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Square Difference Labeling For Tree Related and Degree Splitting Graphs

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Abstract *- In this work, we scrutinize the DS[Z - P_n], DS[braid], DS[Y_{m+1}], DS[moth] graphs affirm Square difference labeling (SDL) and also prove that some tree related graphs are SDG.*

Keywords: *braid, lilly, moth, Square difference graph (SDG), y graph*

AMS classification: *05C78*

I. INTRODUCTION

Throughout the whole, we avail simple, finite and undirected graph and proceed condition and results from [3, 12]. Shiama demonstrated the existence of Square difference labelling in [13, 14]. The approach of degree splitting was bringing out by [10]. Degree splitting of some graphs are established in [2, 7, 8, 9, 11]. Here we examine some tree and cycle associated graphs for SDG. We afford a brief summary of definitions, which are mandatory for the subsisting work.

PRELIMINARIES

Definition 1.1 **[13,6]**

A function of a graph G admits a bijective function $f : V(G) \to \{0,1,2,...p-1\}$ such that the induced function $f^* : E(G) \to N$ for *Square difference graph* is given by $f^*(xy) = |[f(x)]^2 - [f(y)]^2|$, $\forall\, xy \in E(G)$ is distinct.

Definition 1.2.[6]

The *degree splitting* graph is attained from the graph G having at the minimum two nodes of the same degree and T = V $\bigcup_{i=1}^{t} S_i$. by adding vertices $w_1, w_2, \ldots w_t$ and joining to each vertex of S_i, $1 \le i \le t$ and is denoted by $DS(G)$.

Definition 1.3.

Consider G = $(K_{1,n}^{(1)}, K_{1,n}^{(2)})$ is the graph attained by joining apex vertices of stars to a new vertex x.

Definition 1.4.

Lilly graph (I_n) (n ≥ 2) can be acquired by two star graphs $2K_{1,n}$, joining two path graphs $2P_n$, with sharing a common vertex. i.e., $I_n = 2K_{1,n} + 2P_n$.

Defintion 1.5.[6]

Let P_n' and P_n'' be the pair of paths with the vertices v_j and u_j, j = 1, 2, … n – 1, respectively. The graph Z - P_n is procured by joining j^{th} point of path P_n' with $(j + 1)^{th}$ point of path P_n''.

Definition 1.6.[6]

The braid graph B(n), (n ≥ 3), is attained by accompanying j^{th} vertex of P_n' with $(j + 1)^{th}$ vertex of P_n'' and j^{th} vertex of P_n'' with $(j + 2)^{th}$ vertex of P_n' with the new edges for all j = 1, 2, … n-2.

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II. MAIN OUTCOMES

Theorem 2.1

The degree splitting of Z - P_n is Square difference.

Proof

Let G = DS (Z - P_n) with

V(DS (Z - P_n)) = $\{u_j, v_j, w_1, w_2, w_3 / 1 \leq i \leq n\}$

E(DS (Z - P_n)) = $E_1 \cup E_2$, where

$E_1 = \{u_j u_{j+1}, v_j v_{j+1}, v_j u_{j+2} / 1 \leq j \leq n-1, 1 \leq j \leq n-2\}$

$E_2 = \{w_1 u_1, w_1 v_n, w_2 u_j, w_3 v_1, w_3 u_n, w_2 v_j / 1 \leq i \leq n\}$

Obviously, that

$|V| = 2n + 3$ and

$|E| = 5n - 3$

Now, f and f^* are described as

$f(u_j) = 2j - 1$

$f(v_j) = 2(j - 1)$

$f(w_1) = 2n$

$f(w_2) = 2n + 1$

$f(w_3) = 2n + 3$

$f^*(u_j u_{j+1}) = 8j \equiv 0 (\text{mod } 8)$

$f^*(v_j v_{j+1}) = 8j - 4 \equiv 0 (\text{mod } 4)$

$f^*(v_j u_{j+2}) = 12j - 3$

$f^*(w_1 u_1) = 4n^2 - 1$

$f^*(w_1 v_n) = 4n^2 - f(v_n)^2$

$f^*(w_2 u_j) = 4(n^2 + n) - 4(j^2 + j)$

$f^*(w_3 v_1) = (2n + 3)^2$

$f^*(w_3 u_n) = (2n + 3)^2 - f(u_n)^2$

$f^*(w_2 v_j) = (2n + 2)^2 - (2j - 2)^2$

Hence all the above functions are distinctive. Therefore, it is verified.

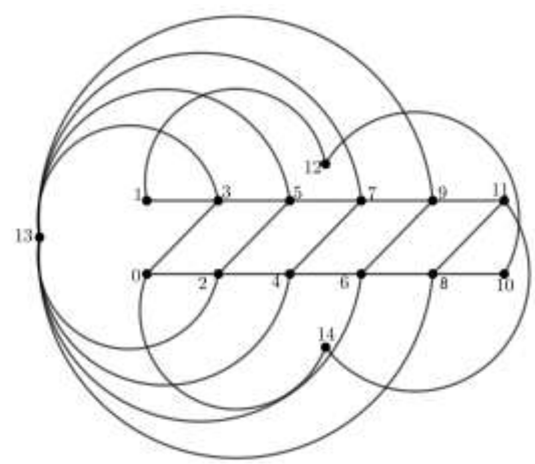

Figure 1. DS(Z – P_6)

Theorem 2.2

DS[B(n)] is SD labeling.

Proof.

Consider G with the vertices u_j, v_j, x, y, z for j = 1,2, …, n and the edges be $u_j u_{j+1}$, $v_j v_{j+1}$, $u_j v_{j+1}$, for j = 1, 2, …, n – 1 ; $v_j u_{j+2}$, for j = 1, 2, …, n – 2 ; xu_1, xu_n, xv_1, xv_n, yu_2, yv_{n-1}; zu_j for $3 \leq j \leq n-1$; zu_j for $2 \leq j \leq n-2$. Obviously, we receive $2n + 3$ nodes and $6n - 5$ edges .

Let illustrate the one – one and onto function g: $V \to \{0, 1, \ldots, 2n + 2\}$as

$g(u_j) = 2j - 1$
$g(v_j) = 2(j - 1)$
$g(x) = 2n$
$g(y) = 2n + 1$
$g(z) = 2n + 2$

and the injective function g^* for the mentioned vertex labeling as

$g^*(u_j u_{j+1}) = 8j$
$g^*(v_j v_{j+1}) = 8j - 4$
$g^*(u_j v_{j+1}) = 4j - 1$
$g^*(v_j u_{j+2}) = 20j + 5$
$g^*(xu_1) = 4n^2 - 1$
$g^*(xu_n) = 4n^2 - f(u_n)^2$
$g^*(xv_1) = 4n^2$
$g^*(xv_n) = 4n^2 - f(v_n)^2$
$g^*(yu_2) = (2n + 1)^2 - 9$
$g^*(yv_{n-1}) = (2n + 1)^2 - f(v_n - 1)^2$
$g^*(zu_j) = (2n + 2)^2 - f(u_j)^2, 3 \leq j \leq n - 1$
$g^*(zv_j) = (2n + 2)^2 - f(v_j)^2, 2 \leq j \leq n - 2$

Clearly the labels defined above are distinct and satisfy the condition of SDL. Hence the theorem.

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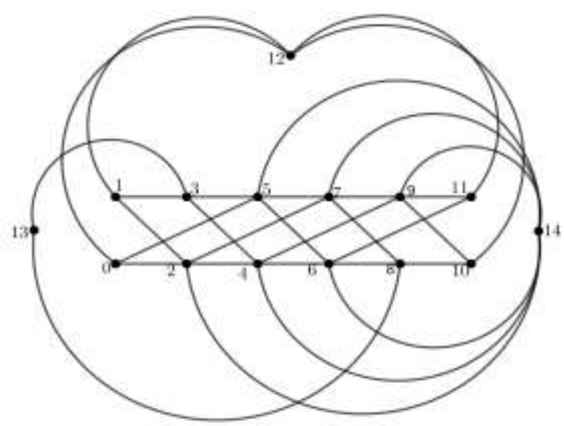

Figure 2. DS[B(6)]

Theorem 2.3.

$DS[Y_{m+1}]$ affirm SDL.

Proof.

Consider $DS[Y_{m+1}]$ be the graph with the vertex set
$V(DS[Y_{m+1}]) = \{u_j, x, y / 1 \leq i \leq n\}$ and
$E(DS[Y_{m+1}])=\{u_j u_{j+1} / 2 \leq j \leq n\}$ $and \{u_1 u_3, xu_j, yu_{n+1}, yu_1, yu_2 / 4 \leq j \leq n\}$
Obviously, we receive $n + 3$ points and $2n$ *lines* respectively.
Specify g and g^* are mentioned below:

$g(u_j) = j - 1$
$g(x) = n + 1$
$g(y) = n + 2$

and

$g^*(u_j u_{j+1}) = 2j - 1$
$g^*(u_1 u_3) = 4$
$g^*(xu_j) = |(n + 1)^2 - (j - 1)^2|$
$g^*(yu_{n+1}) = 4n + 4$
$g^*(yu_1) = (n + 2)^2$

$g^{*}(yu_2) = (n+2)^2 - 1$

both the g and g^{*} are satisfying the limitation of square difference labeling.
Hence DS[Y_{m+1}] admits SDL.

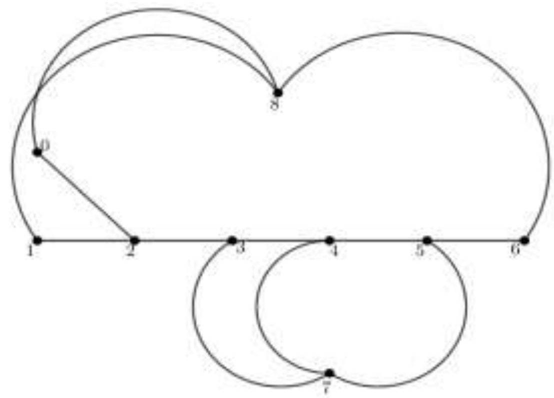

Figure 3. DS[Y_{6+1}]

Theorem 2.4.

DS[moth graph] is SDG.

Proof.

Consider the graph G with
$|V|$ = 8 and
$|E|$ = 11.

Now, consider a function f : V →{0,1,2..., 7} be a bijection defined as follows:

$f(u_j) = j - 1$
$f(x) = 6$
$f(y) = 7$

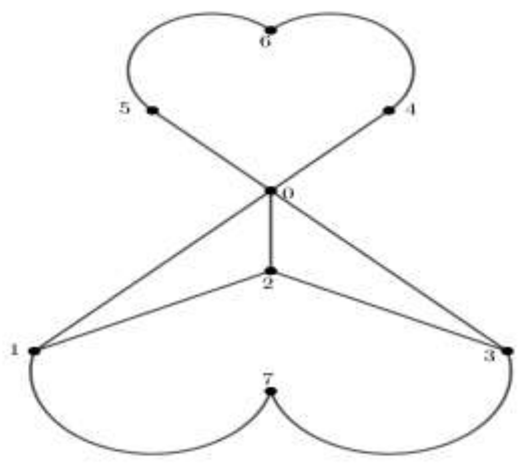

Figure 4. degree splitting of moth graph

By the above function, we receive one - one function f^* as:

$f^{*}(u_1u_2) = 1$
$f^{*}(u_1u_3) = 4$
$f^{*}(u_1u_4) = 9$
$f^{*}(u_1u_5) = 16$
$f^{*}(u_1u_6) = 25$
$f^{*}(u_2u_3) = 3$
$f^{*}(u_3u_4) = 5$
$f^{*}(xu_5) = 20$

$f^*(xu_6) = 11$
$f^*(yu_2) = 48$
$f^*(yu_4) = 40$

It is easily observed that the function satisfies the condition of square difference labeling and also $f^*(uv)$ is an increasing function and disparate.

Theorem 2.5.

Appending two pendant edges to new vertex which subdivides the edge joining the center vertices of $(K_{1,n}^{(1)}, K_{1,n}^{(2)})$ is SDG.

Proof

Let G be the graph with $|V| = 2m + 5$ and $|E| = 2m + 4$.

Define a vertex labeling g: $\{0, 1, 2, \ldots, 2m + 4\}$as

$g(w) = 0$
$g(u) = 1$
$g(v) = 2$
$g(u_i) = i + 2, 1 \le i \le m$
$g(v_i) = m + i + 2$
$g(w') = 2m + 3$
$g(w'') = 2m + 4$

the above labeling satisfies the condition of Square difference labeling and gives the edge labeling g^* as

$g^*(u u_i) = (i + 2)^2 - 1$
$g^*(v v_i) = (m + i + 2)^2 - 4$
$g^*(wu) = 1$
$g^*(wv) = 4$
$g^*(ww') = (2m + 3)^2$
$g^*(ww'') = (2m + 4)^2$

Hence the theorem.

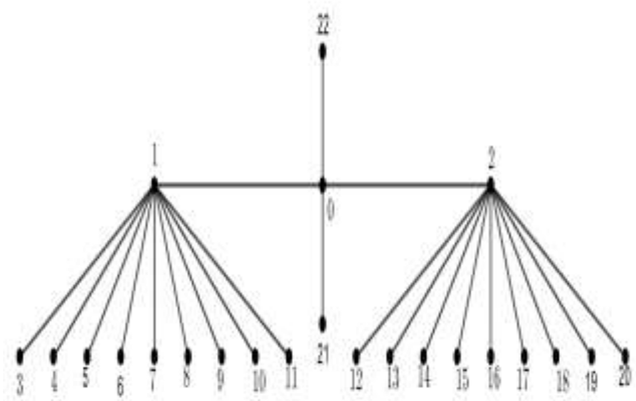

Figure 5. $(K_{1,9}^{(1)}, K_{1,9}^{(2)})$

Theorem 2.6.

$Y_{m+1} \Theta \overline{K_2}$ admits SDL.

Proof.

Let G = $Y_{m+1} \Theta \overline{K_2}$ has $3n + 3$ vertices and $3n + 2$ edges.

We explore the node function f as

$f(v_j) = j - 1$
$f(v_j') = n + 2j - 1$
$f(v_j'') = n + 2j$

and the induced function f^* is defined as

$f^*(v_j v_{j+1}) = 2j - 1, j = 2, 3, \ldots, n$
$f^*(v_1 v_3) = 4$
$f^*(v_j v_j') = |(j-1)^2 - (n+2j-1)^2|$
$f^*(v_j v_j'') = |(j-1)^2 - (n+2j)^2|$
The above-mentioned labeling are distinctive.

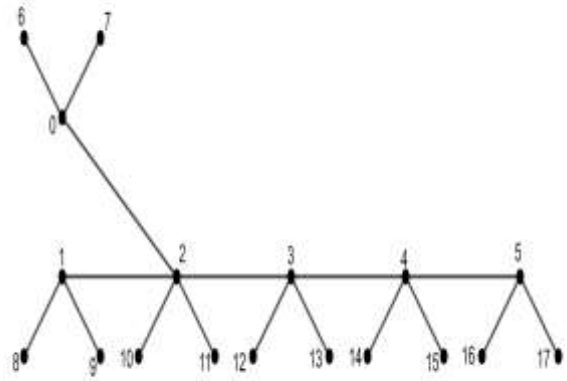

Figure 6. $Y_{5+1}\ \Theta \overline{K_2}$

Theorem 2.7.
Lilly graph I_n admits Square difference labeling.
Proof:
Consider the lilly graph $I_n = 2K_{1,n} + 2P_n$ with the vertex set
$V(I_n) = \{u_i, v_i \ / \ 1 \le i \le n\} \cup \{x_i, y_i, w \ / \ 1 \le i \le n-1\}$
$E(I_n) = \{wu_i, wv_i \ /1 \le i \le n\} \cup \{x_i x_{i+1}, y_i y_{i+1}/1 \le i \le n-2\} \cup \{wu_{n-1}, wv_{n-1}\}$
Clearly $|V| = 4n - 1$
$|E| = 4n - 2$
Now determine the bijective function f as:
$f(u_i) = i - 1$
$f(v_i) = f(u_n) + i$
$f(x_i) = f(v_n) + i$
$f(y_i) = f(x_n) + i$
$f(w) = 4n - 2$

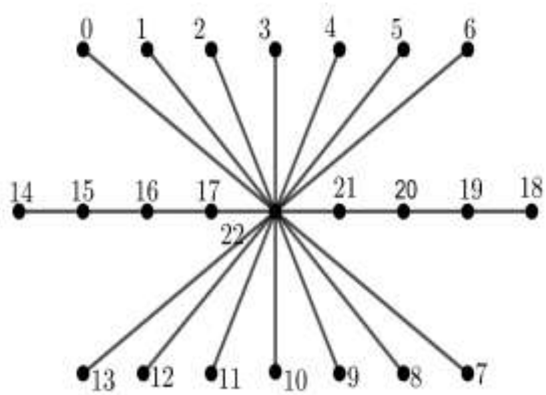

Figure 7. Lilly graph

By the above function, the induced function f^* is given below:
$f^*(wu_i) = (4n-2)^2 - (i-1)^2$
$f^*(wv_i) = (4n-2)^2 - (f(u_n) + i)^2$

$f^*(x_i x_{i+1}) = 4n + 2i - 1$
$f^*(y_i y_{i+1}) = 6n + 2i - 3$
$f^*(wu_{n-1}) = (4n - 2)^2 - f(u_{n-1})^2$
$f^*(wv_{n-1}) = (4n - 2)^2 - f(v_{n-1})^2$

Clearly, the above given labeling satisfies the condition of square difference. Here all the edge labels are distinct and also in increasing sequence. Hence the theorem is verified.

VI. CONCLUSIONS

In this paper, we exemplified that certain tree and cycle relevant graphs are SDG.

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