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# Generalization of new fuzzy Topological Spaces

Arun Kumar Gali

*Department of Mathematics*
*Government First Grade College Sector no-43,Navanagar*
*Bagalkote-587103, Karnataka State , INDIA.*

**Abtract:** *In this paper, we introduce new class of fuzzy sets called fuzzy generalized regular-closed sets and fuzzy generalized regular open sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Also we introduce fuzzy generalized regular-closed sets and fuzzy generalized regular-open sets in fuzzy topological spaces and study some of their properties.*

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## Introduction

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. In the year 1965, L.A.Zadeh [1] introduced the concept of fuzzy subset as a generalization of that of an ordinary subset. The introduction of fuzzy subsets paved the way for rapid research work in many areas of mathematical science. In the year 1968, C.L.Chang [2] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces . Subsequently several researchers contributed to the development of the theory and applications of fuzzy topology. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces. An ordinary subset A or a set X can be characterized by a function called characteristic function

$\mu_A : X \longrightarrow [0,1]$ of A, defined by
$\mu_A(x) = 1$, if $x \in A$.
$= 0$, if $x \notin A$.

Thus an element $x \in X$ isin A if $\mu_A(x) = 1$ and is not in A if $\mu_A(x) = 0$. In general if X is a set and A is a subset of X then A has the following representation. $A = \{ (x, \mu_A(x)) : x \in X\}$, here $\mu_A(x)$ may be regarded as the degree of belongingness of x to A, which is either 0 or 1. Hence A is the class of objects with degree of belongingness either 0 or 1 only. Prof. L.A.Zadeh [1] introduced a class of objects with continuous grades of belongingness ranging between 0 and 1 ; he called such a class as fuzzy subset. A fuzzy subset A in X is characterized as a membership function
$\mu_A : X \longrightarrow [0,1]$ , which associates with each point in x a real number $\mu_A(x)$ between 0 and 1 which represents the degree or grade membership of belongingness of x to A.

The purpose of this paper is to introduce a new class of fuzzy sets called fuzzy gr-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Among many other results it is observed that every fuzzy closed set is fuzzy gr-closed but not conversely.Also we introduce fuzzy gr-open sets in fuzzy topological spaces and study some of their properties.

## I. Preliminaries

**1.1Definition:[1]**A fuzzy subset A in a set X is a function A : X → [0, 1]. A fuzzy subset in X is empty iff its membership function is identically 0 on X andis denoted by 0 or $\mu_\phi$.The set X can be considered as a fuzzy subset of X whosemembership function is identically 1 on X and is denoted by $\mu_x$or $I_x$. In fact everysubset of X is a fuzzy subset of X but not conversely. Hence the concept of a fuzzysubset is a generalization of the concept of a subset.

**1.2 Defnition:[1]** If A and B are any two fuzzy subsets of a set X , thenA is said to be included in B or A is contained in B iff A(x) ≤ B(x) for all x in X. Equivalently, A ≤ B iff A(x) ≤ B(x) for all x in X.

**1.3 Defnition:**[1]Two fuzzy subsets A and B are said to be equal if A(x) = B(x) for every x in X. Equivalently A = B if A(x) = B(x) for every x in X.

**1.4Defnition:[1]** The complement of a fuzzy subset A in a set X, denoted by A′ or 1 − A, is the fuzzy subset of X defined by A′(x) = 1 − A(x) for all x in X. Note that(A′)′ = A.

**1.5 Defnition:[1]** The union of two fuzzy subsets A and B in X, denoted by A ∨ B, is a fuzzy subset in X defined by (A ∨ B)(x) = Max{$\mu_A$(x), $\mu_B$(X)} for all x in X.

**1.6 Defnition:[1]**The intersection of two fuzzy subsets A and B in X, denoted by A ∧ B, is a fuzzy subset in X defined by (A ∧ B)(x) = Min{A(x), B(x)} for all x in X.

**1.7 Defnition:[1]** A fuzzy set on X is ‘Crisp‘ if it takes only the values 0 and 1 on X.

**1.8 Defnition:[2]**Let X be a set and be a family of fuzzy subsets of (X,$\tau$) is called a fuzzy topology on X iff $\tau$ satisfies the following conditions.

(i)$\mu_\phi$ ; $\mu_X \in \tau$: That is 0 and 1 ∈τ

(ii) If $G_i \in \tau$ for i ∈ I then $\bigvee_{i \in I} G_i \in \tau$

(iii) If G,H ∈τ then G ∧ H ∈τ

The pair (X, $\tau$) is called a fuzzy topological space (abbreviated as fts). The membersof $\tau$are called fuzzy open sets and a fuzzy set A in X is said to be closed iff 1 − A is an fuzzy open set in X.

**1.9 Remark**:[2] Every topological space is a fuzzy topological space but not conversely.

**1.10 Defnition:**[2]Let X be a fts and A be a fuzzy subset in X. Then ∧ {B : B is a closed fuzzy set in Xand B ≥ A} is called the closure of A and is denoted by A or cl(A).

**1.11 Defnition:**[2]Let A and B be two fuzzy sets in a fuzzy topological space (X,$\tau$)and let A ≥ B. Then B is called an interior fuzzy set of A if there exists G ∈$\tau$ suchthat A ≥ G ≥ B, the least upper bound of all interior fuzzy sets of A is called theinterior of A and is denoted by $A^0$.

**1.12 Definition[3]** A fuzzy set A in a fts X is said to be fuzzy regular open if and only if there exists a fuzzy open set V in X such that V ≤A ≤cl(V).

**1.13 Definition[3]** A fuzzy set A in a fts X is said to be fuzzy regular-closed if and only if there exists a fuzzy closed set V in X such that int(V)≤ A ≤V. It is seen that a fuzzy set A is fuzzy regularopen if and only if 1-A is a fuzzy regular-closed.

**1.14 Theorem:[3]** The following are equivalent:

(a) $\mu$ is a fuzzy regular closed set,
(b) $\mu^C$ is a fuzzy regular open set,
(c) int(cl($\mu$)) ≤ $\mu$.
(b) int(cl($\mu$)) ≥ $\mu^c$

**1.15 Theorem** [3] (a) Any union of fuzzy regular open sets is a fuzzy regular open set and
(b) any intersection of fuzzy regular closed sets is a fuzzy regular closed.

**1.16 Remark[3]**

(i) Every fuzzy open set is a fuzzy regularopen but not conversely.
(ii) Every fuzzy closed set is a fuzzy regular-closed set but not conversely.
(iii) The closure of a fuzzy open set is fuzzy regularopen set
(iv) The interior of a fuzzy closed set is fuzzy regular-closed set

**1.17Definition:[3]** A fuzzy set $\mu$of a fts X is called a fuzzy regular open set of X if int(cl($\mu$).= $\mu$.

**1.18 Definition:[3]** A fuzzy set $\mu$of fts X is called a fuzzy regular closed set of X if cl(int($\mu$)) = $\mu$.

**1.19 Theorem:[3]** A fuzzy set $\mu$of a fts X is a fuzzy regular open if and only if $\mu^C$ fuzzy regular closed set.

**1.20 Remark:[3]**

(i) Every fuzzy regular open set is a fuzzy open set but not conversely.
(ii) Every fuzzy regular closed set is a fuzzy closed set but not conversely.

**1.21 Theorem:[3]**

(i) The closure of a fuzzy open set is a fuzzy regular closed.
(ii) The interior of a fuzzy closed set is a fuzzy regular open set.

**1.22 Definition:[4]** A fuzzy set $\alpha$in fts X is called fuzzy rw closed
if cl($\alpha$)≤ $\mu$whenever $\alpha \leq \mu$and $\mu$is regular regular-open in X.

## II. Fuzzy generalized regular closed sets and fuzzy generalized regular open sets .

**Definition2.1:** Let (X, T) be a fuzzy topological space. A fuzzy set $\alpha$ of X is called fuzzy generalized regular-closed (briefly, fuzzy gr-closed) if rcl($\alpha$) ≤ $\sigma$ whenever $\alpha \leq \sigma$ and $\sigma$ is fuzzy-open in fts X.

We denote the class of all fuzzy generalized regular closed sets in fts X by FGRC(X).

**Theorem 2.2:** Every fuzzy closed set is a fuzzy gr-closed set in a fts X.

**Proof:** Let $\alpha$ be a fuzzy closed set in a fts X. Let $\beta$ be a fuzzy open set in X such that $\alpha \leq \beta$. Since $\alpha$ is fuzzy closed, r-cl($\alpha$)= $\alpha$. Therefore r-cl($\alpha$) $\leq \beta$. Hence $\alpha$ is fuzzy gr-closed in fts X.

The converse of the above Theorem need not be true in general as seen from the following example.

**Example 2.3:** Let X= {a, b, c}. Define a fuzzy set $\alpha$ in X by

$\alpha$(x) = 1 if x = a

0 othewise

β (x) = 1 if x = a,b

0 otherwise .

Let T= {1, 0, β}. Then (X, T) is a fuzzy topological space. Define a fuzzy set μ in X by

μ(x) = 1 if x = b
0 otherwise .

Then μ is a fuzzy gr-closed set but it is not a fuzzy closed set in fts X.

**Corollary 2.4:** By K.K.Azad we know that, every fuzzy regular closed set is a fuzzy closed set but not conversely. By Theorem 2.2 every fuzzy closed set is a fuzzy gr-closed set but not conversely and hence every fuzzy regular closed set is a fuzzy gr-closed set but not conversely.

**Remark 2.5:** Fuzzy gr closed sets and fuzzy regular-closed sets are independent.

**Example:** (i) Consider Let X= {a, b, c} and the functions $\alpha, \beta$: X $\longrightarrow$ [0, 1] be defined as

$\alpha$(x) = 1 if x = a
0 otherwise

$\beta$(x) = 1 if x = a, b
0 Otherwise

Consider T= {1, 0, $\alpha$ ,$\beta$}. Then (X, T) is a fuzzy topological space. Then the fuzzy set

μ(x) = 1 if x = b
0 otherwise

is a fuzzy gr-closed set but it is not a fuzzy regular-closed set in fts X.

**Example:** (ii)Let X= {a, b, c} and the functions $\alpha, \beta, \gamma$: X $\longrightarrow$ [0, 1] be defined as

$\alpha$ (x) = 1 if x = a
0 otherwise

$\beta$(x) = 1 if x = b
0 therwise

$\gamma$(x) = 1 if x = a,b
0 otherwise

Consider T= {1,0,$\alpha$,$\beta,\gamma$}. Then (X, T) is a fuzzy topological space.

Then the fuzzy set

$\alpha$ (x) = 1 if x = a
0 otherwise.

is a fuzzy regular-closed set but it is not a fuzy gr-closed set in fts X.

**Remark 2.6:** Fuzzy gr-closed sets and fuzzy rw-closed sets are independent.

Example (i): Let X= {a, b, c,d} and the functions $\alpha, \beta, \gamma, \delta$: X $\longrightarrow$ [0, 1] be defined as

$\alpha$ (x) = 1 if x = a
0 otherwise

$\beta$(x) = 1 if x = b
0 Otherwise

$\gamma$(x) = 1 if x = a,b
0 otherwise

$\delta$(x) = 1 if x = a,b,c
0 otherwise

Consider T= {1,0,$\alpha$,$\beta,\gamma,\delta$}. Then (X, T) is a fuzzy topological space.

Then (X, T) is a fts then the fuzzy set

μ(x) = 1 if x = a ,d
0 otherwise.

is a fuzzy gr-closed set but it is not a fuzzy rw closed set in fts X.

**Example:** (ii)Let X= {a, b, c} and the functions $\alpha$: X $\rightarrow$ [0, 1] be defined as

$\alpha$ (x) = 1 if x = a
0 otherwise

Consider T= {1,0,$\alpha$}. Then (X, T) is a fuzzy topological space. Then the fuzzy set

$\alpha$ (x) = 1 if x = a

0 otherwise.

is a fuzzy rw-closed set but it is not a fuzzy gr closed set in fts X.

**Theorem 2.7:** If $\alpha$ and $\beta$ are fuzzy gr-closed sets in fts X, then $\alpha \vee \beta$
fuzzy gr-closed set in fts X.

**Proof:** Let $\sigma$ be a fuzzy open set in fts X such that $\alpha \vee \beta \leq \sigma$. Now $\alpha \leq \sigma$ and $\beta \leq \sigma$.Since $\alpha$and $\beta$are fuzzy gr-closed sets in fts X, r-cl($\alpha$) $\leq \sigma$ and r-cl($\beta$) $\leq \beta$.
Therefore r-cl($\alpha$) $\vee$ r $-$ cl( $\beta$) $\leq \sigma$. But r-cl($\alpha$) $\vee$ r $-$ cl( $\beta$)= r-cl( $\alpha \vee \beta$).Thus r-cl($\alpha \vee \beta$)$\leq \sigma$.
Hence $\alpha \vee \beta$ is a fuzzy gr-closed set in fts X.

**Remark 2.8:** If $\alpha$and $\beta$ are fuzzy gr-closed sets in fts X, then $\alpha \wedge \beta$ need not be a fuzzy gr-closed set in general as seen from the following example.

**Example 2.9:**Consider the fuzzy topological space (X, T) defined

Let X= {a, b, c,d},T= {1,0,$\alpha$,$\beta$,$\gamma$}in this fts X, The fuzzy sets $\delta_1\delta_2$ : X $\rightarrow$ [0, 1] are defined by

$\delta_1$(x) = 1 if x = {a,c,d}
0 otherwise.

$\delta_2$(x) = 1 if x = {a,b,c}
0 otherwise

and $\alpha, \beta, \gamma$: X $\rightarrow$ [0, 1] be defined as

$\alpha$ (x) = 1 if x = a
0 otherwise

$\beta$(x) = 1 if x = c,d
0 otherwise

$\gamma$(x) = 1 if x = a,c,d
0 otherwise

Then $\delta_1 and\ \delta_2$ are fuzzy gr closed sets in fts X.

Let μ = $\delta_1 \wedge \delta_2$, then

μ (x) = 1 if x = a ,c
0 otherwise.
μ = $\delta_1 \wedge \delta_2$, is not a fuzzy gr-closed set in fts X.

**Theorem 2.10:** If a fuzzy set $\alpha$ of fts X is both fuzzy regular open and fuzzy
gr-closed, then α is a fuzzy pre-closed set in fts X.

**Proof:** Suppose a fuzzy set $\alpha$ of fts X is both fuzzy regular open and fuzzy gr-closed. As every fuzzy regular open set is a fuzzy open set and $\alpha \leq \alpha$ we have r-cl($\alpha$ ) $\leq \alpha$ .

Also $\alpha \leq r -$cl( $\alpha$).Therefore r-cl($\alpha$) = $\alpha$ . That is $\alpha$ is fuzzy closed. Since $\alpha$ is fuzzy regular open, int($\alpha$)= $\alpha$. Now cl(int(α))=cl(α) = α. Therefore $\alpha$ is a fuzzy regular- closed set in fts X.

**Theorem 2.11:** If a fuzzy set $\alpha$ of a fts X is both fuzzy open and fuzzy gr closed, then $\alpha$ is a fuzzy closed set in fts X.

**Proof:** Suppose a fuzzy set $\alpha$ of a fts X is both fuzy open and fuzzy gr-closed. Now , $\alpha \leq \alpha$, we have $\alpha \leq r -$cl($\alpha$) .Therefore r-cl($\alpha$) = $\alpha$ and hence $\alpha$ is a fuzzy closed set in fts X.

**Theorem2.12:** If a fuzzy set $\alpha$ of a fts X is both fuzzy open and fuzzy generalized regular-closed, then $\alpha$ is a fuzzy gr-closed set in fts X.

**Proof:** Suppose a fuzzy set $\alpha$ of a fts X is both fuzzy open and fuzzy generalized regular closed. Now $\alpha \leq \alpha$, by hypothesis we have r-cl($\alpha$) $\leq \alpha$.Also $\alpha \leq r -$cl($\alpha$)Therefore r-cl($\alpha$)= $\alpha$. That is $\alpha$ is a fuzzy closed set and hence $\alpha$is a fuzzy gr-closed set in fts X, as every fuzzy closed set is a fuzzy gr-closed set.

**Remark2.13:** If a fuzzy set $\gamma$ is both fuzzy open and fuzzy gr-closed set in a fts X, then $\gamma$ need not be a fuzzy generalized regular closed set in general as seen from the following example.

**Example 2.14:**

Let X= {a, b, c,d} and the functions $\alpha, \beta, \gamma$: X ➜ [0, 1] be defined as

$\alpha$ (x) = 1 if x = a
0 otherwise

$\beta$(x) = 1 if x = c,d
0 otherwise

$\gamma$(x) = 1 if x = a,c,d
0 otherwise

Consider T= {1,0,$\alpha$,$\beta, \gamma$}. Then (X, T) is a fuzzy topological space.In this fts X,$\gamma$is both fuzzy open and gr closed .

**Theorem 2.15:** Let $\alpha$be a fuzzy gr-closed set of a fts X and suppose $\alpha \leq \beta \leq s -$cl($\alpha$). Then $\beta$ is also a fuzzy gr-closed set in fts X.

**Proof:** Let $\alpha \leq \beta \leq r -$cl($\alpha$) and $\alpha$be a fuzzy gr-closed set of fts X. Let $\sigma$be any fuzzy open set such that $\beta \leq \sigma$.Then $\alpha \leq \sigma$ and $\alpha$is fuzzy gr-closed, we have r-cl($\alpha$)$\leq \sigma$.
But r-cl($\beta$) $\leq r -$cl($\alpha$) and thus r-cl($\beta$) $\leq \sigma$. Hence $\beta$is a fuzzy gr-closed set in fts X.

**Theorem2.16:** In a fuzzy topological space X if FO(X) = {1, 0}, where FO(X) is the family of all fuzzy open sets then every fuzzy subset of X is fuzzy gr-closed.

**Proof:** Let X be a fuzzy topological space and FRG$\alpha$(X) = {1, 0}. Let $\alpha$ be any fuzzy subset of X. Suppose $\alpha$= 0. Then 0 is a fuzzy gr-closed set in fts X. Suppose $\alpha \neq 0$. Then 1 is the only fuzzy open set containing $\alpha$and so r-cl($\alpha$) $\leq$1. Hence $\alpha$is a fuzzy gr-closed set in fts X.

**Theorem2.17:** If $\alpha$is a fuzzy gr-closed set of fts X and r-cl($\alpha$)∧(1- r-cl($\alpha$))=0,
then r-cl($\alpha$)- $\alpha$does not contain any non-zero fuzzy openset in fts X.

**Proof:** Suppose $\alpha$ is a fuzzy gr-closed set of fts X and r-cl($\alpha$)∧ (1-r-cl($\alpha$))=0,.We prove the result by contradiction. Let $\beta$ be a fuzzy open set such that r-cl($\alpha$)- $\alpha \geq \beta$ and $\beta \neq 0$.
Now $\beta \leq$r-cl($\alpha$)-$\alpha$ i.e $\beta \leq 1-\alpha$ which implies $\alpha \leq 1-\beta$, since $\beta$is fuzzy open in fts X,then 1- $\beta$ is fuzzy open in X; Since $\alpha$ is a fuzzy gr-closed set in fts X, by definition
r-cl($\alpha$)≤1-$\beta$So $\beta \leq \mathrm{s}-\mathrm{cl}(\alpha)$ Therefore, $\beta \leq$r-cl($\alpha$)∧ (1-r-cl($\alpha$))=0, by hypothesis. This shows that $\beta$=0. which is a contradiction. Hence r-cl($\alpha$)-$\alpha$does not contain any non-zero fuzzy$-open$ set in fts X.

**Corollary 2.18:** If $\alpha$ is a fuzzy gr-closed set of fts X and r-cl($\alpha$)∧ (1-r-cl($\alpha$))=0,
then r-cl($\alpha$)- $\alpha$ does not contain any non-zero fuzzy open set in fts X.

**Proof:** Follows form the corollary 2.18 and the fact that every fuzzy regular
open set is a fuzzy openset in fts X.

**Corollary 2.19:** If α is a fuzzy gr-closed set of a fts X and r-cl($\alpha$)∧ (1-r-cl($\alpha$))=0,
then r-cl($\alpha$)- $\alpha$does not contain any non-zero fuzzy regular closed set in fts X.

**Proof:** Follows form the Theorem 2.18 and the fact that every fuzzy regular open set is a
Fuzzy open set in fts X.

**Theorem2.20:**Let α be a fuzzy gr-closed set of fts X and r-cl($\alpha$)∧ (1-r-cl($\alpha$))=0,Then $\alpha$ is a fuzzy closed set if and only if r-cl($\alpha$)- $\alpha$is a fuzzy −openset in fts X.

**Proof:** Suppose α is a fuzzy closed set in fts X. Then r-cl($\alpha$)= $\alpha$and so r-cl($\alpha$)- $\alpha = 0$, which is a fuzzy open set in fts X. Conversely suppose r-cl($\alpha$)- $\alpha$ is a fuzzy $open$ set in fts X. Since $\alpha$is fuzzy gr-closed, by corollary 2.18 r-cl($\alpha$)- $\alpha$ does not contain any non-zero fuzzy regular open set in fts X then That is r-cl($\alpha$)= $\alpha$and hence $\alpha$is a fuzzy closed set in fts X.

We introduce a fuzzy gr-open set in fuzzy topological space X as follows.

**Definition2.21:** A fuzzy set $\alpha$ of a fuzzy topological space X is called a fuzzy generalized regular-open (briefly, fuzzy gr-open) set if its complement $\alpha^{C}$is a fuzzy gr-closed set in fts X.

We denote the family of all fuzzy gr-open sets in fts X by FO(X).

**Theorem2.22:** If a fuzzy set $\alpha$of a fuzzy topological space X is fuzzy open, then it is fuzzy gr-open but not conversely.

**Proof:** Let $\alpha$be a fuzzy open set of fts X. Then $\alpha^{C}$is fuzzy closed. Now by Theorem 2.2, $\alpha^{C}$ is fuzzy gr-closed. Therefore $\alpha$is a fuzzygr-open set in fts X.

The converse of the above Theorem need not be true in general as seen from the following example.

**Example2.23:**

(ii)Let X= {a, b, c} and define the fuzzy set $\alpha$ in X by $\alpha$: X ⟶ [0, 1] be defined as

$\alpha$ (x) = 1 if x = a
0 otherwise

Let T= {1,0,$\alpha$}Then (X, T) is a fuzzy topological space. Then the fuzzy set $\beta$ in X by

$\beta$ (x) = 1 if x = a,c
0 otherwise.

Then $\beta$ is a fuzzy gr-open but it is not fuzzy open set in fts X.

**Corollary2.24:** By K.K.Azad, we know that, every fuzzy regular open set is a fuzzy open set but not conversely. By Theorem 2.23, every fuzzy open set is a fuzzy gr-open set but not conversely and hence every fuzzy regular open set is a fuzzy gr-open set but not conversely.

**Theorem2.25:** A fuzzy set $\alpha$of a fuzzy topological space X is fuzzy gr-open if and only if $\delta \leq s-$ int($\alpha$) whenever and $\delta \leq \alpha$ and $\delta$ is a fuzzy $open$set in fts X.

**Proof:** Suppose that $\delta \leq s-$int($\alpha$) whenever $\delta \leq \alpha$and $\delta$ is a fuzzy $open$set in fts X. To prove that $\alpha$is fuzzy gr-open in fts X. Let $\alpha^C \leq \beta$and $\beta$is any fuzzy $open$ set in fts X. Then $\beta^c \leq \alpha$, since $\beta$ is fuzzy open in fts X; then 1-$\beta$is also fuzzy-open set in fts X.then $\beta^c$ is also fuzzy-open in fts X By hypothesis $\beta^c \leq$s-int($\alpha$)Which implies [s-int($\alpha$)]$^c \leq \beta$ that is s-int($\alpha^c$)$\leq \beta^c$ since r-cl($\alpha^c$ )= [s $-$ int($\alpha$)]c,Thus$\alpha^C$ is a fuzzy gr-closed and hence $\alpha$ is fuzzy gr-open in X.

Conversely, suppose that $\alpha$ is fuzzy gr-open. Let $\beta \leq \alpha$and $\beta$is any fuzzy $open$ in fts X. Then$\alpha^C \leq$ $\beta^c$since $\beta$ is fuzzy open in fts X,then 1-$\beta$ is fuzzy -open in X;then $\beta^c$ is also fuzzy open in fts X.since $\alpha^C$ is fuzzy gr-closed,we have r-cl($\alpha^C$ ) $\leq \beta^c$and so,$\beta \leq$s-int($\alpha$), since r-cl($\alpha^c$ ) = [s $-$ int($\alpha$)]c.

**Theorem2.26:** If $\alpha$and $\beta$are fuzzy gr-open sets in a fts X, then $\alpha \wedge \beta$ is also
a fuzzy gr-open set in fts X.

**Proof:** Let $\alpha$and $\beta$ be two fuzzy gr-open sets in a fts X. Then $\alpha^C$ and $\beta^C$are fuzzy gr-closed sets in fts X. By Theorem 2.8, $\alpha^C \vee \beta^C$is also a fuzzy grclosed set in fts X.
That is $\alpha^C \vee \beta^C = \alpha^C \wedge \beta^C$is a fuzzy gr-closed set in X. Therefore $\alpha \wedge \beta$is also a fuzzy gr-open set in fts.

**Example2.27:**

Consider the fuzzy topological space (X, T) defined in 2.6

Let X= {a, b, c,d},T= {1,0,$\alpha$,$\beta, \gamma, \delta$}in this fts X, The fuzzy sets $\delta_1 \delta_2$ :X → [ 0, 1] are defined by

$\delta_1$(x) = 1 if x = a
0 otherwise.

$\delta_2$ (x) = 1 if x = c
0 therwise

Then $\delta_1 and\ \delta_2$ are fuzzy gr open sets in fts X.

Let μ = $\delta_1 \vee \delta_2$, then

μ (x) = 1 if x = a ,c
0 otherwise.

μ = $\delta_1 \vee \delta_2$, Is not a fuzzy gr-closed set in fts X.

**Theorem2.28:** If s-int($\alpha$)≤ $\beta \leq \alpha$and $\alpha$ is a fuzzy gr-open set in a fts X, then $\beta$ is also a fuzzy rw-open set in fts X.

**Proof:** Suppose s-int($\alpha$)≤ $\beta \leq \alpha$and $\alpha$ is a fuzzy gr-open set in a fts X. To prove that $\beta$ is a fuzzy gr-open set in fts X. Let $\sigma$be any fuzzy rg$\alpha$-openset in fts X such that $\sigma \leq \beta$.Now $\sigma \leq \beta \leq \alpha$That is $\sigma \leq \alpha$ .
Since $\alpha$ is fuzzy gr-open set of fts X, $\sigma \leq S - int(\alpha )$by Theorem 2.26. By hypothesis s-int($\alpha$)≤ $\beta$.Then int [s-int($\alpha$)] ≤ s-int ($\beta$)That is s-int($\alpha$) ≤s-int($\beta$).
Then $\sigma \leq$ s-int($\beta$) Again by Theorem 2.26 $\beta$ is a fuzzy gr-open set in fts X.

**Theorem 2.29:** If a fuzzy subset $\alpha$of a fts X is fuzzy gr-closed and r-cl($\alpha$) ∧ (1-r-cl($\alpha$)) = 0, then r-cl($\alpha$)- $\alpha$is a fuzzy gr-open set in fts X.

**Proof:** Let $\alpha$be a fuzzy gr-closed set in a fts X and Let r-cl($\alpha$) ∧ (1-r-cl($\alpha$)) = 0,
Let $\beta$ be any fuzzy open set of fts X such that $\beta$ ≤s-int($\alpha$), Then by corollary 2.18 r-cl($\alpha$)- $\alpha$ does not contain any non-zero fuzzy open set and so $\beta$= 0.
Therefore $\beta$ ≤s-int($\alpha$)(cl($\alpha$)- $\alpha$. By theorem 2.26 p-cl($\alpha$)- $\alpha$is a fuzzy gr-open set in fts X.

**Theorem2.30:** Let $\alpha$and $\beta$be two fuzzy subsets of a fts X. If $\beta$is a fuzzy gr-open set and $\alpha$ ≥s-int($\beta$), then$\alpha \wedge \beta$is a fuzzy gr-open set in fts X.

**Proof:** Let $\beta$ be a fuzzy gr-open set of a fts X and $\alpha$ ≥s-int($\alpha$),That is s-int(B) ≤ $\alpha \wedge \beta$ .
Also s-int(B) ≤ $\alpha \wedge \beta$ and $\beta$is a fuzzy gr-open set. By Theorem 2.29,$\alpha \wedge \beta$ is also a fuzzy gr-open set in fts X.

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