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Complete Ideal and n-Ideal of BN-algebras

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Abstract. *In this paper, the notion of complete ideal and n-ideal of BN-algebra are introduced and some of related properties are investigated. Also, we discuss the concept of complete ideal and n-ideal of BN-homomorphism and some of their properties are obtained. In addition, we gave some propositions that explained some relationships between these ideals types*.

Keyword. *BN-algebra, ideal, complete ideal, n-ideal, subalgebra*

I. INTRODUCTION

J. Neggers and H. S. Kim [10] introduce a new algebraic structure is called a *B*-algebra. Furthermore, C. B. Kim and H. S. Kim introduce *BG*-algebra [6], which is the generalization of *B*-algebra. Some of types algebras, such that *BM*-algebra [7] and *BN*-algebra [8] are two specializations of *B*-algebra. The concept of homomorphism is also studied in abstract algebra. A map $\psi : A \to B$ is called a *BN*-homomorphism if $\psi(x * y) = \psi(x) * \psi(y)$ for all $x, y \in A$, where A and B are two *BN*-algebras. The kernel of ψ denoted by ker ψ is defined to be the set $\{x \in A : \psi(x) = 0_B\}$. A *BN*-homomorphism ψ is called a *BN*-monomorphism, *BN*-epimorphism, or *BN*-isomorphism if one-one, onto, or a bijection, respectively. Kim [8] also discuss the concept of coxeter algebra. A coxeter algebra is a non-empty set X with a constant 0 and a binary operation " $*$ " satisfying the following axioms: (B1) $x * x = 0$, (B2) $x * 0 = x$, and $(x * y) * z = x * (y * z)$ for all $x, y, z \in X$.

Fitria et al. [3] discuss the concept of prime ideal of *B*-algebra. The results define an ideal and a prime ideal of *B*-algebra and some of their properties are investigated. A non-empty subset I of *B*-algebra X is called an ideal of X if it satisfies $0 \in X$ and if $y \in I$, $x * y \in I$ implies $x \in I$ for any $x, y \in X$. Moreover, I is called a prime ideal of X if it satisfies $A \cap B \subseteq I$, then $A \subseteq I$ or $B \subseteq I$ for any A and B are two ideals of X. The concept of ideal also discussed in *BN*-algebra by Dymek and Walendziak [2]. They obtain the definition of ideal in *BN*-algebra is equivalent to *B*-algebra, but some of their properties are different. Also, the properties of kernel are obtained, such that the kernel ψ is an ideal in *X*. In addition, Dymek and Walendziak investigate the kernel ψ of *BN*-algebra to *BM*-algebra, such that obtained kernel ψ be a normal ideal of *BN*-algebra.

The concepts of ideals of *B*-algebras are discussed by Abdullah [1], those are a complete ideal (briefly *c*-ideal) and an *n*-ideal in *B*-algebras. The results define a *c*-ideal and an *n*-ideal in *B*-algebra, and some of related properties are investigated. They obtain every normal of *B*-algebra is both *c*-ideal and *n*-ideal. Using the same ideas as previous studies [1] and [2], the concepts of *c*-ideal and *n*-ideal in *B*-algebras to *BN*-algebra will be applied.

The objective of this paper is to construct the concept of complete ideal and *n*-ideal of *BN*-algebras, and then investigate complete ideal and *n*-ideal of normal ideal and *BN*-homomorphism. Finally, we study relationship between these ideals types.

II. PRELIMINARIES

In this section, we recall the notion of *B*-algebra, *BM*-algebra, and *BN*-algebra and review some properties which we will need in the next section. Some definitions and theories related to *c*-ideal and *n*-ideal of *BN*-algebra that have been discussed by several authors [1, 2, 3, 6, 7, 8, 10] will also be presented.

Definition 2.1. [10] A *B*-algebra is a non-empty set X with a constant 0 and a binary operation " $*$ " satisfying the following axioms:

(*B1*) $x * x = 0$,
(*B2*) $x * 0 = x$,
(*B3*) $(x * y) * z = x * \big(z * (0 * y)\big)$,
for all $x, y, z \in X$.

A non-empty subset S of *B*-algebra $(X ; *, 0)$ is called a subalgebra of X if $x * y \in S$, for all $x, y \in S$.

Definition 2.2. [8] An algebra $(X ; *, 0)$ is said to be 0-*commutative* if $x * (0 * y) = y * (0 * x)$ for any $x, y \in X$.

Example 1. Let $A = \{0, 1, 2\}$ be a set with Cayley's table as seen in Table 1.

Table 1: Cayley's table for (A; $*$, 0)

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

From Table 1 we get the value of main diagonal is 0, such that A satisfies $x * x = 0$ for all $x \in A$ (*B1* axiom). In the second column, we see that for all $x \in A$, $x * 0 = x$ (*B2* axiom) and it also satisfies $(x * y) * z = x * (z * (0 * y))$, for all $x, y, z \in A$. Hence, (A; $*$, 0) be a B-algebra. It easy to check that (A; $*$, 0) satisfies $x * (0 * y) = y * (0 * x)$, for all $x, y, z \in A$. Hence, A be a 0-*commutative* B-algebra.

Definition 2.3. [3] A non-empty subset I of B-algebra (X; $*$, 0) is called an ideal of X if it satisfies

(i). $0 \in I$,

(ii). $x * y \in I$ and $y \in I$ imply $x \in I$ for any $x, y \in X$.

Definition 2.4. [1] A non-empty subset I of B-algebra (X; $*$, 0) is said to be *complete* ideal (briefly c-ideal) of X if it satisfies

(i). $0 \in I$,

(ii). $x * y \in I$ for all $y \in I$ such that $y \neq 0$ implies $x \in I$.

Definition 2.5. [1] A non-empty subset I of B-algebra (X; $*$, 0) is said to be n-ideal of X if it satisfies

(i). $0 \in I$,

(ii). $x * y \in I$ and $y \in I$ implies there exist $n \in Z^+$, $x^n \neq 0$ such that $x^n \in I$, where $x^n = \big((x * x) * x\big) * x * \dots * x$.

Definition 2.6. [6] A BG-algebra is a non-empty set X with a constant 0 and a binary operation " $*$ " satisfying the following axioms:

(*B1*) $x * x = 0$,
(*B2*) $x * 0 = x$,
(*BG*) $(x * y) * (0 * y) = x$,
for all $x, y \in X$.

Example 2. Let $X = \{0, 1, 2, 3\}$ be a set with Cayley's table as seen in Table 2.

Table 2: Cayley's table for (X ; $*$, 0)

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then, it can be shown that (X; $*$, 0) is a BG-algebra.

Definition 2.7. [7] A BM-algebra is a non-empty set X with a constant 0 and a binary operation "$*$" satisfying the following axioms:

(*A1*) $x * 0 = x$,
(*A2*) $(z * x) * (z * y) = y * x$ for all $x, y, z \in X$.

Example 3. Let $X = \{0, 1, 2\}$ be a set with Cayley's table as seen in Table 3.

Table 3: Cayley's table for $(X; *, 0)$

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

From the Table 3, we have the values in the second column satisfying $x * 0 = x$ for all $x,\ y \in X$ (*A1* axiom) and they also satisfying $(z * x) * (z * y) = y * x$ for all $x, y, z \in X$ (*A2* axiom). Hence, $(X; *, 0)$ is a *BM*-algebra.

Theorem 2.8. [7] Every *BM*-algebra is a *B*-algebra.

Proof. Theorem 2.8 has been proved in [7].

The converse of Theorem 2.8 does not hold in general.

Proposition 2.9. [7] If $(A; *, 0)$ be a *BM*-algebra, then

(i). $x * (x * y) = y$,

(ii). If $x * y = 0$, then $x = y$,

for all $x, y \in A$.

Proof. Proposition 2.9 has been proved in [7].

Definition 2.10. [8] A *BN*-algebra is a non-empty set X with a constant 0 and a binary operation " $*$ " satisfying the following axioms:

(*BN1*) $x * x = 0$,
(*BN2*) $x * 0 = x$,
(*BN3*) $(x * y) * z = (0 * z) * (y * x)$,
for all $x, y, z \in X$,

Example 4. Let $X = \{0,\ 1,\ 2,\ 3\}$ be a set with Cayley's table as seen in Table 4.

Table 4: Cayley's table for $(X; *, 0)$

$*$	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	1	0	1
3	3	1	1	0

Then, it can be shown that $(X ; *, 0)$ is a *BN*-algebra.

Theorem 2.11. [8] If **(**X ; $*$, 0) is a *BN*-algebra, then X be a 0-*commutative*.

Proof. Theorem 2.11 has been proved in [8].

The converse of Theorem 2.11 does not hold in general.

Definition 2.12. [2] Let $(A; *, 0)$ be a *BN*-algebra. We define a binary relation $\leq$ on A by $x \leq y$ if and only if $x * y = 0$.

It is easy to see that, for any $x \in A$, if $x \leq 0$, then $x = 0$.

Proposition 2.13. [8] If $(A; *, 0)$ be a *BN*-algebra, then

(i). $0 * (0 * x) = x$,

(ii). $0 * (x * y) = y * x$,

(iii). $y * x = (0 * x) * (0 * y)$,

(iv). If $x * y = 0$, then $y * x = 0$,

(v). If $0 * x = 0 * y$, then $x = y$,

for all $x, y \in A$.

Proof. Proposition 2.13 has been proved in [8].

Definition 2.14. [2] A non-empty subset S of *BN*-algebra $(X;\ *,\ 0)$ is called a subalgebra of X if it satisfies $x * y \in S$ for all $x, y \in S$. A non-empty subset *N* of *X* is called a normal if it satisfies $(x * a) * (y * b) \in N$, for any $x * y, a * b \in N$.

Let $(X; *, 0)$ and $(Y; *, 0)$ be two *BN*-algebras. A map $\psi: X \to Y$ is called a *BN*-homomorphism if $\psi(a * b) = \psi(a) * \psi(b)$ for any $a, b \in X$. The kernel of ψ denoted by ker ψ is defined to be $ker\ \psi = \{x \in X: \psi(x) = 0_Y\}$. A *BN*-homomorphism ψ is called a *BN*-monomorphism, *BN*-epimorphism, or *BN*-isomorphism if one-one, onto, or a bijection function, respectively.

Proposition 2.15. [2] Let *A* be a *BN*-algebra and let $S \subseteq A$. Then S is a normal subalgebra of A if and only if S is a normal ideal.

Proof. Proposition 2.15 has been proved in [2].

III. COMPLETE IDEAL OF B*N*-ALGEBRAS

In this section, we get definition of complete ideal briefly *c*-ideal in *BN*-algebra and its properties are obtained. The concept can be extended to the *BN*-homomorphism. Then, we have some of the related properties.

Definition 3.1. A non-empty subset *I* of *BN*-algebra (*X*; $*$, 0) is said to be *complete* ideal (briefly *c*-ideal) of *X* if it satisfies

(i). $0 \in I$,

(ii). $x * y \in I$ for all $y \in I$ such that $y \neq 0$ implies $x \in I$ for any $x, y \in X$.

Example 5. Let $\mathbb{R}$ be the set of real numbers and let $(\mathbb{R}; *, 0)$ be the algebra with the operation $*$ defined by

$$x * y = \begin{cases} x & \text{if } y = 0, \\ y & \text{if } x = 0, \\ 0 & \text{otherwise.} \end{cases}$$

Then, $\mathbb{R}$ is a *BN*-algebra. Moreover, $\{0\}$ and $\mathbb{R}$ be a *c*-ideal of $(\mathbb{R}; *, 0)$.

Example 6. Let $A = \{0,\ 1,\ 2,\ 3\}$ be a set with Cayley's table as seen in Table 5.

Table 5: Cayley's table for $(A; *, 0)$

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	0
2	2	3	0	2
3	3	0	2	0

Then, it can be shown that $(A; *, 0)$ is a *BN*-algebra. We can be prove that $\{0\}$ and *A* are ideals of $(A; *, 0)$, and $\{0\}$, $\{0, 1, 3\}$, and *A* are *c*-ideals of $(A; *, 0)$.

Proposition 3.2. Let $(A; *, 0)$ be a *BN*-algebra and $I \subseteq A$. If I be an ideal then I be a *c*-ideal of A.

Proof. Let I be an ideal of A. Let $x * y \in I$ for all $y \in I$ and $y \neq 0$, then

(i) For $I = \{0\}$, it obviously that I is a *c*-ideal of A.

(ii) For $I \neq \{0\}$ there exist $y \in I$ such that $y \neq 0$ and $x * y \in I$. Since I is an ideal, then $x \in I$. Thus, I is a *c*-ideal of *A*.

Corollary 3.3. Every a normal ideal of *BN*-algebra is a normal *c*-ideal.

Proof. Let $(A; *, 0)$ be a *BN*-algebra and let I be an ideal of A. By Proposition 3.2 we obtain *I* is a *c*-ideal of *A*. Since *I* is a normal, then *I* is a normal *c*-ideal of *A*.

Proposition 3.4. Let $(A; *, 0)$ be a *BN*-algebra and let I be a *c*-ideal of *A*. If $x \leq y$, for all $y \in I$ and $y \neq 0$, then $x \in I$.

Proof. Let $x \leq y$ for all $y \in I$ and $y \neq 0$, then from Definition 2.12 we have $x * y = 0 \in I$, such that $x \in I$.

Example 7. Let $A = \{0, 1, 2, 3\}$ be a set with Cayley's table as seen in Table 6.

Table 6: Cayley's table for $(A; *, 0)$

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	1	0	1
3	3	1	1	0

Then, (*A*; *, 0) is a *BN*-algebra and $\{0\}$, $\{0,2\}$, $\{0,3\}$, $\{0,2,3\}$, $\{0,1,2,3\}$ are all of *c*-ideals of *A*. Moreover, *I* = {0, 2, 3} be a *c*-ideal of *A*, but it is not a subalgebra of *A*, since $2, 3 \in I$, $2 * 3 = 1 \notin I$ and $S = \{0,1\}$ be a subalgebra of *A*, but it is not a subalgebra of *A*, since $2 * 1 = 1 \in S$ and $1 \in S$, but $2 \notin S$.

Example 8. From *BN*-algebra in Example 3, we have $\{0\}$ and *A* = {0, 1, 2, 3} are *c*-ideals of *A* and it can be shown that *A* is a normal of *A*, and $\{0\}$ is also a normal of *A*, however it does not hold in general.

As an illustration the following example is given.

Example 9. Let $A = \{0, 1, 2, 3\}$ be a set with Cayley's table as seen in Table 7.

Table 7: Cayley's table for $(A; *, 0)$

*	0	1	2	3
0	0	1	2	3
1	1	0	3	0
2	2	3	0	2
3	3	0	2	0

It can be shown that (*A* ; *, 0) be a *BN*-algebra. Then, $\{0\}$ and *A* are *c*-ideals of *A*. *A* is a normal, but $\{0\}$ is not a normal, since $2 * 2 = 0 \in I$ and $1 * 3 = 0 \in I$, however $(2 * 1) * (2 * 3) = 3 * 2 = 2 \notin I$.

Theorem 3.5. Let *A* be a *BN*-algebra and $S \subseteq A$. *S* is a normal subalgebra if and only if it is a normal *c*-ideal.

Proof. Let *S* be a normal subalgebra of *A*, it is clearly that $0 \in S$. If $x * y \in S$ and for all $y \in S$, $y \neq 0$, then $0 * y \in S$. Since *S* be a normal obtained $(x * 0) * (y * y) \in S$ and from *BN1* and *BN2* axioms, we get $(x * 0) * (y * y) = x \in S$. Therefore, *S* is a *c*-ideal of *A* and since *S* be a normal, it clearly that *S* is a normal *c*-ideal of *A*. Conversely, let $x, y \in S$, since *S* be a normal, then $(x * y) * (x * 0) \in S$. From *BN2* axiom we get $0 \in S$ and $x * y \in S$. Thus, it shows that *S* is a normal subalgebra of *A*.

Lemma 3.6. Let *I* be a normal *c*-ideal of *BN*-algebra *A* and $x, y \in A$, then

(i) $x \in I \Rightarrow 0 * x \in I$,

(ii) $x * y \in I \Rightarrow y * x \in I$.

Proof.

(i). Let $x \in I$, then $x = x * 0 \in I$. Since I be a normal, then $(0 * x) * (0 * 0) \in I$. From *BN1* and *BN2* axioms we get $(0 * x) * (0 * 0) = 0 * x \in I$.

(ii). Let $x * y \in I$ and by (i) obtained $0 * (x * y) \in I$. From Proposition 2.13 (ii), we obtain $0 * (x * y) = y * x$, such that $y * x \in I$.

Remark 3.7.

1. The intersection of two c-ideals of *BN*-algebra is a c-ideal of *BN*-algebra.
2. The union of ascending sequence of c-ideal is a c-ideal of *BN*-algebra.

Let $(A; *, 0)$ be a *BN*-algebra. If a self-map f be a homomorphism of A, then $f(0) = f(0 * 0) = f(0) * f(0) = 0$ and $\ker f = \{x \in A: f(x) = 0\}$.

Theorem 3.8. If $f: A \to A$ be a homomorphism of A to itself, then $\ker f$ is a c-ideal of A.

Proof. Let f be a homomorphism of A to itself, then it is clearly that $0 \in \ker f$. If $x * y \in \ker f$ and for all $y \in \ker f$, $y \neq 0$ then

$$\begin{aligned} 0 &= f(x * y) \\ &= f(x) * f(y) \\ &= f(x) * 0 \\ 0 &= f(x), \end{aligned}$$

such that $x \in \ker f$. Thus, we get $\ker f$ is a c-ideal of A.

Remark 3.9. The kernel of a homomorphism is not always a normal c-ideal. Let $(A; *, 0)$ be a *BN*-algebra given in Example 6. Clearly, $c\text{-}id_A: A \to A$ is a homomorphism and the c-ideal $\ker(c\text{-}id_A)$ is not normal of A.

Theorem 3.10. Let $(A; *, 0_A)$ be a *BN*-algebra and let $(B; *, 0_B)$ be a *BM*-algebra. Let $f: A \to B$ be a homomorphism from A into B, then $\ker f$ is a normal c-ideal of A.

Proof. Let f be a homomorphism from A into B. From Theorem 3.8 it follows that $\ker f$ is a c-ideal of A. Let $x, y, a, b \in A$ and $x * y,\ a * b \in \ker f$, then $0_B = f(x * y) = f(x) * f(y)$. By Proposition 2.9 (ii) it follows that $f(x) = f(y)$ and $f(a) = f(b)$, such that

$$\begin{aligned} f[(x * a) * (y * b)] &= f(x * a) * f(y * b) \\ &= [f(x) * f(a)] * [f(y) * f(b)] \\ &= [f(x) * f(a)] * [f(x) * f(a)] \\ &= 0_B. \end{aligned}$$

Then, we get $(x * a) * (y * b) \in \ker f$. Hence, it shows that $\ker f$ is a normal c-ideal of A.

Theorem 3.11. Let $(A; *, 0_A)$ be a *BM*-algebra and let $(B; *, 0_B)$ be a *BN*-algebra. If $f: A \to B$ be a homomorphism of A to B, then $\ker f$ is a c-ideal of A.

Proof. Let f be a homomorphism of A to B, then it is clearly that $0_A \in \ker f$. If $x * y \in \ker f$ and for all $y \in \ker f$, $y \neq 0_A$ then

$$\begin{aligned} 0_B &= f(x * y) \\ &= f(x) * f(y) \\ &= f(x) * 0_B \\ 0_B &= f(x), \end{aligned}$$

such that $x \in \ker f$. Thus, we get $\ker f$ is a c-ideal of A.

IV. n-IDEAL OF BN-ALGEBRA

In this section, we get definition of n-ideal in *BN*-algebra and its properties are obtained. Then, we have some of the related properties.

Definition 4.1. A non-empty subset I of *BN*-algebra $(A; *, 0)$ is said to be n-ideal of A if it satisfies

(i). $0 \in I$, and

(ii). $x * y \in I$ and $y \in I$, there exist $n \in Z^+$, $x^n \neq 0$ such that $x^n \in I$, where $x^n = \big((x * x) * x\big) * x * \dots * x$.

Example 10. Let $(A; *, 0)$ be a *BN*-algebra given in Example 6, then $I = \{0, 1, 3\}$ is an n-ideal of A, since $0 \in I$ and $1 * 3 = 0 \in I$, $3 \in I$, there exist $3 \in Z^+$ such that $1^3 = (1 * 1) * 1 = 0 * 1 = 1 \in I$. It follows that $\{0\}, \{0,1,3\}$, and $\{0,1,2,3\}$ are n-ideals of A.

Proposition 4.2. Let $(A; *, 0)$ be a *BN*-algebra and $I \subseteq A$. If I be an ideal then I be an n-ideal of A.

Proof. Let $x * y \in I$ and $y \in I$. Since I is an ideal of A, then $x \in I$. This shows that I is an n-ideal where $n = 1$. This complete the proof.

The converse of Proposition 4.2 is not true in general.

Corollary 4.3. Every a normal ideal of *BN*-algebra is a normal *n*-ideal.

Proof. Let $(A; *, 0)$ be a *BN*-algebra and let I be an ideal of A. By Proposition 4.2 we obtain *I* is an *n*-ideal of *A*. Since *I* is a normal, then *I* is a normal *n*-ideal of *A*.

Proposition 4.4. Every normal subalgebra S of *BN*-algebra A is a normal *n*-ideal.

Proof. It is directly from Proposition 2.15 and Corollary 4.3.

Theorem 4.5. Let $(A; *, 0)$ be a *BN*-algebra. If $f: A \to A$ be a homomorphism of A to itself, then ker f is a n-ideal of A.

Proof. Let f be a homomorphism of A to itself, then it is clearly that $0 \in \ker f$. If $x * y \in \ker f$ and $y \in \ker f$, then

$$\begin{aligned} 0 &= f(x * y) \\ &= f(x) * f(y) \\ &= f(x) * 0 \\ 0 &= f(x). \end{aligned}$$

Thus, there exist $1 \in Z^+$ such that $x^1 = x \in \ker f$. Thus, ker f is an *n*-ideal of A.

Theorem 4.6. Let $(A; *, 0_A)$ be a *BN*-algebra and let $(B; *, 0_B)$ be a *BM*-algebra. Let $f: A \to B$ be a homomorphism from A into B, then ker f is a normal *n*-ideal of *A*.

Proof. Let f be a homomorphism from A into B. Since every *BM*-algebra is a *BN*-algebra, from Theorem 4.5 it follows that ker f is an *n*-ideal of A. Let $x, y, a, b \in A$ and $x * y,\ a * b \in \ker f$, then $0_B = f(x * y) = f(x) * f(y)$. By Proposition 2.9 (ii) it follows that $f(x) = f(y)$ and $f(a) = f(b)$, such that

$$\begin{aligned} f[(x * a) * (y * b)] &= f(x * a) * f(y * b) \\ &= [f(x) * f(a)] * [f(y) * f(b)] \\ &= [f(x) * f(a)] * [f(x) * f(a)] \\ &= 0_B. \end{aligned}$$

Then, we get $(x * a) * (y * b) \in \ker f$. Hence, it shows that ker f is a normal *n*-ideal of A.

Definition 4.7. A non-empty subset *I* of *BN*-algebra (*A*; $*$, 0) is said to be complete *n*-ideal briefly *c*-*n*-ideal of *A*, if it satisfies

(i). $0 \in I$, and
(ii). $x * y \in I$ for all $y \neq 0 \in I \Rightarrow x^n \neq 0 \in I$ for some $n \in Z^+$.

Proposition 4.8. Every *c*-ideal of *BN*-algebra A is a *c*-*n*-ideal of A.

Proof. Let *I* be a *c*-ideal, then $0 \in I$. Let $x * y \in I$ for all $y \neq 0 \in I$, since *I* is a *c*-ideal, then $x \in I$. It follows that *A* is a complete *n*-ideal of *A.*

V. CONCLUSION

In this paper, the notions of *c*-ideal and *n*-ideal of *BN*-algebra are defined and some of their properties are obtained. Furthermore, we define a *c*-*n*-ideal in *BN*-algebra and we have every *c*-ideal of *BN*-algebra is a *c*-*n*-ideal.

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