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Generalized f-Derivation of BP-Algebras

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Abstract: *In this paper, the notions of generalized (l, r)-derivation, generalized (r, l)-derivation, and generalized derivation of BP-agebra are introduced, and some related properties are investigated. Also, we consider generalized (l, r)-f-derivation, generalized (r, l)-f-derivation, and generalized f-derivation of BP-aljabar, where f be an endomorphism of BP-algebra, and their properties are established in details.*

Keyword*: BP-algebra, (l,r)-derivation, (r,l)-derivation, generalized derivation, generalized f-derivation*

I. INTRODUCTION

In 2006, Kim and Kim [6] introduce the notion of *BM*-algebra, which is a non-empty set *X* with a constant 0 and a binary operation "$*$" denoted by $(X;*,0)$, satisfying the following axioms: $(A1)\ x*0=x$ and $(A2)\ (z*x)*(z*y)=y*x$ for all $x,y,z\in X$. They discuss some properties of *BM*-algebra and relation of *BM*-algebra with any other algebras, such as relation between a *BM*-algebra with a 0-commutative *B*-algebra and a *coxeter* algebra. Then, Ahn and Han [1] introduce the notion of *BP*-algebra, which is a non-empty set *X* with a constant 0 and a binary operation "$*$" denoted by $(X;*,0)$, satisfying the following axioms: $(BP1)\ x*x=0$, $(BP2)\ x*(x*y)=y$, and $(BP3)\ (x*z)*(y*z)=x*y$ for all $x,y,z\in X$. Also, some properties of *BP*-algebra and relation of *BP*-algebra with any other algebras, such as *BF*-algebra are discussed. Then, they discuss a quadratic *BP*-algebra and show that the quadratic *BP*-algebra is equivalent to several quadratic algebras. Furthermore, Zadeh et al. [10] introduce relation between *BP*-algebra and any other algebras, such as *BM*-algebra. They prove that the class of *BP*-algebras and *BM*-algebras are equivalent.

The first time, notion of derivation was introduced in prime ring by Posner in 1957. Then, Ashraf et al. [2] introduce the notion of derivation ring and its application. In the development of abstract algebra, the notion of derivation is also discussed in other algebraic structure, such as *BP*-algebra and the concept of f-derivation was introduced too. Kandaraj and Devi [3] have discussed the concept of f-derivation in BP-algebra and its properties. In the same paper, the notion of composition of f-derivation is defined in BP-algebra and some of related properties are investigated. A new notion of derivation and generalized derivation are introduced by some authors. Sugianti and Gemawati [9] introduce the generalized of derivation in BM-algebra. The results define a derivations, a left-right or (l,r)-derivation, a right-left or (r,l)-derivation in BM-algebra, and construct their properties. Then, research on f-derivation and generalization of f-derivation involving an endomorphism *f* has been discussed by Jana et al. [4] on KUS-algebra and by Kim [5] on BE-algebra.

Based on the same idea in the research of Sugianti and Gemawati [9] in constructing the concept of generalized derivation in BM-algebra and as the development of the research of Kandaraj and Devi [3] who discussed the concept of f-derivation in BP-algebra, this article discusses the concept of generalization of derivation and generalization of f-derivation in BP-algebra, and investigated their properties.

II. PRELIMINARIES

In this section, we recall the notion of *BM*-algebra, derivation and generalized derivation of *BM*-algebra, *BP*-algebra and review some properties that we need in the next section. Some definitions and theories related to the generalized of derivation in *BM*-algebra and *BP*-algebra being discussed by several authors [1, 3, 6, 9] are also presented.

Definition 2.1. [6] A *BM*-algebra is a non-empty set *X* with a constant 0 and a binary operation "$*$" satisfying the following axioms:

(A1) $x*0=x$,
(A2) $(z*x)*(z*y)=y*x$,

for all $x,y,z\in X$.

Example 2.1. Let $X=\{0,a,b\}$ be a set with Cayley's table as seen in Table 2.1.

Table 2.1: Cayley's table for $(X;*,0)$

*	0	a	b
0	0	b	a
a	a	0	b
b	b	a	0

From Table 2.1, we have the values in the second column satisfying $x * 0 = x$, for all $x \in X$ (*A1* axiom) and they also satisfy $(z * x) * (z * y) = y * x$, for all $x, y, z \in X$ (*A2* axiom). Hence, $(X; *, 0)$ is a *BM*-algebra.

Lemma 2.2. [6] If $(X; *, 0)$ is a *BM*-algebra, then
- (i) $x * x = 0$,
- (ii) $0 * (0 * x) = x$,
- (iii) $0 * (x * y) = y * x$,
- (iv) $(x * z) * (y * z) = x * y$,
- (v) $x * y = 0$ if and only if $y * x = 0$ for all $x, y, z \in X$,

for all $x, y, z \in X$.

Proof. Lemma 2.2 has been proved in [7]. ■

Let $(X; *, 0)$ be a *BM*-algebra, we denote $x \wedge y = y * (y * x)$ for all $x, y \in X$.

Definition 2.3. [9] . Let $(X; *, 0)$ be a *BM*-algebra. By an (l, r)-derivation of X, a self-map d of X satisfies the identity $d(x * y) = (d(x) * y) \wedge (x * d(y))$, for all $x, y \in X$. If X satisfies the identity $d(x * y) = (x * d(y)) \wedge (d(x) * y)$, for all $x, y \in X$, then we say that d is an (r, l)-derivation. Moreover, if d is both an (l, r)-derivation and an (r, l)-derivation, we say that d is a derivation of X.

Definition 2.4. [9] Let X be a *BM*-algebra. A mapping $D : X \to X$ is called a generalized (l, r)-derivation if there exists an (l, r)-derivation $d : X \to X$ such that $D(x * y) = (D(x) * y) \wedge \left(x * d(y)\right)$ for all $x, y \in X$, if there exists an (r, l)-derivation $d : X \to X$ such that $D(x * y) = \left(x * D(y)\right) \wedge (d(x) * y)$ for all $x, y \in X$, the mapping $D : X \to X$ is called a generalized (r, l)-derivation. Moreover, if D is both a generalized (l, r)-derivation and (r, l)-derivation, we say that D is a generalized derivation.

Definisi 2.5. [1] A *BP*-algebra is a non-empty set *X* with a constant 0 and a binary operation "$*$" satisfying the following axioms:
(*BP1*) $x * x = 0$,
(*BP2*) $x * (x * y) = y$,
(*BP3*) $(x * z) * (y * z) = x * y$,
for all $x, y, z \in X$,

Example 2.2. Let $X = \{0, \mathrm{a}, \mathrm{b}, \mathrm{c}\}$ be a set with Cayley's table as shown in Table 2.2.

Table 2.2: Cayley's table for $(X; *, 0)$

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then, from Table 2.2 it can be shown that $(X ; *, 0)$ is a *BP*-algebra.

Theorem 2.6. [1] If $(X; *, 0)$ a *BP*-algebra, then for all $x, y \in X$,

- (i) $0 * (0 * x) = x$,
- (ii) $0 * (y * x) = x * y$,
- (iii) $x * 0 = x$,
- (iv) If $x * y = 0$, then $y * x = 0$,
- (v) If $0 * x = 0 * y$, then $x = y$,

(vi) If $0 * x = y$, then $0 * y = x$,
(vii) If $0 * x = x$, then $x * y = y * x$.

Proof. The Theorem 2.7 has been proved in [1]. ■

Definition 2.7. [3] Let $(X;*,0)$ be a *BP*-algebra. By a left-right *f*-derivation (briefly, (l,r)-*f*-derivation) on *X*, we mean a self map d_f of *X* satisfies the identity $d_f(x * y) = (d_f(x) * f(y)) \wedge \left(f(x) * d_f(y)\right)$ for all $x, y \in X$. If d_f satisfies the identity $d_f(x * y) = \left(f(x) * d_f(y)\right) \wedge (d_f(x) * f(y))$ for all $x, y \in X$, then it is said that d_f is a right-left *f*-derivation (briefly, (r,l)-*f*-derivation) of X . If d_f is both an (r,l)-*f*-derivation and an (l,r)-*f*-derivation, then d_f is said to be an *f*-derivation.

III. GENERALIZED DERIVATION OF *BP*-ALGEBRA

In this section, a generalized (l,r)-derivation, a generalized (r,l)-derivation, and a generalized derivation in *BP*-algebra are defined by a way similar to the construct of the generalized derivation in *BM*-algebra by Sugianti and Gemawati [9]. Then, also we obtain some related properties.

Let $(X;*,0)$ be a *BP*-algebra, we denote $x \wedge y = y * (y * x)$ for all $x, y \in X$.

Definition 3.1. Let X be a *BP*-algebra. A mapping $D : X \to X$ is called a generalized (l,r)-derivation if there exists an (l,r)-derivation $d : X \to X$ such that $D(x * y) = (D(x) * y) \wedge \left(x * d(y)\right)$ for all $x, y \in X$, if there exists an (r,l)-derivation $d : X \to X$ such that $D(x * y) = \left(x * D(y)\right) \wedge (d(x) * y)$ for all $x, y \in X$, the mapping $D : X \to X$ is called a generalized (r,l)-derivation. Moreover, if D is both a generalized (l,r)-derivation and (r,l)-derivation, we say that D is a generalized derivation.

Example 3.1. Let $X = \{0, 1, 2, 3\}$ be a set with Cayley's table as shown in Table 3.1.

Table 3.1: Cayley's table for $(X;*,0)$

$*$	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

Then, it is easy to show that *X* is a *BP*-algebra. Define a map $d : X \to X$ by $d(x) = x$ and $D : X \to X$ by

$$D(x) = \begin{cases} 2 & if \ x = 0, \\ 3 & if \ x = 1, \\ 0 & if \ x = 2, \\ 1 & if \ x = 3. \end{cases}$$

It can be shown that d is a derivation of *X* and *D* is a generalized derivation of *X*.

Theorem 3.2. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized (l,r)-derivation in *X*, then
(i) $D(x * y) = D(x) * y$ for all $x, y \in X$,
(ii) $D(0) = D(x) * x$ for all $x \in X$,
(iii) $D(x * d(x)) = D(x) * d(x)$ for all $x \in X$,
(iv) $D(x) = D(0) * (0 * x)$ for all $x \in X$.

Proof. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized (l,r)-derivation in *X*.

(i) Since *D* is a generalized (l,r)-derivation in *X*, then by (*BP2*) axiom obtained

$$\begin{aligned} D(x * y) &= (D(x) * y) \wedge (x * d(y)) \\ &= \left(x * d(y)\right) * [\left(x * d(y)\right) * (D(x) * y)] \\ D(x * y) &= D(x) * y. \end{aligned}$$

Hence, it is obtained that $D(x * y) = D(x) * y$ for all $x, y \in X$.
(ii) By (i) we have $D(x * y) = D(x) * y$. Substitution of $y = x$ gives $D(x * x) = D(x) * x$, and by (*BP1*) axiom we get $D(0) = D(x) * x$ for all $x \in X$
(iii) By (i) it is obtained that $D(x * d(x)) = D(x) * d(x)$ for all $x \in X$.
(iv) Since *D* is a generalized (l,r)-derivation in *X*, then by Theorem 2.6 (i) and (*BP2*) axiom we get

$D(x) = D(0 * (0 * x))$
$= (D(0) * (0 * x)) \wedge (0 * d(0 * x))$
$= (0 * d(0 * x) * [(0 * d(0 * x) * (D(0) * (0 * x))]$
$D(x) = D(0) * (0 * x).$

Hence, we obtain $D(x) = D(0) * (0 * x)$ for all $x \in X$. ■

Theorem 3.3. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized (l, r)-derivation in *X*. If $0 * x = x$ for all $x \in X$, then

(i) $D(x) = D(0) * x = x * D(0)$ for all $x \in X$,
(ii) $D(x) * D(y) = x * y$ for all $x, y \in X$.

Proof. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized (*l*, *r*)-derivation in *X*. Since $0 * x = x$, then by Theorem 2.6 (vii) obtained $x * y = y * x$ for all $x, y \in X$.

(i) From Theorem 3.2 (i) we get

$D(x) = D(0 * x)$
$= D(0) * x$
$D(x) = x * D(0).$

Hence, we have $D(x) = D(0) * x = x * D(0)$ for all $x \in X$.

(ii) From (i) we get $D(x) = x * D(0)$ and $D(y) = y * D(0)$. By (*BP3*) axiom obtained

$D(x) * D(y) = (x * D(0)) * (y * D(0))$
$D(x) * D(y) = x * y.$

Therefore, this shows that $D(x) * D(y) = x * y$ for all $x, y \in X$. ■

Theorem 3.4. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized (*r*, *l*)-derivation in *X*, then

(i) $D(x * y) = x * D(y)$ for all $x, y \in X$,
(ii) $D(0) = x * D(x)$ for all $x \in X$,
(iii) $D(d(x) * x) = d(x) * D(x)$ for all $x \in X$.

Proof. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized (r, l)-derivation in *X*.

(i) Since *D* is a generalized (r, l)-derivation in *X*, then by (*BP2*) axiom we get

$D(x * y) = (x * D(y)) \wedge (d(x) * y)$
$= (d(x) * y) * [(d(x) * y) * (x * D(y))]$
$D(x * y) = x * D(y).$

Hence, it is obtained that $D(x * y) = x * D(y)$ for all $x, y \in X$.

(ii) By (i) we have $D(x * y) = x * D(y)$. By substitution of $y = x$ then $D(x * x) = x * D(x)$, and by (*BP1)* axiom we get $D(0) = x * D(x)$ for all $x \in X$.

(iii) From (i) we have $D(d(x) * x) = d(x) * D(x)$ for all $x \in X$. ■

Definition 3.5. Let $(X;*,0)$ be a *BP*-algebra. A sel*f*-map $D: X \to X$ is said to be *regular* if $d(0) = 0$.

From the definition *regular* in *BP*-algebra we have Theorem 3.6, Theorem 3.7, and Theorem 3.8.

Theorem 3.6. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized (l, r)-derivation in *X*, then

(i) If d is a *regular*, then $D(x) = D(x) \wedge x$ for all $x \in X$,
(ii) If *D* is a *regular*, then *D* is an identity function.

Proof. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized (*l*, *r*)-derivation in *X*.

(i) Since d is a *regular*, then $d(0) = 0$ and by Theorem 2.6 (iii) we get

$D(x) = D(x * 0)$
$= (D(x) * 0)) \wedge (x * d(0))$
$= (D(x) * 0) \wedge (x * 0)$
$D(x) = D(x) \wedge x.$

Hence, it is obtained that $D(x) = D(x) \wedge x$ for all $x \in X$.

(ii) Since D is a *regular*, then $D(0) = 0$. From Theorem 3.2 (iv) and Theorem 2.6 (i) we have

$D(x) = D(0) * (0 * x)$
$= 0 * (0 * x)$
$D(x) = x.$

Hence, we obtain $D(x) = x$ for all $x \in X$, such that D is an identity function . ■

Theorem 3.7. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized (*r*, *l*)-derivation in *X*. If D is a *regular*, then D is an identity function.

Proof. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized (r,l)-derivation in *X*. Since D is a *regular*, then $D(0) = 0$. From Theorem 2.6 (iii) and (*BP2*) axiom obtained

$$\begin{aligned} D(x) &= D(x * 0) \\ &= (x * D(0)) \wedge (d(x) * 0) \\ &= (x * 0) \wedge (d(x) * 0) \\ &= x \wedge d(x) \\ &= d(x) * (d(x) * x) \\ D(x) &= x. \end{aligned}$$

Hence, we obtain $D(x) = x$ for all $x \in X$, such that D is an identity function. ■

Theorem 3.8. Let $(X;*,0)$ be a *BP*-algebra and *D* be a generalized derivation in *X*. If D is a *regular* if and only if D is an identity function.

Proof. Let *D* be a generalized (l,r)-derivation in *X* and *D* is a *regular*, then by Theorem 3.6 (ii) we get D is an identity function. If *D* is a generalized (r,l)-derivation in *X* and *D* is a *regular*, then by Theorem 3.7 it shows that *D* is an identity function. Conversely, if *D* is an identity function, then $D(x) = x$ for all $x \in X$, clearly $D(0) = 0$. Hence, *D* is a *regular*. ■

IV. GENERALIZED *f*-DERIVATION OF *BP*-ALJGEBRA

In this section, a generalized (l,r)- *f*-derivation, a generalized (r,l)- *f*-derivation, and a generalized *f*-derivation in *BP*-algebra are defined as a development of the generalized derivation in *BP*-algebra. Then, also we obtain some related properties.

Let (*X*; $*$, 0) and (*Y*; $*$, 0) are two *BP*-algebraA map $f: X \to Y$ to be said a homomorphism of *X* if it satisfied $f(x * y) = f(x) * f(y)$ for all $x, y \in X$. If f is a self-map of X and f is a homomorphism of *X*, then f is a endomorphism of *X*. Note that $f(0) = 0$.

Definition 4.1. Let *X* be a *BP*-algebra and f be an endomorphism of *X*. A mapping $D_f: X \to X$ is called a generalized (*l*, *r*)-*f*-derivation in *X* if there exists an (l,r)- *f*-derivation $d_f: \to X$ such that $D_f(x * y) = (D_f(x) * f(y)) \wedge (f(x) * d_f(y))$ for all $x, y \in X$. If there exists an (r,l)-*f*-derivation $d_f: X \to X$ such that $D_f(x * y) = (f(x) * D_f(y)) \wedge (d_f(x) * f(y))$ for all $x, y \in X$, the mapping D_f is called a generalized (r,l)-*f*-derivation in *X*. Moreover, if D_f is both a generalized (l,r)-*f*-derivation and (r,l)-*f*-derivation, we say that D_f is a generalized *f*-derivation.

Theorem 4.2. Let $(X;*,0)$ be a *BP*-algebra and D_f be a generalized (l,r)-*f*-derivation in *X*, where f be an endomorphism of *X*, then

(i) $D_f(x * y) = D_f(x) * f(y)$ for all $x, y \in X$,
(ii) $D_f(0) = D_f(x) * f(x)$ for all $x \in X$,
(iii) $D_f(x) = D_f(0) * (0 * f(x))$ for all $x \in X$.

Proof. Let $(X;*,0)$ be a *BP*-algebra and D_f be a generalized (l,r)-*f*-derivation in *X*, where f be an endomorphism of *X*.

(i) Since D_f is a generalized (l,r)-*f*-derivation in *X*, then by (*BP2*) axiom obtained

$$\begin{aligned} D_f(x * y) &= (D_f(x) * f(y)) \wedge (f(x) * d_f(y)) \\ &= \left(f(x) * d_f(y)\right) * [\left(f(x) * d_f(y)\right) * (D_f(x) * f(y))] \\ D_f(x * y) &= D_f(x) * f(y). \end{aligned}$$

Hence, it is obtained that $D_f(x * y) = D_f(x) * f(y)$ for all $x, y \in X$.

(ii) By (i) we have $D_f(x * y) = D_f(x) * f(y)$. Substitution of $y = x$ gives $D_f(x * x) = D_f(x) * f(x)$, and by (*BP1*) axiom we get $D_f(0) = D_f(x) * f(x)$ for all $x \in X$.

(iii) Since D_f is a generalized (l,r)-*f*-derivation in *X*, then by Theorem 2.6 (i) and (*BP2*) axiom we get

$$\begin{aligned} D_f(x) &= D_f(0 * (0 * x)) \\ &= (D_f(0) * f(0 * x)) \wedge (f(0) * d_f(0 * x)) \\ &= (f(0) * d_f(0 * x) * [(f(0) * d_f(0 * x) * (D_f(0) * f(0 * x))] \\ D_f(x) &= D_f(0) * (0 * f(x)). \end{aligned}$$

Hence, we obtain $D_f(x) = D_f(0) * (0 * f(x))$ for all $x \in X$. ■

Theorem 4.3. Let $(X;*,0)$ be a *BP*-algebra and D_f be a generalized (*r*, *l*)-*f*-derivation in *X*, where f be an endomorphism of *X*, then

(i) $D_f(x * y) = f(x) * D_f(y)$ for all $x, y \in X$,
(ii) $D_f(0) = f(x) * D_f(x)$ for all $x \in X$.

Proof. Let $(X;*,0)$ be a *BP*-algebra and D_f be a generalized (r,l)-*f*-derivation in *X*, where f be an endomorphism of *X*.

(i) Since D_f is a generalized (r,l)-*f*-derivation in *X*, then by (*BP2*) axiom we get

$$D_f(x * y) = (f(x) * D_f(y)) \wedge (d_f(x) * f(y))$$
$$= (d_f(x) * f(y)) * [(d_f(x) * f(y)) * (f(x) * D_f(y))]$$
$$D_f(x * y) = f(x) * D_f(y).$$

Hence, it is obtained that $D_f(x * y) = f(x) * D_f(y)$ for all $x, y \in X$.

(ii) By (i) we have $D_f(x * y) = f(x) * D_f(y)$. By substitution of $y = x$ then $D_f(x * x) = f(x) * D_f(x)$ and by (*BP1)* axiom we get $D_f(0) = f(x) * D_f(x)$ for all $x \in X$. ■

From the notion of *regular* in *BP*-algebra we obtain Theorem 4.4, Theorem 4.5, and Theorem 4.6.

Theorem 4.4. Let $(X;*,0)$ be a *BP*-algebra and D_f be a generalized (l,r)-*f*-derivation in *X*, where f be an endomorphism of *X*, then

(i) If d_f is a *regular*, then $D_f(x) = D_f(x) \wedge f(x)$ for all $x \in X$,
(ii) If D_f is a *regular*, then $D_f(x) = f(x)$ for all $\in X$. .

Proof. Let $(X;*,0)$ be a *BP*-algebra and D_f be a generalized (*l*, *r*)-derivation in *X*, where f be an endomorphism of *X*.

(i) Since d_f is a *regular*, then $d_f(0) = 0$ and by Theorem 2.6 (iii) we get

$$D_f(x) = D_f(x * 0)$$
$$= (D_f(x) * f(0)) \wedge (f(x) * d_f(0))$$
$$= (D_f(x) * 0) \wedge (f(x) * 0)$$
$$D_f(x) = D_f(x) \wedge f(x).$$

Hence, it is obtained that $D_f(x) = D_f(x) \wedge f(x)$ for all $x \in X$.

(ii) Since D_f is a *regular*, then $D_f(0) = 0$. From Theorem 4.2 (iii) and Theorem 2.6 (i) we have

$$D_f(x) = D_f(0) * (0 * f(x))$$
$$= 0 * (0 * f(x))$$
$$D_f(x) = f(x).$$

Hence, we obtain $D_f(x) = f(x)$ for all $x \in X$. ■

Theorem 4.5. Let $(X;*,0)$ be a *BP*-algebra and D_f be a generalized (*r*, *l*)-*f*-derivation in *X*, where f be an endomorphism of *X*. If D_f is a *regular*, then $D_f(x) = f(x)$ for all $x \in X$.

Proof. Let $(X;*,0)$ be a *BP*-algebra and D_f be a generalized (r,l)-*f*-derivation in *X*, where f be an endomorphism of *X*. Since D_f is a *regular*, then$D_f(0) = 0$. From Theorem 2.6 (iii) and (*BP2*) axiom obtained

$$D_f(x) = D_f(x * 0)$$
$$= (f(x) * D_f(0)) \wedge (d_f(x) * f(0))$$
$$= (f(x) * 0) \wedge (d_f(x) * 0)$$
$$= f(x) \wedge d_f(x)$$
$$= d_f(x) * (d_f(x) * f(x))$$
$$D_f(x) = f(x).$$

Hence, we obtain $D_f(x) = f(x)$ for all $x \in X$. ■

Theorem 4.6. Let $(X;*,0)$ be a *BP*-algebra and D_f be a generalized derivation in *X*, where f be an endomorphism of *X*. D_f is a *regular* if and only if $D_f(x) = f(x)$ for all $x \in X$.

Proof. Let D_f be a generalized (l,r)-f-derivation in X and D_f is a *regular*, then by Theorem 4.4 (ii) we get $D_f(x) = f(x)$ for all $x \in X$. If D_f is a generalized (r,l)- f-derivation in X and D_f is a *regular*, then by Theorem 4.5 it shows that $D_f(x) = f(x)$ for all $x \in X$. Conversely, if $D_f(x) = f(x)$, then $D_f(0) = f(0) = 0$. Hence, D_f is a *regular*. ■

V. CONCLUSION

The definition of a generalized derivation in *BP*-algebra is equivalent to a generalized derivation in *BM*-algebra, and all the properties of the generalized derivation in *BM*-algebra also satisfied in *BP*-algebra. However, there is a property of the generalized derivation in *BP*-algebra, which is not true in *BM*-algebra, i.e. if D is a generalized (l,r)-derivation in *BP*-algebra $(X; *,0)$ and $0 * x = x$, then $D(x) = D(0) * x = x * D(0)$ and $D(x) * D(y) = x * y$ for all $x, y \in X$. Furthermore, the properties of the generalized derivation in *BP*-algebra are different to the properties of the generalized *f*-derivation in *BP*-algebra.

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