabelle 1: The two by two table of Bernoulli random variables

2.1.13 Two by two table of Binomial random variables

Definition 13 (Two by two table of Binomial random variables).

Under conditions where *the probability of an event, an outcome, a success et cetera is constant from Bernoulli trial to Bernoulli trial t*, it is

$$\begin{aligned} A &= N \times E(U_t) \\ &\equiv N \times (U_t \times p(U_t)) \\ &\equiv N \times (p(U_t) + p(W_t)) \\ &\equiv N \times p(U_t) \end{aligned} \tag{34}$$

and

$$\begin{aligned} B &= N \times E(W_t) \\ &\equiv N \times (W_t \times p(W_t)) \\ &\equiv N \times (p(U_t) + p(c_t)) \\ &\equiv N \times p(W_t) \end{aligned} \tag{35}$$

where N denotes the population size. Furthermore, it is

$$a \equiv N \times (E(U_t)) \equiv N \times (p(U_t)) \tag{36}$$

and

$$b \equiv N \times (E(W_t)) \equiv N \times (p(W_t)) \tag{37}$$

and

$$c \equiv N \times (E(c_t)) \equiv N \times (p(c_t)) \tag{38}$$

and

$$d \equiv N \times (E(d_t)) \equiv N \times (p(d_t)) \tag{39}$$

and

$$a + b + c + d \equiv A + \underline{A} \equiv B + \underline{B} \equiv N \tag{40}$$

Table 2 provides an overview of a two by two table of Binomial random variables.

		Conditioned W_t		
		TRUE	FALSE	
Condition	TRUE	a	b	A
U_t	FALSE	c	d	$\underline{A}$
		B	$\underline{B}$	N

Tabelle 2: The two by two table of Binomial random variables

2.1.14 Index Of Unfairness

Definition 14 (Index Of Unfairness).

The quality of collected may depend upon several factors. Therefore, it is appropriate to quantify possible collection or extraction bias due to the method used. The index of unfairness (IOU) is defined [see 17] as

$$\begin{aligned} p(IOU) &\equiv Absolute\left(\left(\frac{A+B}{N}\right) - 1\right) \\ &\equiv Absolute\left(\left(\frac{\underline{A}+\underline{B}}{N}\right) - 1\right) \end{aligned} \tag{41}$$

2.1.15 Index Of Independence

Definition 15 (Index Of Independence).

bias due to collection methods and other factors cannot be excluded completely. The index of independence (IOI) is of use in this context and defined [see 16] as

$$\begin{aligned} p(IOI) &\equiv Absolute\left(\left(\frac{A+\underline{B}}{N}\right) - 1\right) \\ &\equiv Absolute\left(\left(\frac{\underline{A}+B}{N}\right) - 1\right) \end{aligned} \tag{42}$$

2.1.16 Placebo controlled analysis

Lemma 1 (Placebo controlled analysis.)**.** *A well-conducted sample analysis with an appropriate approach is of general importance. A placebo determined sample is a sample designed not to have a certain event. Placebo-controlled samples analysis are of great value for standard investigations of different relationships. A sample experimental group (A) which contains an event (allocated blindly, randomly et cetera)* ***(verum group)*** *can be compared to another control group ($\underline{A}$) which is missing especially such a certain event* ***(placebo group)****. In order to obtain optimal, reproducible results, many times investigators ensure that $A \equiv \underline{A}$. Under these circumstances, it is given that*

$$p(IOU) \equiv p(IOI) \tag{43}$$

Direct proof. Sometimes, sample analysis are grounded on the demand that

$$A \equiv \underline{A} \tag{44}$$

Adding B, it is

$$A + B \equiv \underline{A} + B \tag{45}$$

Dividing by N, we obtain

$$\frac{A + B}{N} \equiv \frac{\underline{A} + B}{N} \tag{46}$$

Rearranging, it is

$$\frac{A + B}{N} - 1 \equiv \frac{\underline{A} + B}{N} - 1 \tag{47}$$

Taking the absolute, it is

$$Absolute\left(\left(\frac{A + B}{N}\right) - 1\right) \equiv Absolute\left(\left(\frac{\underline{A} + B}{N}\right) - 1\right) \tag{48}$$

and finally

$$p\left(IOU\right) \equiv p\left(IOI\right) \tag{49}$$

Quod erat demonstrandum.

sample size calculation is an important part of conducting an appropriate analysis. However, sample studies grounded on $p\left(IOU\right) \equiv p\left(IOI\right)$ need not produce automatically scientifically validated knowledge and systematic bias is not excluded completely.

2.1.17 Independence

Definition 16 (Independence).

Historically, logic and probability theory which by time derived from the former are two of the main pillars in the modern study of human reasoning. At first sight, combining logic and probability theory in the same mathematical framework, as done in this publication, might look a little bit strange because probability theory deals more or less with uncertainties whereas logic as such is concerned with absolutely certain inferences or truths. In this context, it is important to note that we will steer clear of the scientific debate over the exact nature and the meaning of probability. However, it is possible to treat **the probability of an event as the truth value of probability theory**. Thus far and in contrast to Fuzzy logic and other trials of non-classical logic, such an approach opens the strategic possibility to develop a logically consistent multi-valued logic. In this context, the concept of independence is of fundamental [65] importance in (natural) sciences as such and as old as human mankind itself. The first documented mathematical approach to the concept of independence can be ascribed preliminary to the French mathematician and equally a friend of Isaac Newton (1642 - 1726, the Julian calendar), Abraham de Moivre (1667 – 1754). Abraham de Moivre demands the following: " *Two Events are independent, when they have no connexion one with the other, and that the happening of one neither forwards nor obstructs the happening of the other. Two Events are dependent, when they are connected together as that the Probability of either's happening is altered by the happening of the other … therefore, those two Events being independent, the Probability of their both happening will be* $\frac{1}{13} \times \frac{1}{13} \equiv \frac{1}{169}$ "[see 74, p. 6/7]. The tremendous improvement of the concept of independence is undoubtedly due to the contributions of many scientists. Andrei Nikolajewitsch Kolmogorow (1903-1987), a Russian mathematician and one of the most important mathematicians of the 20th century mathematics, elaborates on the meaning of concept of independence too. "*The concept of mutual independence of two or more experiments holds, in a certain sense, a central position in the theory of probability … In consequence, one of the most important problems in the philosophy of the natural sciences is … to make precise the premises which would make it possible to regard any given real events as independent.*"[see 66, p. 8/9]. In fact, it is insightful to recall Einstein's theoretical approach to the concept of independence before the mind's eye. "*Ohne die Annahme einer … Unabhängigkeit der … Dinge voneinander … wäre physikalisches Denken … nicht möglich.*"[38]. In other words, the existence or the occurrence of an event U_t at the Bernoulli trial t need not but can be independent of the existence or of the occurrence of another event W_t *at the same* Bernoulli trial t. Mathematically, independence [65], [74] in terms of probability theory is defined at the same (period of) time t (i. e. Bernoulli trial t) as

$$p\left(U_t \cap W_t\right) \equiv p\left(U_t\right) \times p\left(W_t\right) \tag{50}$$

In a narrower sense, the conditio sine qua non relationship concerns itself at the end only with the case whether the presence of an event U_t (condition) enables or guarantees the presence of another event W_t (conditioned). As a result of these thoughts, another question worth asking concerns the relationship between the independence of an event U_t (a condition) and another event W_t (conditioned) and the necessary condition relationship. To be confronted with the danger of bias and equally with the burden of inappropriate conclusions drawn, another fundamental question at this stage is whether is it possible that an event U_t (a condition) is a necessary condition of event W_t (conditioned) even under circumstances where the event U_t (a condition) (a necessary condition) is independent of an event W_t (conditioned)? This question is already answered more or less to the negative [15]. An event U_t which is a necessary condition of another event W_t is equally an event without which another event (W_t) could not be, could not occur and implies as such already a kind of a dependence. Thus far, **data which provide evidence of a significant conditio sine qua non relationship between two events like U_t and W_t and equally support the hypothesis that U_t and W_t are independent of each other are more or less self-contradictory and of very restricted or of none value for further analysis**. In fact, if the opposite view would be taken as plausible, contradictions are more or less inescapable.

2.1.18 Dependence

Definition 17 (Dependence).

The dependence of events [see 3, p. 57-61] is defined as

$$p\left(\underbrace{U_t \cap W_t \cap C_t \cap \ldots}_{n}\right) \equiv \sqrt[n]{\underbrace{p(U_t) \times p(W_t) \times p(C_t) \times \ldots}_{n}} \tag{51}$$

2.1.19 Exclusion relationship

Definition 18 (Exclusion relationship [*EXCL*]).

Mathematically, the exclusion (EXCL) relationship, denoted by $p(U_t \mid W_t)$ in terms of probability theory, is defined as

$$\begin{aligned} p(U_t \mid W_t) &\equiv p(b_t) + p(c_t) + p(d_t) \\ &\equiv \frac{N \times (p(b_t) + p(c_t) + p(d_t))}{N} \\ &\equiv \frac{b+c+d}{N} \\ &\equiv 1 - (p(U_t \cap W_t) \equiv 0) \\ &\equiv 1 - (p(a_t) \equiv 0) \\ &\equiv (p(U_t \rightarrow \neg W_t)) \cap (p(W_t \rightarrow \neg U_t)) \\ &\equiv +1 \end{aligned} \tag{52}$$

Conjunction, disjunction, and negation are one of the simplest logical operators. To some extent, exclusion is determined by the negation of a conjunction and can be expressed equivalently in terms of a conditio per quam relationship (definition 32) as $p(U_t \mid W_t) \equiv (p(U_t \rightarrow \neg W_t)) \cap (p(W_t \rightarrow \neg U_t)) \equiv +1$. In spoken English, **if U_t then $\neg$ W_t and equally vice versa. If W_t then $\neg$ U_t** . Table 3 demonstrates the theoretical distribution of an exclusion relationship in terms of a sufficient condition as if U_t then $\neg$ W_t.

Tabelle 3: Exclusion and sufficient condition I.

		Conditioned W_t		
		NO	YES	
Condition U_t	YES	1	0	U_t
	NO	1	1	$\underline{U_t}$
		$\underline{W_t}$	W_t	1

Table 4 demonstrates the theoretical distribution of an exclusion relationship in terms of a sufficient condition as if W_t then $\neg$ U_t.

Tabelle 4: Exclusion and sufficient condition I.

		Conditioned U_t		
		NO	YES	
Condition W_t	YES	1	0	W_t
	NO	1	1	$\underline{W_t}$
		$\underline{U_t}$	U_t	1

Furthermore, consider, for example, that the two events **being a male human being** (U_t = TRUE) and equally **being a pregnant human being** (W_t = TRUE) are excluding each other at the same Bernoulli trial t. Even if **such a relationship is investigated inside a sample**, the definition of the exclusion relationship need to hold true at every single event. Mathematically, let $p(a_t) \equiv p((U_t = \text{TRUE}) \cap (W_t = \text{TRUE}))$ denote the joint probability distribution function of an event U_t and an event W_t. One determining feature of an exclusion relationship is the fact that $p(a_t) \equiv p((U_t = \text{TRUE}) \cap (W_t = \text{TRUE})) \equiv 0$. In other words, in case of an exclusion relationship it is not possible to observe an event U_t and at the same (period of) time or Bernoulli trial t an event W_t. Table 5 provide us with an overview of this example and equally one possible theoretical distribution of an exclusion relationship. Examinations of the protective effects and long-term benefits of commonly used statin therapy in both primary and secondary prevention of cardiovascular disease should be able to provide clear evidence of an exclusion relationship between statin therapy and death due to any (including cardiovascular) cause [18].

		Conditioned (pregnant) W_t		
		TRUE	FALSE	
Condition (male)	TRUE	**+0**	$p(b_t)$	$p(U_t)$
U_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{U_t})$
		$p(W_t)$	$p(\underline{W_t})$	+1

Tabelle 5: U_t excludes W_t and vice versa.

2.1.20 The goodness of fit test of an exclusion relationship

Definition 19 (**The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship**).

Under some well known circumstances, testing hypothesis about an exclusion relationship $p(U_t \mid W_t)$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship [12], [13] with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}((U_t \mid W_t) \mid A) &\equiv \frac{(b-(a+b))^2}{A} + \\ &\quad \frac{((c+d)-\underline{A})^2}{\underline{A}} \\ &\equiv \frac{a^2}{A} + 0 \\ &\equiv \frac{a^2}{A}\end{aligned} \tag{53}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}((U_t \mid W_t) \mid B) &\equiv \frac{(c-(a+c))^2}{B} + \\ &\quad \frac{((b+d)-\underline{B})^2}{\underline{B}} \\ &\equiv \frac{a^2}{B} + 0 \\ &\equiv \frac{a^2}{B}\end{aligned} \tag{54}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of an exclusion relationship/distribution p(U_t | W_t), in which case the null hypothesis to be accepted. Yate's [110] continuity correction has not been used under these circumstances.

2.1.21 The left-tailed p Value of an exclusion relationship

Definition 20 (The left-tailed p Value of an exclusion relationship).

It is known that as a sample size, N, increases, a sampling distribution of a special test statistic approaches the normal distribution (central limit theorem). Under these circumstances, the left-tailed (lt) p Value [19] of an exclusion relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}(U_{\mathrm{t}} \mid W_{\mathrm{t}}) &\equiv 1-e^{-(1-p(U_{\mathrm{t}}|W_{\mathrm{t}}))} \\ &\equiv 1-e^{-(a/N)} \end{aligned} \tag{55}$$

A low p-value may provide some evidence of statistical significance. Table 6 demonstrates another example of the distribution of an exclusion relationship.

Tabelle 6: Exclusion relationship.

		Conditioned W_t		
		YES	NO	
Condition U_t	YES	0	1	1
	NO	1	1	2
		1	2	3

2.1.22 Either or conditions

Definition 21 (Either U_t or W_t conditions [*NEQV*]).

Mathematically, an either U_t or W_t condition relationship (NEQV), denoted by p(U_t >-< W_t) in terms of probability theory, is defined as

$$\begin{aligned} p(U_{\mathrm{t}} >-< W_{\mathrm{t}}) &\equiv p(b_{\mathrm{t}})+p(c_{\mathrm{t}}) \\ &\equiv \frac{N \times (p(b_{\mathrm{t}})+p(c_{\mathrm{t}}))}{N} \\ &\equiv \frac{b+c}{N} \\ &\equiv +1 \end{aligned} \tag{56}$$

2.1.23 The Chi-square goodness of fit test of an either or condition relationship

Definition 22 (The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship).

An either or condition relationship p(U_t >-< W_t) can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship [12], [13] with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}((U_{\mathrm{t}} >-< W_{\mathrm{t}}) \mid A) &\equiv \frac{(b-(a+b))^2}{A} + \\ &\quad \frac{c-((c+d))^2}{\underline{A}} \\ &\equiv \frac{a^2}{A}+\frac{d^2}{\underline{A}} \end{aligned} \tag{57}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left(\left(U_{\text{t}} > - < W_{\text{t}}\right) \mid B\right) &\equiv \frac{\left(c-\left(a+c\right)\right)^2}{B} + \\ &\frac{b-\left(\left(b+d\right)\right)^2}{\underline{B}} \\ &\equiv \frac{a^2}{B} + \frac{d^2}{\underline{B}}\end{aligned} \tag{58}$$

Yate's [110] continuity correction has not been used in this context.

2.1.24 The left-tailed p Value of an either or condition relationship

Definition 23 (The left-tailed p Value of an either or condition relationship).

The left-tailed (lt) p Value [19] of an either or condition relationship can be calculated as follows.

$$\begin{aligned}pValue_{\text{lt}}\left(U_{\text{t}} > - < W_{\text{t}}\right) &\equiv 1 - e^{-\left(1-p\left(U_{\text{t}} > - < W_{\text{t}}\right)\right)} \\ &\equiv 1 - e^{-\left(\left(a+d\right)/N\right)}\end{aligned} \tag{59}$$

In this context, a low p-value indicates again a statistical significance. Table 7 provides an illustration of the theoretical distribution of an either U_t or W_t relationship.

Tabelle 7: Either U_t or W_t relationship.

		Conditioned W_t		
		YES	NO	
Condition U_t	YES	0	1	1
	NO	1	0	1
		1	1	2

2.1.25 Neither nor conditions

Definition 24 (Neither U_t nor W_t conditions [*NOR*]).

Mathematically, a neither U_t nor W_t condition relationship (NOR), denoted by $p(U_t \uparrow W_t)$ in terms of probability theory, is defined as

$$\begin{aligned}p\left(U_{\text{t}} \uparrow W_{\text{t}}\right) &\equiv p\left(d_{\text{t}}\right) \\ &\equiv \frac{N \times \left(p\left(d_{\text{t}}\right)\right)}{N} \\ &\equiv \frac{d}{N} \\ &\equiv +1\end{aligned} \tag{60}$$

2.1.26 The Chi square goodness of fit test of a neither nor condition relationship

Definition 25 (The $\tilde{\chi}^2$ goodness of fit test of a neither U_t nor W_t condition relationship).

A neither U_t nor W_t condition relationship $p(U_t \uparrow W_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution). The $\tilde{\chi}^2$ goodness of fit test of a neither U_t nor W_t condition relationship [12], [13] with degree of freedom (d. f.) of d. f. = 1 may be calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left(\left(U_{\text{t}} \uparrow W_{\text{t}}\right) \mid A\right) &\equiv \frac{\left(d-\left(c+d\right)\right)^2}{\underline{A}} + \\ &\frac{\left(\left(a+b\right)-A\right)^2}{A} \\ &\equiv \frac{c^2}{\underline{A}} + 0\end{aligned} \tag{61}$$

or equally as

$$\tilde{\chi}^2{}_{\text{Calculated}}\left(\left(U_{\mathrm{t}} \uparrow W_{\mathrm{t}}\right) \mid B\right) \equiv \frac{(d-(b+d))^2}{\underline{B}} + \frac{((a+c)-B)^2}{B} \equiv \frac{b^2}{\underline{B}} + 0 \tag{62}$$

Yate's [110] continuity correction has not been used in this context.

2.1.27 The left-tailed p Value of a neither nor B condition relationship

Definition 26 (**The left-tailed p Value of a neither $\mathbf{U_t}$ nor $\mathbf{W_t}$ condition relationship**).

The left-tailed (lt) p Value [19] of a neither U_t nor W_t condition relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}\left(U_{\mathrm{t}} \uparrow W_{\mathrm{t}}\right) &\equiv 1 - e^{-(1-p(U_{\mathrm{t}} \uparrow W_{\mathrm{t}}))} \\ &\equiv 1 - e^{-p(U_{\mathrm{t}} \cup W_{\mathrm{t}})} \\ &\equiv 1 - e^{-((a+b+c)/N)} \end{aligned} \tag{63}$$

where $\cup$ may denote disjunction or logical or. In this context, a low p-value indicates again a statistical significance. Table 8 provides an illustration of the theoretical distribution of a neither U_t nor W_t relationship.

Tabelle 8: Neither U_t nor W_t relationship.

		Conditioned W_t		
		YES	NO	
Condition U_t	YES	0	0	0
	NO	0	1	1
		0	1	1

2.1.28 Necessary condition

Definition 27 (**Necessary condition [*Conditio sine qua non*]**).

Scientific knowledge and objective reality are deeply interrelated. As mentioned at the start of the article, the specification of necessary conditions has traditionally been part of the philosopher's investigations of different phenomena. Behind the need of a detailed evidence it is justified to consider that philosophy as such has certainly not **a monopoly on the truth** and other areas such as medicine as well as other sciences and technology may transmit truths as well and may be of help to move beyond one's selfenclosed unit. Seemingly **the law's concept of causation** justifies to say few words on this subject, to put some light on some questions. Are there any criteria in law for deciding whether one action or an event U_t has caused another (generally harmful) event W_t? What are these criteria? May causation in legal contexts differ from causation outside the law, for example, in science or in our everyday life and to what extent? Under which circumstances is it justified to tolerate such differences as may be found to exist? To understand just what is the law's concept of causation it is useful to know how the highest court of states are dealing with causation. In the case *Hayes v. Michigan Central R. Co., 111 U.S. 228*, the U.S. Supreme Court defined 1884 conditio sine qua non as follows: "... **causa sine qua non – a cause which, if it had not existed, the injury would not have taken place**". [62] The German Bundesgerichtshof für Strafsachen stressed once again the importance of conditio sine qua non relationship in his decision by defining the following: "**Ursache eines strafrechtlich bedeutsamen Erfolges jede Bedingung, die nicht hinweggedacht werden kann, ohne daß der Erfolg entfiele**"[32] Another lawyer elaborated on the basic issue of **identity and difference between cause and condition**. Von Bar was writing: "Die erste Voraussetzung, welche erforderlich ist, damit eine Erscheinung als die Ursache einer anderen bezeichnet werden könne, ist, daß jene eine der Bedingungen dieser sein. Würde die zweite Erscheinung auch dann eingetreten sein, wenn die erste nicht vorhanden war, so ist sie in keinem Falle Bedingung und noch weniger Ursache. Wo immer ein Kausalzusammenhang behauptet wird, da muß er wenigstens diese Probe aushalten ... **Jede Ursache ist nothwendig auch eine Bedingung eines**

Ereignisses; aber nicht jede Bedingung ist Ursache zu nennen."[2] Von Bar's position translated into English: *The first requirement, which is required, thus that something could be called as the cause of another, is that the one has to be one of the conditions of the other. If the second something had occurred even if the first one did not exist, so it is by no means a condition and still less a cause. Wherever a causal relationship is claimed, the same must at least withstand this test… Every cause is necessarily also a condition of an event too; but not every condition is cause too.* Thus far, let us consider among other the following in order to specify necessary conditions from another, probabilistic point of view. An event (i. e. U_t) which is a necessary condition of another event or outcome (i.e. W_t) must be given, must be present for a conditioned, for an event or for an outcome W_t to occur. A necessary condition (i. e. U_t) is a requirement which must be fulfilled **at every single Bernoulli trial t**, in order for a conditioned or an outcome (i.e. W_t) to occur but it alone does not determine the occurrence of an event. In other words, if a necessary condition (i. e. U_t) is given, an outcome (i.e. W_t) need not to occur. In contrast to a necessary condition, a 'sufficient'condition is the one condition which 'guarantees'that an outcome will take place or must occur for sure. Under which conditions we may infer about the unobserved and whether observations made are able at all to justify predictions about potential observations which have not yet been made or even general claims which my go even beyond the observed (*the 'problem of induction')* is not the issue of the discussion at this point. Besides of the principal necessity meeting such a challenge, a necessary condition of an event can but need not to be at the same Bernoulli trial t a sufficient condition for an event to occur. However, theoretically it is possible that an event or an outcome is determined by many necessary conditions. Let us focus to some extent on what this means or in other words how much importance can we attribute to such a special case. *Example*. A human being cannot live without oxygen. A human being cannot live without water. A human being cannot live without a brain. A human being cannot live without kidneys. A human being cannot live without ... et cetera. Thus far, even if oxygen is given, if water is given, if a brain is given, without functioning kidney's (or something similar) a human being will not survive on the long run. This example is of use to reach the following conclusion. Although it might seem somewhat paradoxical at first sight, **even under circumstances where a condition or an outcome depends on several different necessary conditions it is particularly important that every single of these necessary conditions for itself must be given otherwise the conditioned (i.e. the outcome) will not occur**. Finally, mathematically, the necessary condition (SINE) relationship, denoted by $p(U_t \leftarrow W_t)$ in terms of probability theory, is defined as

$$\begin{aligned} p(U_t \leftarrow W_t) &\equiv p(a_t) + p(b_t) + p(d_t) \\ &\equiv \frac{N \times (p(a_t) + p(b_t) + p(d_t))}{N} \\ &\equiv \frac{a+b+d}{N} \\ &\equiv +1 \end{aligned} \tag{64}$$

Table 9 provides an overview of the definition of the necessary condition.

		Conditioned W_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	$p(b_t)$	$p(U_t)$
U_t	FALSE	**+0**	$p(d_t)$	$p(\underline{U}_t)$
		$p(W_t)$	$p(\underline{W}_t)$	+1

Tabelle 9: Necessary condition.

2.1.29 The Chi-square goodness of fit test of a necessary condition relationship

Definition 28 (**The $\tilde{\chi}^2$ goodness of fit test of a necessary condition relationship**).

The data as obtained by investigations can vary extremely across studies as well as among and within individuals. Some (experimental) studies may support a hypothesis of a conditio sine qua non relationship between two factors while other may fail on the same matter. An appropriate study design is of essential importance for a successful execution of research. However, each design has its own strengths and weaknesses, and the data achieved need not to guarantee to arrive at correct conclusions. Besides of all, under some known circumstances, testing hypothesis about the conditio sine qua non relationship $p(U_t \leftarrow W_t)$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution), first described by the German statistician Friedrich Robert Helmert [51] and later rediscovered by Karl Pearson [81] in the context of a goodness of fit test. The $\tilde{\chi}^2$ goodness of fit test of a conditio sine qua non relationship [12], [13] with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}(U_{\text{t}} \leftarrow W_{\text{t}} \mid B) &\equiv \frac{(a-(a+c))^2}{B} + \\ &\quad \frac{((b+d)-\underline{B})^2}{\underline{B}} \\ &\equiv \frac{c^2}{B} + 0 \\ &\equiv \frac{c^2}{B}\end{aligned} \tag{65}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}(U_{\text{t}} \leftarrow W_{\text{t}} \mid \underline{A}) &\equiv \frac{(d-(c+d))^2}{\underline{A}} + \\ &\quad \frac{((a+b)-A)^2}{A} \\ &\equiv \frac{c^2}{\underline{A}} + 0 \\ &\equiv \frac{c^2}{\underline{A}}\end{aligned} \tag{66}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. It has not yet been finally clarified whether the use of Yate's [110] continuity correction is necessary at all.

2.1.30 The left-tailed p Value of the conditio sine qua non relationship

Definition 29 (The left-tailed p Value of the conditio sine qua non relationship).

The left-tailed (lt) p Value [19] of the conditio sine qua non relationship can be calculated as follows.

$$\begin{aligned}pValue_{\text{lt}}(U_{\text{t}} \leftarrow W_{\text{t}}) &\equiv 1 - e^{-(1-p(U_{\text{t}} \leftarrow W_{\text{t}}))} \\ &\equiv 1 - e^{-(c/N)}\end{aligned} \tag{67}$$

A low p-value indicates statistical significance.

From another point of view, table 10 provides an example of the theoretical distribution of a necessary condition too.

Tabelle 10: Necessary condition.

		Conditioned W_t		
		YES	NO	
Condition U_t	YES	1	1	2
	NO	0	1	1
		1	2	3

2.1.31 The expected Chi-Square value of a cell

Definition 30 (The expected Chi-Square value of a cell).

Chi-square is a statistical test commonly used to compare observed data with data we would expect to obtain according to a specific hypothesis. Historically, the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution), first described by the German statistician Friedrich Robert Helmert [51] was rediscovered later by Karl Pearson

[81] in the context of a $\tilde{\chi}^2$ goodness of fit test. One of the assumptions of the Chi-square test is not that the observed value in each cell is greater than 5 but that the expected value in each cell is greater than 5. The expected Chi-Square value of **the cell a** of the table 11 is calculated as follows:

$$E(a) \equiv \frac{(A \times B)}{N} \tag{68}$$

In other words, for each cell (i. e. a, b c, d), its row (A, $\underline{A}$) marginal is multiplied by its column (B, $\underline{B}$) marginal, and that product is divided by the sample size (N).

		Conditioned W_t		
		TRUE	FALSE	
Condition	TRUE	a	b	A
U_t	FALSE	c	d	$\underline{A}$
		B	$\underline{B}$	N

Tabelle 11: Chi square and a 2x2 table

2.1.32 Fisher's exact one sided right tailed test of a necessary condition relationship

Definition 31 (Fisher's exact one sided right tailed test of a necessary condition relationship).

Under some circumstances, a certain sampling distribution of a test statistic (like necessary condition relationship) is only approximately equal to the theoretical chi-squared distribution and a chi-squared goodness of fit test [16], [17] might provide only approximate significance values. In point of fact, if **the expected values calculated** are too low or below 5, Fisher's Exact Test is an alternative to a chi-square test and it is more appropriate to consider the use Fisher's Exact test in place of chi-square test especially for 2×2 tables. Fisher's exact test is used especially when sample sizes are small, but the same is valid for all sample sizes. However, Fisher's exact test can be used even for tables that are larger than 2×2. Sir Ronald Aylmer Fisher (1890 – 1962) published an exact statistical significance test ("*Fisher's exact test*") [42] for the analysis of contingency tables *valid for all sample sizes*.
The null hypothesis of Fisher's Exact test is that the rows and the columns of the 2 × 2 table are independent, such that the probability of a subject being in a particular row is not influenced by being in a particular column.
Table 12 may provide an overview of the foundation of Fisher's Exact test.

		Conditioned W_t		
		TRUE	FALSE	
Condition	TRUE	a	b	A
U_t	FALSE	c	d	$\underline{A}$
		B	$\underline{B}$	N

Tabelle 12: Two by two table and Fisher's exact test

Fisher's exact test is a conservative test which is based on the hyper geometric distribution and not on the calculation of probabilities from a distribution (as in t-tests or chi-square). The hyper geometric (HGD) probability mass function is given by

$$\begin{aligned} p_{\text{HGD}}(X=a) &\equiv \left(\frac{\binom{A}{a} \times \binom{N-A}{B-a}}{\binom{N}{B}}\right) \\ &\equiv \left(\frac{\binom{A}{a} \times \binom{\underline{A}}{c}}{\binom{N}{B}}\right) \end{aligned} \tag{69}$$

Fisher's exact test can be used on more robust data sets too. Consider sampling a population of size N that has B objects with O and $\underline{B}$ with $\underline{O}$. Draw a sample of A objects and find a objects with O (see table 13).

Then there are

$$\binom{N}{A} \tag{70}$$

		Sampling a population		
		O	<u>O</u>	
In Sample	YES	a	b	A
(Not In Sample)	NO	c	d	<u>A</u>
		B	<u>B</u>	N

Tabelle 13: Two by two table and Fisher's exact test II

possible samples. Of these,

$$\binom{B}{a} \tag{71}$$

is the number of ways of choosing O in a sample of size B, while

$$\binom{\underline{B}}{b} \tag{72}$$

is the number of ways of choosing not-O or <u>O</u> in a sample of size

$$N - B = \underline{B} \tag{73}$$

Because these are independent, there are

$$\binom{B}{a} \times \binom{\underline{B}}{b} \tag{74}$$

ways of choosing a Os and b not-Os.

Therefore, the probability of choosing a

$$\begin{aligned} Os &\equiv \frac{\binom{B}{a} \times \binom{\underline{B}}{b}}{\binom{N}{A}} \\ &\equiv \frac{\frac{B!}{a! \times c!} \times \frac{\underline{B}!}{b! \times d!}}{\frac{N!}{A! \times \underline{A}!}} \\ &\equiv \frac{B! \times \underline{B}! \times A! \times \underline{A}!}{N! \times a! \times b! \times c! \times d!} \end{aligned} \tag{75}$$

which is Fisher's exact test formula given usually. In order to calculate the significance of the observed data, i.e. the total probability of observing data as extreme or more extreme if the null hypothesis true, we have to calculate the P value of a one-tailed test.

The *one sided right tailed (rt) P Value* under conditions of the validity of the hyper-geometric [45], [61], [80] distribution (HGD) is calculated according to the following formula [22], [90].

$$pValue(HGD)_{\mathrm{rt}}\,(X \geq a) \equiv 1 - \sum_{t=0}^{a-1} \left(\frac{\binom{A}{t} \times \binom{N-A}{B-t}}{\binom{N}{B}} \right) \tag{76}$$

2.1.33 Sufficient condition

Definition 32 (Sufficient condition [*Conditio per quam*]).

Mathematically, the sufficient condition (IMP) relationship, denoted by $p(U_t \rightarrow W_t)$ in terms of probability theory, is defined as

$$\begin{aligned} p\left(U_{\mathrm{t}} \rightarrow W_{\mathrm{t}}\right) &\equiv p\left(a_{\mathrm{t}}\right) + p\left(c_{\mathrm{t}}\right) + p\left(d_{\mathrm{t}}\right) \\ &\frac{N \times \left(p\left(a_{\mathrm{t}}\right) + p\left(c_{\mathrm{t}}\right) + p\left(d_{\mathrm{t}}\right)\right)}{N} \\ &\equiv \frac{a + c + d}{N} \\ &\equiv +1 \end{aligned} \tag{77}$$

Let us assume the relationship $p(U_t \rightarrow W_t)$ as proofed, secured and given. Let $p(C_t)$ denote the probability of another event D_t. The conditio per quam relationship is one of the many foundations of mathematical techniques for an industrial mass-identifications of antidotes too. An event which can counteract the occurrence of another event can be understood something as an anti-dot event. Under conditions where $p(U_t \rightarrow W_t) + p(D_t) \equiv +1$, event D_t is an anti-dot of event U_t.

2.1.34 The Chi square goodness of fit test of a sufficient condition relationship

Definition 33 (**The $\tilde{\chi}^2$ goodness of fit test of a sufficient condition relationship**).

Under some well known circumstances, testing hypothesis about the conditio per quam relationship $p(U_t \rightarrow W_t)$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a conditio per quam relationship [12], [13] with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}(U_t \rightarrow W_t \mid A) &\equiv \frac{(a-(a+b))^2}{A} + \\
&\quad \frac{((c+d)-\underline{A})^2}{\underline{A}} \\
&\equiv \frac{b^2}{A} + 0 \\
&\equiv \frac{b^2}{A}
\end{aligned} \tag{78}$$

or equally as

$$\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}(U_t \rightarrow W_t \mid \underline{B}) &\equiv \frac{(d-(b+d))^2}{\underline{B}} + \\
&\quad \frac{((a+c)-B)^2}{B} \\
&\equiv \frac{b^2}{\underline{B}} + 0 \\
&\equiv \frac{b^2}{\underline{B}}
\end{aligned} \tag{79}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of the conditio per quam relationship/distribution $p(U_t \rightarrow W_t)$, in which case the null hypothesis accepted. Yate's [110] continuity correction has not been used in this context.

2.1.35 The left-tailed p Value of the conditio per quam relationship

Definition 34 (**The left-tailed p Value of the conditio per quam relationship**).

The left-tailed (lt) p Value [19] of the conditio per quam relationship can be calculated as follows.

$$\begin{aligned}
pValue_{\text{lt}}(U_t \rightarrow W_t) &\equiv 1 - e^{-(1-p(U_t \rightarrow W_t))} \\
&\equiv 1 - e^{-(b/N)}
\end{aligned} \tag{80}$$

Again, a low p-value indicates a statistical significance.

Table 14 demonstrates the theoretical distribution of a sufficient condition.

Tabelle 14: Sufficient condition.

		Conditioned W_t		
		YES	NO	
Condition U_t	YES	1	0	1
	NO	1	1	2
		2	1	3

2.1.36 Necessary and sufficient conditions

Definition 35 (**Necessary and sufficient conditions [*EQV*]**).

Mathematically, the necessary and sufficient condition (EQV) relationship, denoted by p($U_t \leftrightarrow W_t$) in terms of probability theory, is defined as

$$\begin{aligned} p(U_t \leftrightarrow W_t) &\equiv p(a_t) + p(d_t) \\ &\equiv \frac{N \times (p(a_t) + p(d_t))}{N} \\ &\equiv \frac{a+d}{N} \\ &\equiv +1 \end{aligned} \tag{81}$$

2.1.37 The Chi square goodness of fit test of a necessary and sufficient condition relationship

Definition 36 (**The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship**).

Even the necessary and sufficient condition relationship p($U_t \leftrightarrow W_t$) can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship [12], [13] with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}(U_t \leftrightarrow W_t \mid A) &\equiv \frac{(a-(a+b))^2}{A} + \\ &\quad \frac{d-((c+d))^2}{\underline{A}} \\ &\equiv \frac{b^2}{A} + \frac{c^2}{\underline{A}} \end{aligned} \tag{82}$$

or equally as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}(U_t \leftrightarrow W_t \mid B) &\equiv \frac{(a-(a+c))^2}{B} + \\ &\quad \frac{d-((b+d))^2}{\underline{B}} \\ &\equiv \frac{c^2}{B} + \frac{b^2}{\underline{B}} \end{aligned} \tag{83}$$

The calculated $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship can be compared with a theoretical chi-square value at a certain level of significance α. Under conditions where the observed values are equal to the expected/theoretical values of a necessary and sufficient condition relationship/distribution p($U_t \leftrightarrow W_t$), the $\tilde{\chi}^2$-distribution equals zero. It is to be cleared whether Yate's [110] continuity correction should be used at all.

2.1.38 The left-tailed p Value of a necessary and sufficient condition relationship

Definition 37 (**The left-tailed p Value of a necessary and sufficient condition relationship**).

The left-tailed (lt) p Value [19] of a necessary and sufficient condition relationship can be calculated as follows.

$$\begin{aligned} pValue_{\text{lt}}(U_t \leftrightarrow W_t) &\equiv 1 - e^{-(1-p(U_t \leftrightarrow W_t))} \\ &\equiv 1 - e^{-((b+c)/N)} \end{aligned} \tag{84}$$

In this context, a low p-value indicates again a statistical significance. Table 15 may provide an overview of the theoretical distribution of a necessary and sufficient condition.

Tabelle 15: Necessary and sufficient condition.

		Conditioned W_t		
		YES	NO	
Condition U_t	YES	1	0	1
	NO	0	1	1
		1	1	2

For the purposes at hand, as should be immediately apparent, it is obviously clear, straightforward, deeply important and beyond any question that in analytic philosophy, accurate specifications of necessary and sufficient conditions (NSC) play already a central and vital role while logical fallacies cannot be excluded in general. In analytic philosophy, the concept of necessary and sufficient conditions is based on notions like an antecedent and a consequent too whilst analytic philosophy is not ensuring permanently that an antecedent and a consequent are given or treated at the same (period of) time t. Finally, some known and invalid inferential forms reasoning may follow (affirming the consequent, denying the antecedent et cetera). In contrast to analytic philosophy, the probability based concept of necessary and sufficient conditions is grounded on **events occurring at the same (period of) time t**. Another important clarification regarding necessary and sufficient conditions is the fact that NSC are equally converses of each other. In this case, there is a kind of strange mirroring as follows: $U_{\mathrm{t}} \leftarrow W_{\mathrm{t}} \equiv W_{\mathrm{t}} \rightarrow U_{\mathrm{t}}$. On this account, U_t being a sufficient condition of W_t is logically equivalent to W_t being a necessary condition of U_t (and vice versa). However, this has no influence on the definition of necessary and sufficient conditions. Necessary and sufficient conditions are defined as $(U_{\mathrm{t}} \leftrightarrow W_{\mathrm{t}}) \equiv (U_{\mathrm{t}} \leftarrow W_{\mathrm{t}}) \cap (U_{\mathrm{t}} \rightarrow W_{\mathrm{t}})$ and not as $(U_{\mathrm{t}} \leftrightarrow W_{\mathrm{t}}) \equiv (U_{\mathrm{t}} \leftarrow W_{\mathrm{t}}) \cap (W_{\mathrm{t}} \rightarrow U_{\mathrm{t}})$ where $\cap$ denote conjugation. The account of necessary and sufficient conditions just outlined is particularly different from the concept of logical conditions. It is, then, worth making the obvious point that a causal relationship may posses many different features and a very serious and fundamental question may arise: **can an effect$_t$ as such occur without a cause$_t$**? If we answer this question to the positive, we must accept equally that events can occur without a cause or that a causeless effect may exist or that a causeless change is possible in principle. In last consequence, such a scientific attitude ultimately demand us to abandon **the principle of causality** in general. In contrast to such an **anti causal position** it is clear that the principle of causality implies too that a cause$_t$ is needed for an effect$_t$ to occur. A cause$_t$ is a necessary condition of an effect$_t$. In other words, **without** a cause$_t$ **no** effect$_t$ or a cause and a necessary condition are identical. However, it is inappropriate to treat a necessary condition of an event (effect$_t$) as being at the same (period of) time t a sufficient condition for the same event (effect$_t$) to occur. Such an attitude may end up at a causal fallacy. A necessary condition of an event is a condition which must be present for another event (effect$_t$) to occur. A necessary condition must be given in order for event (effect$_t$) to occur, but it alone does not provide sufficient cause for the occurrence of the event (effect$_t$). In contrast to a necessary condition, a sufficient condition is a condition that will produce the event (effect$_t$) to occur. Therefore and besides of **the identity of a cause and a necessary condition**, a cause as such cannot be reduced only to a necessary condition, a cause at the same (period of) time t is equally different from a necessary condition, both are logically not equivalent. **The difference between a cause and a necessary condition** determines the fact, that a cause is equally much more than only a necessary condition. In contrast to a necessary condition, an event as such, which is a cause$_t$ of another event should ensure too that the other event (effect$_t$) need to occur. To bring it to the point, a cause is at the same (period of) time t a sufficient condition of an effect too. In the light of these considerations, another determining part of a causal relationship is the relation **if** cause$_t$ **then** effect$_t$. Therefore, let us notice again what is strangest about the fundamental relationship between a cause and an effect. A cause$_t$ at a certain (period of) time t is both, a necessary and sufficient condition of an effect$_t$. What is interesting, however, is that with all respect for such a clear scientific position, the same is not uncontested and is not accepted without objection. Even if it almost seems impossible to bring forward an appropriate comment to all **anti causal authors** who wrote on the relation between cause and effect it is nevertheless still required to make at least few cautionary remarks on J. L. Mackie's position. J. L. Mackie's effectively anti causal position may serve as an example and as a representative for the numerous others. J. L. Mackie's theoretically very inappropriate approach to the notion cause and effect can be found in his paper 'Causes and Conditions'[see 73]. Completely in line with David Hume's (1711 - 1776) meanwhile outdated account to the relation of a cause and an effect, Mackie writes: "…a cause is … an event which precedes the event of which it is the cause …"[see 73, p. 245] In other sense, we must accept a logical fallacy as the foundation of causation: a cause is temporally prior in time to an effect. Based on his flawed approach to the nature of causation, Mackie is inventing enthusiastically a logical fallacy abbreviated as **INUS**, a very special, artificial, logically inconsistent and unrealistic approach to the relationship between a cause and an effect. "The so-called cause is … an insufficient but necessary part of a condition which is itself unnecessary but sufficient for the result [effect, author]. "[see 73, p. 245] More or less, Mackie himself reduces defectively a cause as such only to a sufficient condition. Mackie is trying to convince the reader that a cause is not a necessary condition. In other words, an effect can occur without a cause. Mackie demands: "The so-called cause is ... a condition which is itself unnecessary ... "[see 73, p. 245] In contrast to a necessary condition, following Mackie, a "... cause is ... a condition which is ... sufficient for the result [effect, author]. "[see 73, p. 245]. However, this doesn't necessarily mean that both are really to be equated according to Mackie. Besides of all, Mackie is compelled to admit that a cause is at the end quite different from a sufficient condition too even if not a necessary condition. In contrast to a pure sufficient condition, a cause is only "... an insufficient but necessary part of a condition ... "[see 73, p. 245], whatever this may mean. Whether it is possible or not to decompose a sufficient condition into single parts like a sufficient part and a non sufficient part or into a necessary part and into a not necessary part like $U_t \equiv (sufficient\ part_t \cup not\ sufficient\ part_t) \cap (necessary\ part_t \cup not\ necessary\ part_t)$ is not an issue which appears to be able to affect the nature of a sufficient condition. A sufficient condition is a sufficient condition or it is not a sufficient condition independently of any single parts which may determine the same. Unfortunately, this not the point where Mackie's completely unrealistic and unnecessary narration ends. Mackie tries to convince us that a "... cause is ... a condition which is itself unnecessary ... for the result [effect, author]. "[see 73, p. 245] Mackie imposes its own flawed understanding of the relation between cause and effect on others so thoughtlessly, that even the toughest among the patient is hardly able to bear. According to Mackie, **a cause is not a necessary condition of an effect**. To bring it to the extreme, according to Mackie, **an effect can occur without a cause**! In last consequence, Mackie is giving up the principle of causality. Last but not least, Mackie's so-called INUS logical fallacy is an insufficient but necessary part of a failed, brutal theoretical attack on the principle of causality which is itself unnecessary but sufficient for non-sense produced by the author himself. A final assessment of the issue necessary and sufficient conditions and causation and of the need for further action to be taken with regard to the recognition or the detection of causal relationships (from data) is the fundamental credo that a necessary and sufficient condition relationship is able to recognise or to detect causal relationships (from data). Table 16 provides an illustration of the theoretical distribution of a necessary and sufficient condition with respect to the causal relationship.

Tabelle 16: Causal relationship.

		Effect W_t		
		YES	NO	
Cause U_t	YES	1	0	1
	NO	0	1	1
		1	1	2

Many times, studies or experiments may be the next best method of addressing questions about the causal relationship between two factors like U_t and W_t. However, when performing real-word experiments or other investigations, bias of different kind (subjective and objective factors) including logical fallacies need to be considered in detail and the possibility to recognise the causal relationship between the two factors U_t and W_t while relying only on the **necessary and sufficient condition** relationship may be very rarely the case. Therefore and in general, it is necessary that the same study or different studies independently of each other provide significant evidence of a necessary condition relationship between the factors factors like U_t and W_t and equally of a sufficient condition relationship between the same factors U_t and W_t and of course, if possible, of a necessary and sufficient condition relationship between factors U_t and W_t. At least for these reasons and in order to avoid misconceptions about a causal relation between the factors U_t and W_t, we always require additional tools like the **causal relationship** k to be able to recognise a causal relationship between factors like U_t and W_t from data.

2.1.39 Causal relationship k

Definition 38 (**Causal relationship k**).

Nonetheless, mathematically, the causal relationship [3]–[5], [9]–[11] between a cause U_t and an effect W_t, denoted by $k(U_t, W_t)$ in terms of probability theory, is defined *at each single [96] Bernoulli trial t* as

$$k(U_t, W_t) \equiv \frac{\sigma(U_t, W_t)}{\sigma(U_t) \times \sigma(W_t)} \equiv \frac{p(U_t \cap W_t) - p(U_t) \times p(W_t)}{\sqrt[2]{(p(U_t) \times (1 - p(U_t))) \times (p(W_t) \times (1 - p(W_t)))}} \tag{85}$$

where σ (U_t , W_t) denotes the co-variance between a cause U_t and an effect W_t *at every single Bernoulli trial t*, σ (U_t) denotes the standard deviation of a cause U_t at the same single Bernoulli trial t, σ (W_t) denotes the standard deviation of an effect W_t at same single Bernoulli trial t. Table 17 provides an overview of the definition of the causal relationship k.

		Effect W_t		
		TRUE	FALSE	
Cause	TRUE	p(U_t)	p(W_t)	p(U_t)
U_t	FALSE	p(c_t)	p(d_t)	p($\underline{A}_t$)
		p(W_t)	p($\underline{B}_t$)	+1

Tabelle 17: the causal relationship k

2.1.40 Fisher's exact test and the causal relationship k

Definition 39 (Fisher's exact test and the causal relationship k).

Under some circumstances, the significance of a causal relationship k can be tested by Fisher's exact statistical significance test ("*Fisher's exact test*") [42] for the analysis of contingency tables too.
The null hypothesis of Fisher's Exact test is that a cause and an effect as illustrated by the 2 × 2 table 18 are independent.

		Effect W_t		
		TRUE	FALSE	
Cause	TRUE	a	b	A
U_t	FALSE	c	d	$\underline{A}$
		B	$\underline{B}$	N

Tabelle 18: Fisher's exact test and causation

The observed data are determined by several factors one of which is the study design too. In order to evaluate the significance of the observed data, i.e. the total probability of observing data as extreme or more extreme if the null hypothesis true, it is necessary to calculate a P value i. e. of a one-tailed test.
The *one sided right tailed (rt) P Value* under conditions of the validity of the hyper-geometric [45], [61], [80] distribution (HGD) is calculated according to the following formula [22], [90].

$$pValue(HGD)_{\text{rt}}(X \geq a) \equiv 1 - \sum_{t=0}^{a-1} \left(\frac{\binom{A}{t} \times \binom{N-A}{B-t}}{\binom{N}{B}} \right) \tag{86}$$

The *one sided left tailed (lt) P Value* under conditions of the validity of the hyper-geometric [45], [61], [80] distribution (HGD) is calculated according to the following formula.

$$pValue(HGD)_{\text{lt}}(X \leq a) \equiv \sum_{t=0}^{a} \left(\frac{\binom{A}{t} \times \binom{N-A}{B-t}}{\binom{N}{B}} \right) \tag{87}$$

B. Axioms

Axiom I. Lex identitatis

$$+1 = +1 \tag{88}$$

Axiom II. Lex contradictionis

$$+0 = +1 \tag{89}$$

Axiom III. Lex negationis

$$\neg(0) \times 0 = 1 \tag{90}$$

where $\neg$ denotes (logical [29] or natural) negation [1], [43], [48], [49], [59], [60], [63], [69], [76], [88], [94], [104]. In this context, there is some evidence that $\neg(1) \times 1 = 0$. In other words, it is $(\neg(1) \times 1) \times (\neg(0) \times 0) = 1$

III. RESULTS

A. The correlation relationship

Theorem 1 (The correlation relationship)**.** *Let Y denote a quantity. let E(Y) denote the expectation value of Y. Let X denote another quantity, let E(X) denote the expectation value of X. The correlation coefficient is based on a quantity dominated, mechanical understanding of the relationship between two factors like X and Y.*

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{91}$$

is true, **then** the conclusion

$$\begin{aligned} \rho(Y,X) &\equiv \frac{E((Y - E(Y)) \times (X - E(X)))}{E(Y - E(Y)) \times E(X - E(X))} \\ &\equiv \frac{\sigma(Y,X)}{\sigma(Y) \times \sigma(X)} \equiv +1 \end{aligned} \tag{92}$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{93}$$

is true. Multiplying this premise (i. e. axiom) by Y, it is

$$Y \equiv Y \tag{94}$$

Bravais [31] (1811-1863) - Pearson's (1857-1936) "*product-moment coefficient of correlation*"([44], [79]) in contrast to the causal relationship k ([3]–[5], [9]–[11]) is based on the demand that **Y = X**. Based on this fundamental assumption, equation 92 can be rearranged as

$$Y \equiv X \tag{95}$$

Equation 95 leads to

$$E(Y) \equiv E(X) \tag{96}$$

Equation 95 demands too that

$$Y^2 \equiv X^2 \tag{97}$$

Equation 97 demands that

$$E\left(Y^2\right) \equiv E\left(X^2\right) \tag{98}$$

Equation 95 can be rearranged as

$$Y - E(Y) \equiv X - E(Y) \tag{99}$$

According to equation 96, equation 99 changes to

$$Y - E(Y) \equiv X - E(X) \tag{100}$$

In other words, we must accept the equality of

$$E(Y - E(Y)) \equiv E(X - E(X)) \tag{101}$$

By squaring equation 101, it is

$$E(Y-E(Y))^2 \equiv E(X-E(X))^2 \tag{102}$$

or

$$E(Y-E(Y)) \times E(Y-E(Y)) \equiv E(X-E(X)) \times E(X-E(X)) \tag{103}$$

Based on equation 101, equation 103 can be rearranged as

$$E(Y-E(Y)) \times E(X-E(X)) \equiv E((X-E(X)) \times (X-E(X))) \tag{104}$$

Based on equation 95 and equation 96, equation 104 can be rearranged as

$$E(Y-E(Y)) \times E(X-E(X)) \equiv E((Y-E(Y)) \times (X-E(X))) \tag{105}$$

Rearranging equation 105, Bravais [31] (1811-1863) - Pearson's (1857-1936) "*product-moment coefficient of correlation*"[44], [79] follows [see 89, p. 496] as

$$\begin{aligned} \rho(Y,X) &\equiv \frac{E((Y-E(Y)) \times (X-E(X)))}{E(Y-E(Y)) \times E(X-E(X))} \\ &\equiv \frac{\sigma(Y,X)}{\sigma(Y) \times \sigma(X)} \equiv +1 \end{aligned} \tag{106}$$

Quod erat demonstrandum.

Remark 2. *In point of fact, it has always been this way, theories about the relationship between a cause and an effect of different kind are and have been around us for a long time. In other words, the attention on causal inference or the problem of causality in general is continuously growing. In this context, it is relatively easy to get convinced that correlation and causation are not identical in order to draw reliable conclusions (from observational data). Many times, mathematical examples or proofs are able to illustrate the truth of a statement. However and conversely, one single counterexample, experiment et cetera is enough and posses the theoretical potential to demonstrate the falsity of a theory, of a theorem et cetera. Meanwhile, there are more than enough counterexamples which where able to provide evidence that causation is not identical with causation, correlation is not enough for causal inference. But the question remains what makes the difference between causation and correlation?*
Bravais [31] (1811-1863) - Pearson's (1857-1936) "product-moment coefficient of correlation"[44], [79] is based on the assumption that ***a quantity Y is equivalent to a quantity X****. Only under these circumstance, it is possible to derive product-moment coefficient of correlation in a technically correct way. However this implies to, that the a product-moment coefficient of correlation is not identical with causation. The product-moment coefficient of correlation demands us to accept a simple and mechanical relationship between two quantities which is not identical with causation. Karl Pearson (1857-1936) himself "rejected causal thinking."[see 26, p. 39]. Pearson's rather delicate skepticism about causation is difficult to understand. Pearson elaborates on cause and effect as follows: "Beyond such discarded fundamentals ... lies still another fetish amidst the inscrutable arcana of even modern science, namely, the category of cause and effect."[see 83, p. vi] Following Pearson, there is none scientific value of the notion cause. "For science, cause, ..., is meaningless "[see 83, p. 128] Pearson directly and blantly advocates correlation instead of causation. "It is this conception of correlation ... which is the wider category by which we have to replace the old idea of causation. "[see 83, p. 157] Following Pearson, causation is not needed at all. On the contrary, if Pearson is right, "... Association, as replacing Causation "[see 83, p. 156] is all what is needed in science.*

B. The law of nature relationship g

From one point of view, determinism [84], [85] as such is deeply connected with the physical sciences. Planck himself went so far to demand that 'an event is then causally determined ... if it can be predicted with certainty'.

> "ein Ereignis dann kausal bedingt ...
> wenn es mit Sicherhaeit vorausgesagt werden kann "[see 84, p. 6]

Many times, laws of nature [see 92] are mere descriptions of the way objective reality is. On the other account, the same laws of nature are determined by the process of the self-organisation of objective reality too. The scientific dispute between Regularists and the Necessitarians is very much in trouble because none of both provided anything valuable to the problem whether there exist any eternal laws of nature or are the laws of nature itself only relative? However it may be, sometimes, a clarification and mathematical analysis of the laws of nature or scientific laws or natural laws can be described mathematically too.

Theorem 2 (The law of nature relationship g)**.** *In this theorem, we are specifying the probability measure on the sample space of an experiment as being equal to p = 1. In other words, it is for sure that an event occurred. Thus far, let the sample space Y_t denote the set of all possible outcomes of an experiment at a certain Bernoulli trial t. Let y_t denote a random variable, a real-valued function defined on a single element of the sample space Y_t at a Bernoulli trial t. In general, it is $Y_t \equiv \{y_{1\,t}, y_{2\,t}, \ldots, y_{n\,t}, \}$. Let E($y_t$) denote the expectation value of y_t. Let the sample space X_t denote the set of all possible outcomes of X at a certain Bernoulli trial t. Let x_t denote a random variable, a real-valued function defined on a single element of the sample space X_t at a Bernoulli trial t. In general, it is $X_t \equiv \{x_{1\,t}, x_{2\,t}, \ldots, x_{n\,t}, \}$. Let E($y_t$) denote the expectation value of y_t. Let f(x_t) denote a mathematical function which describes the behaviour of each element of a set X_t, let E(f(x_t)) denote the expectation value of f(x_t). The law of nature relationship is based on a quantity dominated, mechanical understanding of the relationship between two factors like y_t and f(x_t). Let g(y_t, f(x x_t)) denote the law of nature relationship, 'der gesetzmäßige Zusammenhang'. The law of nature relationship is defined as*

$$g(y_t, f(x_t)) \equiv \frac{\sigma(y_t, f(x_t))}{\sigma(y_t) \times \sigma(f(x_t))} \equiv +1 \tag{107}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{108}$$

is true, **then** the conclusion

$$\begin{aligned} g(y_t, f(x_t)) &\equiv \frac{E((y_t - E(y_t)) \times (f(x_t) - E(f(x_t))))}{E(y_t - E(y_t)) \times E(f(x_t) - E(f(x_t)))} \\ &\equiv \frac{\sigma(y_t, f(x_t))}{\sigma(y_t) \times \sigma(f(x_t))} \equiv +1 \end{aligned} \tag{109}$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{110}$$

is true. Multiplying this premise (i. e. axiom) by y_t, it is

$$y_t \equiv y_t \tag{111}$$

The law of nature relationship $\mathbf{g(y_t, f(x_t))}$ is based on the demand that an outcome, denoted as $\mathbf{y_t}$ is determined exactly by f(x_t) at every run of an experiment, at every Bernoulli trial t. In other words, it is $\mathbf{y_t = f(x_t)}$. Based on this fundamental assumption, equation 109 can be rearranged as

$$y_t \equiv f(x_t) \tag{112}$$

Equation 112 leads to

$$E(y_t) \equiv E(f(x_t)) \tag{113}$$

Equation 112 demands too that

$$y_t^2 \equiv f(x_t)^2 \tag{114}$$

Equation 114 demands that

$$E\left(y_t^2\right) \equiv E\left(f(x_t)^2\right) \tag{115}$$

Equation 112 can be rearranged as

$$y_t - E(y_t) \equiv f(x_t) - E(y_t) \tag{116}$$

According to equation 113, equation 116 changes to

$$y_t - E(y_t) \equiv f(x_t) - E(f(x_t)) \tag{117}$$

In other words, we must accept the equality of

$$E(y_t - E(y_t)) \equiv E(f(x_t) - E(f(x_t))) \tag{118}$$

By squaring equation 118, it is

$$E(y_t - E(y_t))^2 \equiv E(f(x_t) - E(f(x_t)))^2 \tag{119}$$

or

$$E\left(y_{\mathrm{t}}-E\left(y_{\mathrm{t}}\right)\right)^{2} \equiv E\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right) \times E\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right) \tag{120}$$

or

$$E\left(y_{\mathrm{t}}-E\left(y_{\mathrm{t}}\right)\right) \times E\left(y_{\mathrm{t}}-E\left(y_{\mathrm{t}}\right)\right) \equiv E\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right) \times E\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right) \tag{121}$$

Based on equation 118, equation 121 can be rearranged as

$$E\left(y_{\mathrm{t}}-E\left(y_{\mathrm{t}}\right)\right) \times E\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right) \equiv E\left(\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right) \times\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right)\right) \tag{122}$$

Based on equation 112 and equation 113, equation 122 can be rearranged as

$$E\left(y_{\mathrm{t}}-E\left(y_{\mathrm{t}}\right)\right) \times E\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right) \equiv E\left(\left(y_{\mathrm{t}}-E\left(y_{\mathrm{t}}\right)\right) \times\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right)\right) \tag{123}$$

Rearranging equation 123, the law of nature relationship g(y_t, f(x x_t)) follows [see 89, p. 496] as

$$\begin{aligned} g\left(y_{\mathrm{t}}, f\left(x_{\mathrm{t}}\right)\right) & \equiv \frac{E\left(\left(y_{\mathrm{t}}-E\left(y_{\mathrm{t}}\right)\right) \times\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right)\right)}{E\left(y_{\mathrm{t}}-E\left(y_{\mathrm{t}}\right)\right) \times E\left(f\left(x_{\mathrm{t}}\right)-E\left(f\left(x_{\mathrm{t}}\right)\right)\right)} \\ & \equiv \frac{\sigma\left(y_{\mathrm{t}}, f\left(x_{\mathrm{t}}\right)\right)}{\sigma\left(y_{\mathrm{t}}\right) \times \sigma\left(f\left(x_{\mathrm{t}}\right)\right)} \equiv+1 \end{aligned} \tag{124}$$

Quod erat demonstrandum.

Remark 3. *A paradigmatic shift is necessary to be undertaken in moving away from inconsistent and unnecessary theories of causation based on the structural equation modelling or counterfactual claims et cetera in order to provide a coherent mathematical foundation for the analysis of cause and effect relationships. The earliest attempt to formulate a kind of a structural equation modelling, a multivariate statistical analysis technique of causation was made in the 1920's by Sewall Wright [see 109, p. 557] However, it is important to point out that Wright derived the structural equation modelling from the coefficient of correlation. Wright is writing: "The present paper is an attempt to present a method of measuring the direct influence along each separate path in such a system and thus of finding the degree to which variation of a given effect is determined by each particular cause. The method depends on the combination of knowledge of the degrees of correlation among the variables in a system with such knowledge as may be possessed of the causal relations "[see 109, p. 557]. Wright himself points out that "**The method depends on the** … **correlation among the variables** … "[see 109, p. 557]. In contrast to Pearl's do(X=x) operator [see 78, p. 204], the law of nature relationship $g\left(y_t, f\left(x_t\right)\right)$ provides a logically consistent mathematical alternative to the structure equation modelling proposed analysis of dependencies between endogenous and exogenous variables. Even if multiplied by N=the sample size, the relationship need not to change. We obtain*

$$g\left(y_{t}, f\left(x_{t}\right)\right) \equiv \frac{N \times N \times \sigma\left(y_{t}, f\left(x_{t}\right)\right)}{N \times \sigma\left(y_{t}\right) \times N \times \sigma\left(f\left(x_{t}\right)\right)} \equiv+1 \tag{125}$$

C. Something and its own other

Theorem 3 (Something and its own other)**.** *Let ${}_RU_t$ denote something, a random variable as seen from the point of view of a stationary observer R, a quantum mechanical entity et cetera existing independently and outside of human mind and consciousness. Let the probability of ${}_RU_t$ be $p\left({}_RU_t\right)$, let the n-th moment expectation value of ${}_RU_t$ be*

$$E\left({}_RU_t{}^n\right) \equiv p\left({}_RU_t\right) \times\left({}_RU_t{}^n\right) \tag{126}$$

Let the n-th moment expectation value of the other of ${}_RU_t$,of 'the local hidden variable'of ${}_RU_t$, of the complementary of ${}_RU_t$, of the opposite of ${}_RU_t$ of the anti ${}_RU_t$ be

$$E\left({}_R\underline{U_t}{}^n\right) \equiv\left(1-p\left({}_RU_t\right)\right) \times\left({}_RU_t{}^n\right) \tag{127}$$

In general, it is

$${}_RU_t{}^n \equiv E\left({}_RU_t{}^n\right)+E\left({}_R\underline{U_t}{}^n\right) \tag{128}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{129}$$

is true, **then** the conclusion

$$_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \equiv E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}}\right)+E\left({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}{}^{\mathrm{n}}\right) \tag{130}$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{131}$$

is true. Multiplying this premise (i. e. axiom) by ${}_{\mathrm{R}}\mathrm{U_t}^{\mathrm{n}}$ it is

$$_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \equiv {}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \tag{132}$$

Equation 132 can be rearranged as

$$_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \equiv (+1)\times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \tag{133}$$

too or as

$$_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \equiv (1+0)\times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \tag{134}$$

and equally as

$$_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \equiv \left(1+p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)-p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)\times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \tag{135}$$

or as

$$_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \equiv \left(p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)+\left(1-p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)\right)\times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \tag{136}$$

too. Equation 136 simplifies as

$$_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \equiv \left(p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}}\right)+\left(\left(1-p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)\times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}}\right) \tag{137}$$

Equation 137 simplifies further according to the definition given (see equation 3) as

$$_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \equiv E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}}\right)+\left(\left(1-p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)\times {}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}}\right) \tag{138}$$

However, equation 138 itself simplifies again according to the definition given (see equation 7) as

$$_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}} \equiv E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{\mathrm{n}}\right)+E\left({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}{}^{\mathrm{n}}\right) \tag{139}$$

In other words, even inside a set n, ${}_{\mathrm{R}}\mathrm{U_t}^{\mathrm{n}}$ is determined by itself (E ${}_{\mathrm{R}}\mathrm{U_t}^{\mathrm{n}}$) and and the other of itself (E(${}_{\mathrm{R}}\underline{\mathrm{U}}_{\mathrm{t}}{}^{\mathrm{n}}$)) with the consequence that our conclusion is true.

Quod erat demonstrandum.

D. The inner contradiction

Theorem 4 (The inner contradiction)**.** *In general, it is*

$$\sigma({}_{R}U_{t})^{2} \equiv E\left({}_{R}U_{t}\right)\times E\left({}_{R}\underline{U}_{t}\right) \tag{140}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{141}$$

is true, **then** the conclusion

$$\sigma({}_{\mathrm{R}}U_{\mathrm{t}})^{2} \equiv E\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\times E\left({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}\right) \tag{142}$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{143}$$

is true. Multiplying this premise (i. e. axiom) by ${}_{\mathrm{R}}\mathrm{U_t}$

$$_{\mathrm{R}}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}U_{\mathrm{t}} \tag{144}$$

According to theorem 3, equation 139 it is ${}_{\mathrm{R}}U_{\mathrm{t}} \equiv E({}_{\mathrm{R}}U_{\mathrm{t}}) + E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}})$. Equation 144 changes to

$$ {}_{\mathrm{R}}U_{\mathrm{t}} \equiv E({}_{\mathrm{R}}U_{\mathrm{t}}) + E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}) \tag{145} $$

Rearranging equation 145, it is

$$ {}_{\mathrm{R}}U_{\mathrm{t}} - E({}_{\mathrm{R}}U_{\mathrm{t}}) \equiv E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}) \tag{146} $$

Taking the expectation value, it is [see 66, p. 42]

$$ E({}_{\mathrm{R}}U_{\mathrm{t}} - E({}_{\mathrm{R}}U_{\mathrm{t}})) \equiv \sigma({}_{\mathrm{R}}U_{\mathrm{t}}) \equiv E(E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}})) \tag{147} $$

Squaring equation 147, it is

$$ \sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 \equiv E({}_{\mathrm{R}}U_{\mathrm{t}} - E({}_{\mathrm{R}}U_{\mathrm{t}}))^2 \equiv E(E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}))^2 \tag{148} $$

According to Kolmogroff [see 66, p. 42], it is easy to calculate that $\sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 \equiv \sigma({}_{\mathrm{R}}U_{\mathrm{t}}) \times \sigma({}_{\mathrm{R}}U_{\mathrm{t}}) \equiv E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right) - (E({}_{\mathrm{R}}U_{\mathrm{t}}))^2$. In general, we obtain

$$ \sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 \equiv E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right) - (E({}_{\mathrm{R}}U_{\mathrm{t}}))^2 \equiv E(E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}))^2 \tag{149} $$

However, equation 149 can be simplified further. The expectation value of ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ is defined (see equation 16) as $E({}_{\mathrm{R}}U_{\mathrm{t}}) \equiv p({}_{\mathrm{R}}U_{\mathrm{t}}) \times ({}_{\mathrm{R}}U_{\mathrm{t}})$. The expectation value of ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}{}^2$ is defined (see equation 18) as $E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right) \equiv p({}_{\mathrm{R}}U_{\mathrm{t}}) \times \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right) \equiv p({}_{\mathrm{R}}U_{\mathrm{t}}) \times ({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}U_{\mathrm{t}})$. Equation 149 changes to

$$ \begin{aligned} \sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 &\equiv (p({}_{\mathrm{R}}U_{\mathrm{t}}) \times ({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}U_{\mathrm{t}})) - (p({}_{\mathrm{R}}U_{\mathrm{t}}) \times ({}_{\mathrm{R}}U_{\mathrm{t}}))^2 \\ &\equiv E(E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}))^2 \end{aligned} \tag{150} $$

and equally to

$$ \begin{aligned} \sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 &\equiv \left(p({}_{\mathrm{R}}U_{\mathrm{t}}) \times \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right)\right) - \left(p({}_{\mathrm{R}}U_{\mathrm{t}})^2 \times ({}_{\mathrm{R}}U_{\mathrm{t}})^2\right) \\ &\equiv E(E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}))^2 \end{aligned} \tag{151} $$

Equation 151 can be simplified as

$$ \begin{aligned} \sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 &\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right) \times \left(p({}_{\mathrm{R}}U_{\mathrm{t}}) - p({}_{\mathrm{R}}U_{\mathrm{t}})^2\right) \\ &\equiv E(E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}))^2 \end{aligned} \tag{152} $$

or as

$$ \begin{aligned} \sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 &\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right) \times (p({}_{\mathrm{R}}U_{\mathrm{t}}) \times (1 - p({}_{\mathrm{R}}U_{\mathrm{t}}))) \\ &\equiv E(E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}))^2 \end{aligned} \tag{153} $$

We rearrange equation 153 further. It is

$$ \begin{aligned} \sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 &\equiv {}_{\mathrm{R}}U_{\mathrm{t}} \times (p({}_{\mathrm{R}}U_{\mathrm{t}}) \times {}_{\mathrm{R}}U_{\mathrm{t}} \times (1 - p({}_{\mathrm{R}}U_{\mathrm{t}}))) \\ &\equiv E(E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}))^2 \end{aligned} \tag{154} $$

Equation 154 simplifies (see definition 9, equation 17) as

$$ \begin{aligned} \sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 &\equiv ({}_{\mathrm{R}}U_{\mathrm{t}} \times p({}_{\mathrm{R}}U_{\mathrm{t}})) \times E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}) \\ &\equiv E(E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}))^2 \end{aligned} \tag{155} $$

Furthermore, equation 155 simplifies (see definition 9, equation 16) as

$$ \begin{aligned} \sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 &\equiv E({}_{\mathrm{R}}U_{\mathrm{t}}) \times E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}) \\ &\equiv E(E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}))^2 \end{aligned} \tag{156} $$

At the end, it is

$$ \sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2 \equiv E({}_{\mathrm{R}}U_{\mathrm{t}}) \times E({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}) \tag{157} $$

In other words, our conclusion is true.

***Quod erat demonstrandum*.**

Remark 4. *The variance is determined by the relationship between something and its own other as* $\sigma({}_RU_t)^2 \equiv E({}_RU_t) \times E({}_R\underline{U_t})$ *and appears to be equally a measure of the inner contradiction between something and its own other too.* ***The greater the variance, the greater the inner contradiction*** *between something and its own other, between the rich and the non-rich, between the healthy and the non-healthy, between the local and the non-local et cetera. The expectation value of the anti* ${}_RU_t$*, denoted as* ${}_R\underline{U_t}$*, of sets follows as* $E({}_R\underline{U_t}) \equiv \dfrac{\sigma({}_RU_t)^2}{E({}_RU_t)}$.

E. The probability of single events

Theorem 5 (The probability of single events)**.** *The complex conjugate is of use to find the probability of a single event. A wave-function which defines the probability amplitude may be determined as a complex function like* $\Psi({}_RU_t) \equiv (a({}_RU_t) + (i \times b({}_RU_t)))$ *while the complex conjugate of the wave-function is determined as* $\Psi^*({}_RU_t) \equiv (a({}_RU_t) - (i \times b({}_RU_t)))$ *The probability of a single event is defined in terms of the complex conjugate as*

$$\begin{aligned}\Psi({}_RU_t) \times \Psi^*({}_RU_t) &\equiv p({}_RU_t)\\ &\equiv (a({}_RU_t) + (i \times b({}_RU_t))) \times (a({}_RU_t) - (i \times b({}_RU_t)))\\ &\equiv \left(a({}_RU_t)^2\right) - \left(i^2 \times b({}_RU_t)^2\right)\\ &\equiv \left(a({}_RU_t)^2\right) + \left(b({}_RU_t)^2\right)\end{aligned} \tag{158}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{159}$$

is true, **then** the conclusion

$$\Psi({}_\mathrm{R}U_\mathrm{t}) \times \Psi^*({}_\mathrm{R}U_\mathrm{t}) \equiv \left(a({}_\mathrm{R}U_\mathrm{t})^2\right) + \left(b({}_\mathrm{R}U_\mathrm{t})^2\right) \tag{160}$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{161}$$

is true. Multiplying this premise (i. e. axiom) by$\Psi({}_\mathrm{R}U_\mathrm{t}) \times \Psi^*({}_\mathrm{R}U_\mathrm{t})$ it is

$$\Psi({}_\mathrm{R}U_\mathrm{t}) \times \Psi^*({}_\mathrm{R}U_\mathrm{t}) \equiv \Psi({}_\mathrm{R}U_\mathrm{t}) \times \Psi^*({}_\mathrm{R}U_\mathrm{t}) \tag{162}$$

The wave-function is determined as a complex function like $\Psi({}_\mathrm{R}U_\mathrm{t}) \equiv (a({}_\mathrm{R}U_\mathrm{t}) + (i \times b({}_\mathrm{R}U_\mathrm{t})))$ while the complex conjugate of the wave-function is determined as $\Psi^*({}_\mathrm{R}U_\mathrm{t}) \equiv (a({}_\mathrm{R}U_\mathrm{t}) - (i \times b({}_\mathrm{R}U_\mathrm{t})))$. Substituting these relationships into equation 162 we obtain

$$\Psi({}_\mathrm{R}U_\mathrm{t}) \times \Psi^*({}_\mathrm{R}U_\mathrm{t}) \equiv (a({}_\mathrm{R}U_\mathrm{t}) + (i \times b({}_\mathrm{R}U_\mathrm{t}))) \times (a({}_\mathrm{R}U_\mathrm{t}) - (i \times b({}_\mathrm{R}U_\mathrm{t}))) \tag{163}$$

or

$$\begin{aligned}\Psi({}_\mathrm{R}U_\mathrm{t}) \times \Psi^*({}_\mathrm{R}U_\mathrm{t}) &\equiv p({}_\mathrm{R}U_\mathrm{t})\\ &\equiv \left(a({}_\mathrm{R}U_\mathrm{t})^2\right) - \left(i^2 \times b({}_\mathrm{R}U_\mathrm{t})^2\right)\\ &\equiv \left(a({}_\mathrm{R}U_\mathrm{t})^2\right) + \left(b({}_\mathrm{R}U_\mathrm{t})^2\right)\end{aligned} \tag{164}$$

Quod erat demonstrandum.

F. Anti Chebyshev - The Chebyshev equality

Theorem 6 (Anti Chebyshev - The Chebyshev equality)**.** *The Pafnuty Lvovich Chebyshev's (1821 – 1894) inequality (also called the Irénée-Jules Bienaymé [25] (1796 – 1878) – Chebyshev inequality) was proved by Chebyshev [98] in 1867 and later by his student Andrey Markov (1856–1922) in his 1884 Ph.D. thesis. Chebyshev's inequality [see 66, p. 42] is defined as*

$$p\left(|\,{}_RU_t - E({}_RU_t)\,| \geq \sqrt[2]{E({}_RU_t{}^2)}\right) \leq \frac{\sigma({}_RU_t)^2}{E({}_RU_t{}^2)} \tag{165}$$

However, Chebyshev's inequality [see 66, p. 42] in this form provides only an approximate value of ***the exact probability of a single event****. The exact value of the probability of a single event (***Chebyshev's equality***) is given by*

$$p({}_RU_t) \equiv 1 - \frac{\sigma({}_RU_t)^2}{E({}_RU_t{}^2)} \tag{166}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{167}$$

is true, **then** the conclusion

$$p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \equiv 1-\frac{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})^2}{E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right)} \tag{168}$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{169}$$

is true. Multiplying this premise (i. e. axiom) by the variance $\sigma({}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}})^2$

$$\sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)^2 \equiv \sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)^2 \tag{170}$$

Equation 170 can be rearranged (see definition 9, equation 19) as

$$E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right)-\left(E\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)^2 \equiv \sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)^2 \tag{171}$$

or as

$$E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right) \equiv \left(E\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)^2+\sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)^2 \tag{172}$$

The normalised form of the variance follows as

$$\frac{\left(E\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)^2}{E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right)}+\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)^2}{E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right)} \equiv +1 \tag{173}$$

Rearranging equation 173, it is

$$\frac{\left(E\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)^2}{E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right)} \equiv 1-\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)^2}{E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right)} \tag{174}$$

Equation 174 simplifies (see definition 7, equation 12) as

$$p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \equiv 1-\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)^2}{E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^2\right)} \tag{175}$$

Quod erat demonstrandum.

G. The causal relationship k

Theorem 7 (Causal relationship k)**.** *Thus far, let $p({}_RU_t)$ represent the probability from the point of view of a stationary observer R of a certain cause ${}_RU_t$, i. e. a random variable or a quantum mechanical observable or a cluster inside a set, at a certain Bernoulli trial t. Let $E({}_RU_t{}^2)$ denote the expectation value of the cause ${}_RU_t{}^2$. Let $E({}_RU_t)$ denote the expectation value of the cause ${}_RU_t$. Let $\sigma({}_RU_t)$ denote the standard deviation of the cause ${}_RU_t$. Let $\sigma({}_RU_t)^2$ denote the variance of the cause ${}_RC_t$. Let $p({}_RW_t)$ represent the probability from the point of view of a stationary observer R of its own effect ${}_RW_t$, i. e. a random variable or a quantum mechanical observable or a cluster inside a set, at a certain Bernoulli trial t. Let $E({}_RW_t{}^2)$ denote the expectation value of the effect ${}_RW_t{}^2$. Let $E({}_RW_t)$ denote the expectation value of the effect ${}_RW_t$. Let $\sigma({}_RW_t)$ denote the standard deviation of the effect ${}_RW_t$. Let $\sigma({}_RW_t)^2$ denote the variance of the effect ${}_RW_t$. Let $\sigma({}_RU_t, {}_RW_t)$ denote the co-variance of cause ${}_RU_t$ and effect ${}_RW_t$. The causal relationship, denoted as $k\left({}_RU_t,{}_RW_t\right)$, inside a sets can be calculated as*

$$\begin{aligned}
k\left({}_RU_t,{}_RW_t\right) &\equiv \frac{\sigma\left({}_RU_t,{}_RW_t\right)}{\sqrt[2]{\sigma\left({}_RU_t\right)^2\times\sigma\left({}_RW_t\right)^2}}\\
&\equiv \frac{\sigma\left({}_RU_t,{}_RW_t\right)}{\sigma\left({}_RU_t\right)\times\sigma\left({}_RW_t\right)}\\
&\equiv \frac{\left({}_RU_t\times{}_RW_t\right)\times\left(p\left({}_RU_t,{}_RW_t\right)-\left(p\left({}_RU_t\right)\times p\left({}_RW_t\right)\right)\right)}{\sqrt[2]{\left(\left({}_RU_t{}^2\right)\times\left(p\left({}_RU_t\right)\times\left(1-p\left({}_RU_t\right)\right)\right)\times\left({}_RW_t{}^2\right)\times\left(p\left({}_RW_t\right)\times\left(1-p\left({}_RW_t\right)\right)\right)\right)}}\\
&\equiv \frac{\left({}_RU_t\times{}_RW_t\right)\times\left(p\left({}_RU_t,{}_RW_t\right)-\left(p\left({}_RU_t\right)\times p\left({}_RW_t\right)\right)\right)}{\left({}_RU_t\times{}_RW_t\right)\times\sqrt[2]{\left(\left(p\left({}_RU_t\right)\times\left(1-p\left({}_RU_t\right)\right)\right)\times\left(p\left({}_RW_t\right)\times\left(1-p\left({}_RW_t\right)\right)\right)\right)}}\\
&\equiv \frac{\left(p\left({}_RU_t,{}_RW_t\right)-\left(p\left({}_RU_t\right)\times p\left({}_RW_t\right)\right)\right)}{\sqrt[2]{\left(\left(p\left({}_RU_t\right)\times\left(1-p\left({}_RU_t\right)\right)\right)\times\left(p\left({}_RW_t\right)\times\left(1-p\left({}_RW_t\right)\right)\right)\right)}}
\end{aligned} \tag{176}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{177}$$

is true, **then** the conclusion

$$k\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right) \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right)}{\sigma({}_{\mathrm{R}}U_{\mathrm{t}}) \times \sigma({}_{\mathrm{R}}W_{\mathrm{t}})} \tag{178}$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{179}$$

is true. Multiplying this premise (i. e. axiom) by $({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}})$ it is

$$({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}}) \equiv ({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}}) \tag{180}$$

According to equation 21 it is

$${}_{\mathrm{R}}U_{\mathrm{t}} \equiv \frac{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})}{\sqrt[2]{p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \times \left(1 - p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)}} \tag{181}$$

Equation 180 changes to

$$({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}}) \equiv \left(\frac{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})}{\sqrt[2]{p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \times \left(1 - p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)}}\right) \times {}_{\mathrm{R}}W_{\mathrm{t}} \tag{182}$$

According to equation 22 it is

$${}_{\mathrm{R}}W_{\mathrm{t}} \equiv \frac{\sigma({}_{\mathrm{R}}W_{\mathrm{t}})}{\sqrt[2]{p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right) \times \left(1 - p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right)\right)}} \tag{183}$$

Equation 182 changes to

$$({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}}) \equiv \left(\frac{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})}{\sqrt[2]{p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \times \left(1 - p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)}}\right) \times \left(\frac{\sigma({}_{\mathrm{R}}W_{\mathrm{t}})}{\sqrt[2]{p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right) \times \left(1 - p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right)\right)}}\right) \tag{184}$$

According to definition 11, equation 26, it is

$${}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}} \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right)}{\left(p\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \times p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right)\right)\right)} \tag{185}$$

Simplifying equation 184 it is

$$\left(\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right)}{\left(p\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \times p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right)\right)\right)}\right) \equiv \left(\frac{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})}{\sqrt[2]{p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \times \left(1 - p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right)}}\right) \times \left(\frac{\sigma({}_{\mathrm{R}}W_{\mathrm{t}})}{\sqrt[2]{p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right) \times \left(1 - p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right)\right)}}\right) \tag{186}$$

Further rearrangement of equation 186 yields the causal relationship between the cause ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ and the effect ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{t}}$, denoted as $k\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right)$, as

$$\begin{aligned} k\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right) &\equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right)}{\sigma({}_{\mathrm{R}}U_{\mathrm{t}}) \times \sigma({}_{\mathrm{R}}W_{\mathrm{t}})} \\ &\equiv \frac{\left(p\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \times p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right)\right)\right)}{\sqrt[2]{p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \times \left(1 - p\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\right) \times p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right) \times \left(1 - p\left({}_{\mathrm{R}}W_{\mathrm{t}}\right)\right)}} \end{aligned} \tag{187}$$

Quod erat demonstrandum.

IV. DISCUSSION

In point of fact, it has always been this way, various amounts of data of different kind are and have been around us for a long time. However, the nature of inquiry of data by information technologies has changed dramatically and equally the extent to which torrents are captured inexpensively. It is therefore no wonder that an explosion of the amount of data is becoming more and more a challenge than a sound of opportunity knocking. However, data collected are especially of use for testing scientific human-generated hypotheses and other questions too. To exploit the data flood and too address key questions related to data science need many more than before than the strategy **one man, one (statistical) method** is not appropriate enough. There is a drift towards a data-driven decision-making and discovery and human knowledge itself is becoming increasingly data-intensive. Therefore, looking for insights into data demands simple to use and logically consistent mathematical tools and methods. **The causal relationship k as proofed and derived by theorem 7** is one among the many statistical methods which is of use to analyse data for a cause effect relationship. Nonetheless, a well-informed, reasonably observant and circumspect reader might be inclined to believe to recognise the trace of Bravais [31] (1811-1863) - Pearson's (1857-1936) "*product - moment coefficient of correlation*"[44], [79] inside the causal relationship k [3]–[5], [9]–[11]. Whereby, it is vital and of very great importance to bear the fundamental differences between **Bravais - Pearson's product-moment coefficient of correlation** and the causal relationship k in mind. Historically, according to Pearson himself it is necessary to consider that "*The fundamental theorems of correlation were for the first time and almost exhaustively discussed by B r a v a i s ('Analyse mathematique sur les probabilities des erreurs de situation d'un point.' Memoires par divers Savans, T. IX., Paris, 1846, pp. 255-332) nearly half a century ago.*"[79] In this context, neither is it epistemologically justified to elaborate once again on the issue **causation [26] versus correlation, both are not identical** [93] nor does it make any sense to insist on the fact that "*Pearson's philosophy discouraged him from looking too far behind phenomena.*"[46]. Whereas it is essential to consider that the causal relationship k, in contrast to Pearson's product-moment coefficient of correlation [79] or to Pearson's phi coefficient [82], is **defined, derived and valid at every single Bernoulli trial t**. Indeed, this might be a very *small* difference. However, a small difference can make a big impact. In this special context and in any case, this small difference *makes* [14] the difference.

In particular, causal relationship k can help us to resolve the contradiction that has arisen between the quantity of data available and the quality of human knowledge generated. Otherwise, the march into contradictions and conflicts, made possible by enormous new sources of data, will sweep through human lives with no area that is going to be untouched.

V. CONCLUSION

Experimental and non-experimental data can be analysed for causal relationships too.

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