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Determining The hypocycloid area By using the circle of assist

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Abstract — *This paper discusses the hypocycloid area, starting with determining the auxiliary circle to determine the hypocycloid area. Furthermore, to determine the hypocycloid area with the auxiliary circle, it is obtained from the reduction of the area of the circle forming the hypocycloid and the area of the auxiliary circle. From determining the hypocycloid area, the hypocycloid area is obtained for n = 3, n = 4, n = 5 and n = 6 and the relationship between the circle that pertains to the hypocycloid and the circle that forms the hypocycloid for all n.*

Keywords — *Hypocycloid, hypocycloid area, astroid.*

I. INTRODUCE

Hypocycloid was first introduced by Nasir al-din al-tusi in 1247 as a Persian astronomer and mathematician [11, 13]. Hypocycloid is one of the parametric curves, there is some literature that describes the use of parametric curves in engineering, for example analyzing the topological characteristics of the epicycloid structure of the planetary gear type where there is a hypocycloid mechanism using constraints and design criteria to produce a new epicycloid (hypocycloid) mechanism design. and simple [5]. Likewise, the hypocycloid and epicycloid parametric equations can influence the waveform of the velocity reducer through UG three-dimensional entity modeling [15]. There is also literature that describes the expansion of a two-dimensional hypocycloid parametric equation into three dimensions using a spherical coordinate system [10].

Some architectural designs in modern buildings are often found that use curve and surface innovation [1]. Apart from this, there is also the use of hypocycloid and epicycloid parametric equations to create attractive decorative motifs [12]. Meanwhile, the hypocycloid formation process and its parametric equations are discussed in [14] as well as the application of the astroid at the bus door when the door is open. Furthermore, there is literature compiling several theorems relating to hypocycloids and epicycloids, and finding the relationship between hypocycloids, hypotrochoids and polar curves [4]. The hypocycloid area n = 4 which is better known to astroids has been previously determined using the integral [2].

Many articles have discussed hypocycloids, but previous authors only discussed the use of hypocycloids in engineering or art. So, in this paper we will show how to determine the area of the hypocycloid n = 3, n = 4, n = 5 and n = 6 using the auxiliary circle and the relationship between the circle that touches the hypocycloid and the circle with radius a and the circle with radius b.

II. LITERATURE REVIEW

Hypocycloid curve is a curve that results from a fixed point P that is on the circumference of a small circle of radius b that revolves around the inside of a larger circle with radius a, where this curve has a parametric equation [2, 3, 6, 11]. As shown in equations (1) and (2),

$$x = (a-b)cos\theta + bcos\left[\left(\frac{a-b}{b}\right)\theta\right], \tag{1}$$

$$y = (a-b)sin\theta - bsin\left[\left(\frac{a-b}{b}\right)\theta\right]. \tag{2}$$

This equation is a general hypocycloid equation that has existed in the literature [5, 14]. Equations (1) and (2) consider the relationship between the values of a and b as the radius of two circles where a and b are real number parameters [9]. So far it is still limited to $a > b > 0$, it is intended to obtain a hypocycloid curve $n > 2$ and $n\epsilon Z^+$. To calculate the area of the parametric curve *(A)* analytically by calculus, it is known

$$A = \int_{\theta_1}^{\theta_2} g(\theta)\ f`(\theta)\ d\theta, \qquad \theta_1 \le \theta \le \theta_2. \tag{3}$$

This formulation has been used to calculate the area of the astroid written in parametric analytics [2]. The astroid parametric equation (*hypocycloid n = 4*) can be obtained by plugging $n = \frac{a}{b} = 4$ into the general hypocycloid equation, so that

$$x = f(\theta) = \frac{a(3\cos\theta + \cos 3\theta)}{4}, \tag{4}$$

$$y = g(\theta) = \frac{a(3\sin\theta - \sin 3\theta)}{4}. \tag{5}$$

Furthermore, equations (4) and (5) are substituted into equation (3), in order to obtain

$$A = \int_0^{2\pi} \frac{a(3\sin\theta + \sin 3\theta)}{4} \left(\frac{a(-3\sin\theta - \sin 3\theta)}{4}\right) d\theta,$$

$$A = \frac{3}{8}\pi a^2.$$

Based on Figure 1, in [7, 8, 16] the area of the juring and the area of the tembereng namely

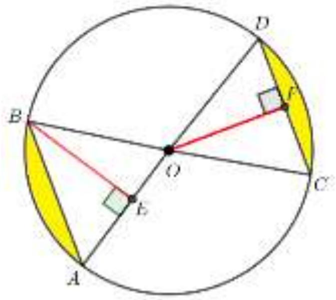

Figure 1. Circle Illustration

$$\text{Area of a section} = \frac{\text{the size of the central}}{360°} \cdot \pi r^2. \tag{6}$$

$$\text{Area of a segment} = \text{Area of a section } AOB - \text{Area of a triangle } AOB. \tag{7}$$

III. RESULTS

Determining the hypocycloid area is to determine the circle through two hypocycloid vertices, one of which is the point forming the hypocycloid, then connect the center point of the circle formed to the hypocycloid corner that is traversed, then determine the area of the circle, the area of the triangle and the area of the formed wall.

In Figure 2, one of the hypocycloid angles already has a point P, where point P is a point that forms a hypocycloid. The first step taken to determine the area of the hypocycloid is to make a point at each hypocycloid angle, so that point A and point D are obtained as in Figure 2.

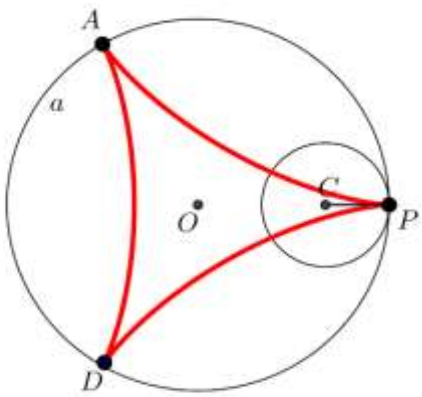

Figure 2. Illustration Point of Angle *Hypocycloid n=3*

The second step, draw a line from point A to point P so that the AP line is formed, as in Figure 3

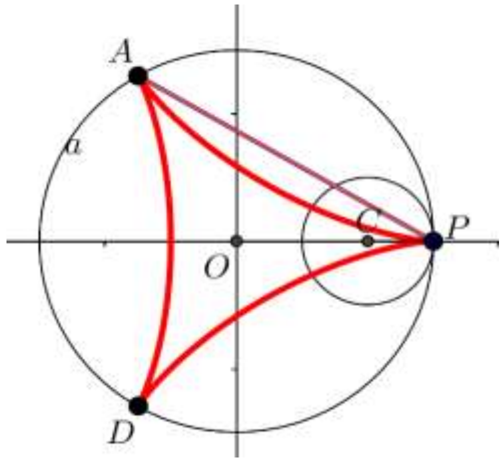

Figure 3. Illustration Line *AP* of *Hypocycloid n=3*

From Figure 3 it is known that the ordinate of point A and the ordinate of point P, is

$$\text{Point } A = \left(-\frac{1}{2}a, \frac{\sqrt{3}}{2}a\right).$$
$$\text{Point } P = (a, 0).$$

After knowing the ordinate of point A and point P the next is to determine the length of the AP line using the formula for the distance between two points, so that it is obtained

$$|AP| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$
$$|AP| = \sqrt{\left(a - \left(-\frac{1}{2}a\right)\right)^2 + \left(0 - \frac{\sqrt{3}}{2}a\right)^2},$$
$$|AP| = \sqrt{\left(\frac{3a}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}a\right)^2},$$
$$|AP| = a\sqrt{3}. \tag{8}$$

Next, make an auxiliary circle whose radius is along the AP and the auxiliary circle through two points, namely point A and point P as in Figure 4.

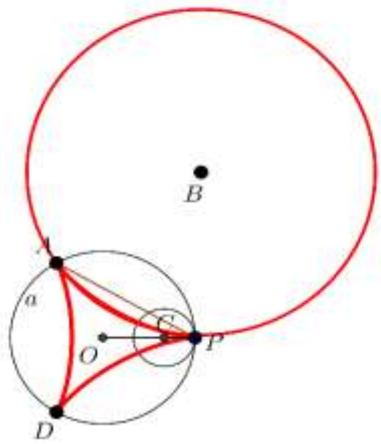

Figure 4. Illustration Circle of Assist *Hypocycloid n=3*

The next step is to draw a line from the center of circle *B* to point *A* and point *P* to form the lines *BA, BP*, section *PBA* and triangle *PBA* as in Figure 5. Because *BA* and *BP* radius of circle *B* so that *BA*=*BP*=*AP*=$a\sqrt{3}$, and angle $PBA = 60°$.

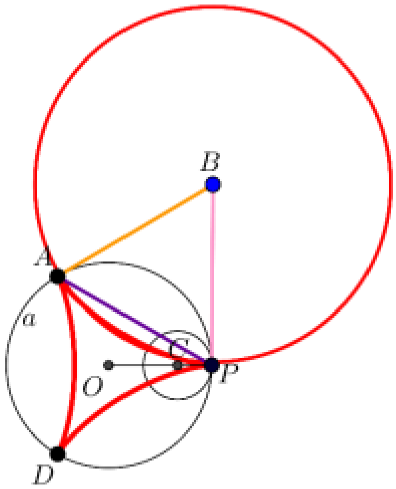

Figure 5. Illustration Section and Angle *PBA* *Hypocycloid n=3*

to find area of section *PBA* use the equation (6), obtained

$$\text{Area of Section } PBA = \frac{60^\circ}{360^\circ} \cdot \pi\left(a\sqrt{3}\right)^2,$$

$$\text{Area of Section } PBA = \frac{a^2}{2}\pi. \tag{9}$$

And area of angle *PBA*, obtained

$$\text{Area of angle } PBA = \frac{1}{2} \cdot a\sqrt{3} \cdot a\sqrt{3} \cdot \sin 60^\circ,$$

$$\text{Area of angle } PBA = \frac{a^2}{4} 3\sqrt{3}. \tag{10}$$

The next step is determine area 1 ilustrated on Figure 6.

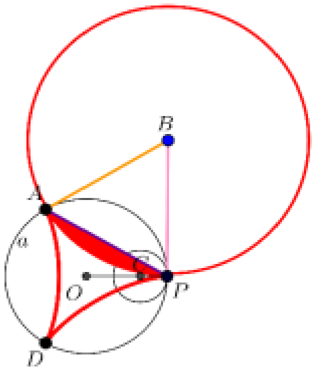

Figure 6. Ilustration Segment 1 *Hypocycloid n=3*

Substitusion equation (9) and aquation (10) in to equation (7) obtained area of segment 1, is

$$\text{Area of Segment 1} = \text{Area of Section } PBA - \text{Area of Angle } PBA,$$

$$\text{Area of Segment 1} = \frac{a^2}{4}\left(2\pi - 3\sqrt{3}\right). \tag{11}$$

The next step draws a line from point *A* to center *O* and from point *P* to center *O* so that the *PO*, *AO*, angle *POA* and section *POA*. Because *PO* and *AO* is radius of circle *O* then *AO*=*PO*=*a*, and *n=3* so that angle *POA*=$\frac{360^\circ}{3} = 120^\circ$, ilustrated of Figure 7. Determine area of section *POA* from Figure 7 used equation (6) so that obtained

$$\text{Area of Section } POA = \frac{120^\circ}{360^\circ} \cdot \pi a^2,$$

$$\text{Area of Section } POA = \frac{a^2}{3}\pi. \tag{12}$$

And area of triangle *POA*, is

$$\text{Area of Triangle } POA = \frac{1}{2} \cdot a \cdot a \cdot \sin 120^\circ,$$

$$\text{Area of Triangle } POA = \frac{a^2}{4}\sqrt{3}. \tag{13}$$

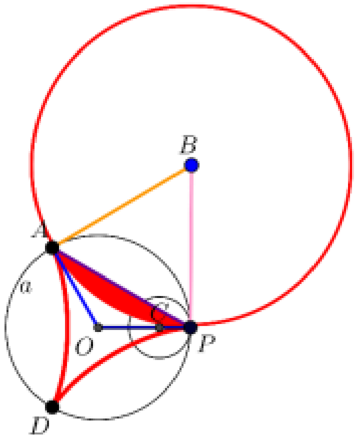

Figure 7. Ilustration Secton and Angle *POA Hypocycloid n=3*

After obtaining the area of the *POA* and area of triangle *POA*, the next find area of segment 2 on Figure 8.

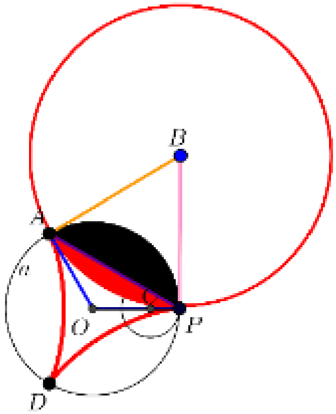

Figure 8. Ilustration Segment 2 *Hypocycloid n=3*

Substituting equation (12) and equation (13) into equation (7) is obtained

$$\text{Area of Segment 2} = \text{Area of Section } POA - \text{Area of Triangle } POA,$$

$$\text{Area of Segment 2} = \frac{a^2}{12}\left(4\pi - 3\sqrt{3}\right). \quad (14)$$

Based on Figure 8, determining the hypocycloid area is obtained by subtracting the area of the radius a and the results of the total area of tembereng 1 and area of tembereng 2 that have been obtained in equations (11) and (14). So that the hypocycloid area is obtained

$$\mathrm{L}_{H3} = \text{Area of Circle} - 3(\text{Area of Segment 1} + \text{Area of Segment 2}),$$

$$\mathrm{L}_{H3} = \pi a^2 - 3\left[\left(\frac{a^2}{4}\left(2\pi - 3\sqrt{3}\right)\right) - \left(\frac{a^2}{12}\left(4\pi - 3\sqrt{3}\right)\right)\right],$$

$$\mathrm{L}_{H3} = \pi a^2 - \left(\frac{10a^2\pi - 12a^2\sqrt{3}}{4}\right),$$

$$\mathrm{L}_{H3} = \frac{a^2}{2}\left(6\sqrt{3} - 3\pi\right).$$

In the same way it is obtained

$$\mathrm{L}_{H4} = \frac{a^2}{3}\left(6\sqrt{3} + 6 - 4\pi\right).$$

$$\mathrm{L}_{H5} = \frac{a^2}{12}\left(20{,}72823\sqrt{3} + 28{,}53 - 13{,}81881\pi\right).$$

$$\mathrm{L}_{H6} = a^2\left(3\sqrt{3} - \pi\right).$$

Furthermore, to determine the relationship of the circle which pertains to the hypocycloid and the circle that forms the hypocycloid is to determine the length of the radius of the circle which pertains to the hypocycloid. Consider Figure 9.

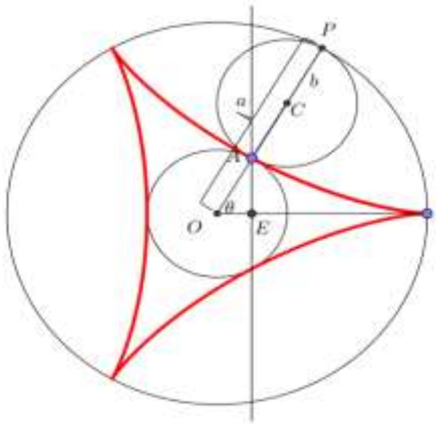

Figure 9. Ilustration the Offending Circle Hypocycloid *n=3*

From Figure 9 it is known that the ordinate x and ordinate y point A is $\left(-\frac{b}{2}, \frac{\sqrt{3}}{2}b\right)$ with the radius of the circle which alludes to the hypocycloid is OA which is symbolized by r_3 and is centered at point (0,0). So that the radius is obtained

$$r_3 = \sqrt{\left(-\frac{b}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}b\right)^2},$$
$$r_3 = b.$$

In the same way it is obtained

$$r_4 = 2b.$$
$$r_5 = 3b.$$
$$r_6 = 4b.$$
$$r_7 = 5b.$$

ext we will look for the general form for the radius length if n is hypocycloid. Based on the length of the radius of the circle which pertains to the hypocycloid for *n=3*, *n=4*, *n=5*, *n=6* dan *n=7* obtaned

$$r_n = (n-2)b, \text{for } n = 3, 4, 5, 6, 7, \cdots$$

IV. CONCLUSION

From determining the area of the hypocycloid using the auxiliary circle it only applies to *n=3*, *n=4*, *n=5* and *n=6* because for *n=7* the formed auxiliary circle does not coincide with the hypocycloid side, as well as for the other *n*. However, there is a relationship between the circle which pertains to the hypocycloid and the circle which forms the hypocycloid for all *n*.

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