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Cube Difference Labeling For Generalized Theta Graph

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Abstract *- In this article we prove that generalized theta graph and hanging of generalized theta graph are cube difference graph (CDG).*

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Introduction

Labeling of graphs is the assignment of values to vertices, edges or both subject to certain conditions. There exist different labeling and its detailed report is given in [1]. The CDL was introduced by [4] and also he proved CDL for some standard graphs like paths, cycle, stars and trees. [3] defined generalized theta graph and [2] has proved on pair sum labeling of some special graphs.

I. Preliminaries

1. Consider a graph G said to be CDL if there exist a one to one correspondence f from v to $\{0,1,2,\ldots p-1\}$ such that the procured function f^* given by $f^*(uv) = \left| f(u)^3 - f(v)^3 \right|$ is injective.
2. A generalized theta graph consist a pair of end vertices joined by n internal disjoint paths of length atleast 2, we denote it as $\theta[l^{[m]}]$, $l \geq 3, m \geq 1$.
3. Hanging generalized theta graph is attained by joining the apex of a $\theta[l^{[m]}]$ to an edge connecting a leaf (Pendant Edge) and is denoted by H[$\theta[l^{[m]}]$].

Theorem 2.1
The generalized theta graph admits CDL.

Proof: Let G = $\theta[l^{[m]}]$ with $m(l-1)$ vertices and lm edges. The subsequent cases are observed here:

Case (1): when m is odd

$V(G) = V_1 \cup V_2 \cup V_3$, where

$$V_1 = \left\{ u_{i,j}, v_{i,j} \,/\, 1 \leq i \leq l-1, 1 \leq j \leq \left\lfloor \frac{m}{2} \right\rfloor \right\}$$

$V_2 = \{u, v\}$

$V_3 = \{w_i \,/\, 1 \leq i \leq l-1\}$ and

$E(G) = \bigcup_{k=1}^{4} E_k$, where

$$E_1 = \left\{ u_{i,j}u_{i+1,j} / 1 \le i \le l-2, 1 \le j \le \frac{m}{2} \right\}$$

$$E_2 = \left\{ v_{i,j}v_{i+1,j} / 1 \le i \le l-2, 1 \le j \le \frac{m}{2} \right\}$$

$$E_3 = \{ uu_{1,j}, vu_{l-1,j}, uv_{1,j}, vv_{l-1,j} \}$$

$$E_4 = \{ w_i w_{i+1}, uw_1, vw_{l-1} / 1 \le i \le l-2 \}$$

Define a vertex valued function $f : v \to \{0,1,2,\ldots m(l-1)+1\}$ as follows

$$f(u) = m(l-1)+1$$

$$f(v) = m(l-1)$$

$$f(u_{i,j}) = 2(i-1) + 2(l-1)(j-1)$$

$$f(v_{i,j}) = 2i - 1 + 2(l-1)(j-1), \ for\ 1 \le i \le l-1, 1 \le j \le m/2$$

$$f(w_i) = f(v_{l-1,\lfloor m/2 \rfloor}) + i$$

The edge labels are:

(i) When *l* is odd

It is easily observed that

$$f^*(u_{i,j}u_{i+1,j}) < f^*(vu_{l-1,j}) < f^*(uv_{1,j})$$

Similarly, $f^*(v_{i,j}v_{i+1,j}) < f^*(vv_{l-1,j}) < f^*(uu_{i,j})$ and $f^*(w_i w_{i+1}) < f^*(vw_{l-1})$

Hence $f^*(e_i) \ne f^*(e_j)$ for all $e_i, e_j \in E(G)$.

(ii) When *l* is even

$f^*(u_{i,j}u_{i+1,j})$, $f^*(v_{i,j}v_{i+1,j})$, $f^*(w_i w_{i+1})$ and $f^*(vw_{l-1})$ are same as mentioned in subcase (i) in case (1).

$$f^*(uu_{1,j}) \equiv 0 (\bmod\ 8)$$

$$f^*(vu_{l-1,j}) \equiv 3 (\bmod\ 8)$$

$$f^*(uv_{1,j}) \equiv 0 (\bmod\ 5)$$

$$f^*(vv_{l-1,j}) \equiv 0 (\bmod\ 8)$$

$$f^*(uw_1) \equiv 0 (\bmod\ 8)$$

Case (2): m is even

Let

$$V = \{ u_{i,j}, v_{i,j} / 1 \le i \le l-1, 1 \le j \le m/2 \} \cup \{u, v\}$$

$$E(G) = \bigcup_{k=1}^{3} E_k$$

As defined in case(1)

The vertex labeled function is defined as same in case (1) except for $f(w_i)$

Now the edge label induces f^* by $f^*(uv) = \left| [f(u)]^3 - [f(v)]^3 \right|$

For all $uv \in E$

(i) When *l* is odd

$f^{*}(u_{i,j}u_{i+1,j}) \equiv 0(\text{mod } 8)$

$f^{*}(v_{i,j}v_{i+1,j}) \equiv 2(\text{mod } 8)$

$f^{*}(uu_{1,j}) \equiv 5(\text{mod } 8)$

$f^{*}(uv_{1,j}) \equiv 0(\text{mod } 3)$

$f^{*}(vu_{l-1,j}) \equiv 0(\text{mod } 8)$

$f^{*}(vv_{l-1,j}) \equiv 5(\text{mod } 8)$

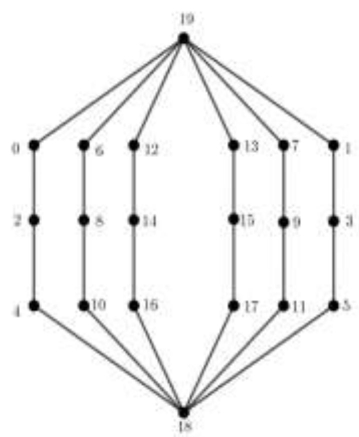
(ii) When l is even

The induced function of $(u_{i,j}u_{i+1,j})$ and $(v_{i,j}v_{i+1,j})$ has same edge labeling as in case (2) subcase(i).

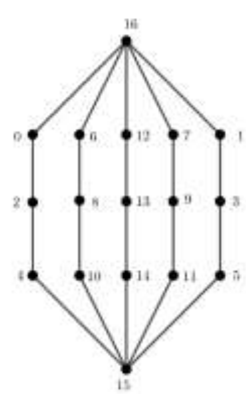
In addition to this the procured function $f^{*}(uu_{1,j})$, $f^{*}(vu_{l-1,j})$, $f^{*}(uv_{1,j})$ and $f^{*}(vv_{l-1,j})$ attains the cube difference of odd and even integer values at end of the vertices gives us odd values which is strictly in increasing sequence.
Thus, all the edge labeling are distinct.

Hence $\theta[l^{[m]}]$ is cube difference graph.

For example, the graph $\theta[4^{[6]}]$ and $\theta[4^{[5]}]$ are given in figure 1(a) and 1(b) respectively.



$\theta[4^{[6]}]$



$\theta[4^{[5]}]$

Figure1: CDL for $\theta[4^{[6]}]$ and $\theta[4^{[5]}]$

Theorem 2.2

The graph H[$\theta[l^{[m]}]$] is a CDG.

Proof: let G be a hanging generalized theta graph with *m(l-1)+3* vertices and *lm+1* edges. Let us follow the two class:

Class 1: when m is even.

Let $V = \left\{ u_{i,j}, v_{i,j} / 1 \le i \le l-1, 1 \le j \le \left\lfloor \frac{m}{2} \right\rfloor \right\}$

$\cup \{u, v\} \cup \{y\}$

And $E(G) = \bigcup_{k=1}^{5} E_k$, where for $1 \le i \le l-1, 1 \le j \le m/2$

$E_1 = \{u_{i,j} u_{i+1,j}\}$

$E_2 = \{v_{i,j} v_{i+1,j}\}$

$E_3 = \{uu_{1,j}, uv_{1,j}\}$ $E_4 = \{vu_{l-1,j}, vv_{l-1,j}\}$

$E_5 = \{yu\}$

And the vertex labeled function f on v is defined as for $1 \le i \le l-1, 1 \le j \le m/2$

f(y) = m(l-1)+2

f(u) = m(l-1)+1

f(v) = m(l-1)

$f(u_{i,j}) = 2(i-1)+2(l-1)(j-1)$

$f(v_{i,j}) = 2i-1+2(l-1)(i-1)$

Class 2: when m is odd

Define $V(G) = V_1 \cup V_2 \cup V_3 \cup V_4$, where V_1, V_2 and V_3 are same as defined in case(1) and $V_4=\{w_i\}$

Let the vertex labeled function are same as mentioned in class 1 added to $f(w_i) = f(u_{l-1,\lfloor m/2 \rfloor}) + i$

Similarly $E(G) = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6$, where E_1, E_2, E_3, E_4 and E_5 are same as in case (1) and $E_6 = \{w_i w_{i+1} / i=1 \text{ to } l-2\}$.

Now there exist a induced function f^* for the edge labeling which are labeled as above discussed in theorem1 except for the apex $f^*(yu) \equiv 1 (\mathrm{mod}\ 6)$. Thus the above labeling are distinct. Hence the H[$\theta[l^{[m]}]$] admits CDL.

II. Conclusion

We have proved that the $\theta[l^{[m]}]$ and H[$\theta[l^{[m]}]$] admits CDL.

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