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# Accelerating Anisotropic Bianchi Type-V Model with Barotropic Matter and Viscous Dark Energy in Lyra Geometry

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***Abstract***
*The present study deals with spatially homogeneous and anisotropic Bianchi type-V universe filled with barotropic matter and viscous dark energy in Lyra geometry. To get the solutions of the field equations, we consider the hybrid expansion law (HEL) for the average scale factor proposed by Akarsu et al. [69]. To differentiate between different dark energy models, we perform a geometrical diagnostic with the help of statefinder pair* $\{r, s\}$. *The physical and geometrical behaviors of the model are studied in detail.*

***Keywords:*** *Bianchi types-V metric, viscous dark energy, statefinder parameters, Lyra geometry.*

## I. Introduction

Today's reality is that, we are living in an accelerating and expanding universe. Firstly, Type Ia Supernovae gives direct proof for such acceleration and expansion of the universe [1],[2]. After some time, observations like measurements of CMB (Cosmic Microwave Background) [3]-[5] and galaxy power spectrum [6]-[8] have shown that universe is expanding rapidly. The accelerating phase of cosmic expansion might be due to the presence of mysterious energy known as dark energy [9]. Dark energy occupies 68.3% of total energy density. On the other hand, there is other exotic matter so-called dark matter consists 26.8% of the total energy density and barotropic matter is 4.9 % of the total energy density.

The equation of state (EoS) parameter of dark energy is $\omega^{de} = p^{de}/\rho^{de}$,where $\rho^{de}$ is density and $p^{de}$ is pressure of the dark energy respectively, gives the study of DE. Firstly, the observed cosmic acceleration has been explained by cosmological constant ($\Lambda$) having equation of state parameter $\omega^{de} = -1$. But it has coincidence problem and fine tuning problem. To overcome these problems, many authors have considered dynamical $\omega^{de}$ like quintessence ($-1 < \omega^{de} < -1/3$) [10],[11], quintom ($\omega^{de} < -1/3$) [12], Phantom ($\omega^{de} < -1$) [13], chaplygin gas and generalized chaplygin gas models [14]-[16] etc. Quintessence dark energy models introduce a scalar field $\phi$ that is minimally coupled to the gravity. Cosmic coincidence problem's solution provided by modified version of quintessence, but this field with $\omega^{de} > -1$ isn't in agreement with recent observational data [17]-[20], which denotes that $\omega^{de} < -1$ allows at 68% confidential level. As today's observational evidence include the possibility of $\omega^{de} < -1$, a new type of scalar field model having negative energy, called as a phantom field model have been introduced [13]. Two important problems namely, the Big-Rip [21],[22] of the universe as well as the ultraviolet quantum instabilities problem [23] have been encountered by this given scenario. A new scenario of DE named as a quintom was putforword for the model with transition from $\omega^{de} > -1$ to $\omega^{de} < -1$ be partially favored by current observational data [24]. This model may be assumed to be amalgamation of quintessence as well as phantom dark energy models, which may attain the transition from $\omega^{de} > -1$ to $\omega^{de} < -1$. To avoid the occurrence of the Big-Rip in the dissipative DE models having negative pressure, can be considered to be responsible for the present acceleration, so an effective bulk viscosity have been put forward [25]. Cosmic media is not a perfect fluid, has been indicated by some astrophysical observational data such as remarkable degree of the isotropy of CMBR (Cosmic microwave background radiation) as well as the large entropy per baryon [26]. Since, the viscosity effect can be concerned in the universe evolution. Zimdhal et al. [27] and Balakin et al. [28] discussed the role of viscous pressure as an agent that drives the current acceleration of the universe. Padmanabhan and Chitre [29] have already discussed the possibility of a viscosity dominated lake epoch of the universe with accelerated expansion. From different point of view, many authors investigated the effect of bulk viscosity on the background of the expansion of the universe [30]-[36]. Recently viscous dark energy and thermodynamics of viscous dark energy as well as

generalized second law of thermodynamics have been investigated by Sheykhi and Setare [37]. Many authors have studied dark energy models in general relativity and alternative theories of gravitation [38]-[44].
The restriction of general relativity in giving satisfactory explanation of evolution of phase of the universe has been led cosmologists to acquire various hypothesis as well as study their inference in this context. These hypothesis contain those allocating i) the time-dependence of the cosmological term and the gravitational constant ii) other physical or geometrical field with the universe iii) modified theories of gravity. Such theories are expected to disclose a number of aspects of physical and mathematical interests associated with them. To generalize the objective of geometrizing the gravitation, to involve an electromagnetism's geometrical description, many researchers have used their effects as the Einstein's theory of GR has based on the geometrical description of gravitation. In Lyra's geometry , Wely's concept of gauge , which is essential a metrial concept , is modified by the introduction of a gauge function into the structure less manifold , as a result of which the cosmological constant arises naturally from the geometry. Halford [45] studied cosmological theory within the framework of Lyra's geometry plays similar role of cosmological constant in general theory of relativity. Soleng [46] has pointed out that the constant displacement field in Lyra geometry will either include a creation field or contain a special vacuum field which together with the gauge vector term be considered as a cosmological term. Weyl [47] had suggested a new kind of gauge theory which includes a metric tensor to geometrize the gravitation and the electromagnetism. Lyra [48] putforword a geometry, called as Lyra geometry, which is nothing but the modification of Riemannian geometry in which gauge function having structureless manifold is presented, as a result of which the cosmological constant appears naturally from the geometry. Sen and Dunn [49] and Sen [50] have developed a new scalar tensor-theory based on Lyra geometry and similar to Einstein theory of GR. Using constant displacement vector, cosmological model based on Lyra geometry was studied by Soleng [51]. Gad [52] has presented an axially symmetric cosmological mesonic stiff fluid model in Lyra's geometry. Adhav [53] studied LRS Bianchi type–I universe with anisotropic dark energy cosmological model with early deceleration as well as late-time acceleration in Lyra geometry. Pradhan and Singh [54] studied anisotropic Bianchi type-I string cosmological model in normal gauge for Lyra geometry with constant deceleration parameter. Katore and Hatwar [55] have presented a Kaluza-Klein universe filled with magnetized anisotropic dark energy in Lyra geometry. Shri Ram et al. [56] have investigated a hypersurface and homogeneous cosmological model using dynamical EoS parameter in the text of Lyra geometry. Shri Ram [57] has investigated a Kantowski-Sachs universe with anisotropic dark energy in Lyra geometry. Recently, many authors have studied cosmological models in different context in Lyra's geometry [58]-[63].Recently Kadam [64] explored Holographic Dark Energy Model in Brans - Dicke Theory of Gravitation, Edlabadkar et al. [65] have studied Behaviour of the Strange Quark Matter with Domain walls in 5D Kaluaza-Klein Theory of Gravitation, Hypersurface Homogeneous Cosmological Models with Stiff-Matter in General Relativity have been investigated by Mantu Kumar [66].
Here, we study the anisotropic Bianchi type-V universe filled with barotropic matter and viscous dark energy in Lyra geometry. We have used hybrid expansion law for average scale factor, to obtain solutions of the field equations. The Statefinder diagnostic is applied. The physical and geometrical behaviors of the model are also studied.

## II. Metric and field equations

The spatially homogeneous and anisotropic Bianchi type-V metric can be written as

$$ds^2 = -dt^2 + A^2dx^2 + e^{2\alpha x}(B^2dy^2 + C^2dz^2), \tag{1}$$

where $A$, $B$ and $C$ are the cosmic scale factors and are functions of the cosmic time $t$ only and $\alpha \neq 0$ is an arbitrary constant.
The average scale factor $a$ for Bianchi type-V model (1) is defined as

$$a = (ABC)^{1/3}. \tag{2}$$

The field equation in normal gauge for Lyra geometry is given by Sen [50] as

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\left(\phi_i\phi_j - \frac{1}{2}g_{ij}\phi_\nu\phi^\nu\right) = {}^{m}T_{ij} + {}^{de}T_{ij}, \tag{3}$$

where $R_{ij}$ is the Ricci tensor, $R$ is the Ricci scalar, $\phi_i$ refers to the time-varying displacement field vector, which is defined as,

$$\phi_i = (0,0,0,\beta), \tag{4}$$

where $\beta$ is called the displacement vector.

Here ${}^{m}T_{ij}$ and ${}^{de}T_{ij}$ are the energy momentum tensors of barotropic matter and viscous dark energy respectively. These are given as

$$^{m}T_{ij} = \text{diag}[-\rho^{m}, p^{m}, p^{m}, p^{m}] = \text{diag}[-1, \omega^{m}, \omega^{m}, \omega^{m}]\rho^{m}, \tag{5}$$

and

$$^{de}T_{ij} = \text{diag}\,[-\rho^{de}, p^{de}, p^{de}, p^{de}] = \text{diag}[-1, \omega^{de}, \omega^{de}, \omega^{de}]\rho^{de}, \tag{6}$$

where $\rho^m$ and $p^m$ denote the energy density and pressure of the barotropic matter and $\omega^m = p^m / \rho^m$ ($0 \le \omega^m \le 1$) is its EoS parameter. Similarly $\rho^{de}$ and $p^{de}$ denote the energy density and pressure of the viscous dark energy while $\omega^{de} = p^{de} / \rho^{de}$ is its EoS parameter.

The effective pressure of viscous dark energy is given by [67]

$$p_{eff}^{de} = p^{de} + \Pi, \tag{7}$$

where $\Pi = -\xi(\rho^{de})3H$ is the viscous coefficient and $H = \frac{1}{3}\frac{\dot{V}}{V}$ is the mean Hubble parameter. It is vital to point out that as a consequence of the positive sign of the entropy change in an irreversible process, $\xi$ has to be positive [68].In general $\xi(\rho^{de}) = \xi_0(\rho^{de})^{\tau}$, where $\xi_0(>0)$ and $\tau$ are the constant parameters [34]-[36].

In a co-moving coordinate system, for metric (1), the field equation (3) using equations (4)-(7) subsequently leads to following set of equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} + \frac{3\beta^2}{4} = -\omega^m\rho^m - \omega_{eff}^{de}\rho^{de} + \Pi \tag{8}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{\alpha^2}{A^2} + \frac{3\beta^2}{4} = -\omega^m\rho^m - \omega_{eff}^{de}\rho^{de} + \Pi \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} + \frac{3\beta^2}{4} = -\omega^m\rho^m - \omega_{eff}^{de}\rho^{de} + \Pi \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{C}\dot{A}}{CA} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\alpha^2}{A^2} - \frac{3\beta^2}{4} = \rho^m + \rho^{de} \tag{11}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0, \tag{12}$$

$$\frac{3}{2}\beta\dot{\beta} + \frac{3\beta^2}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \tag{13}$$

The energy conservation equation $(T_{ij;j} = 0)$ yields

$$\dot{\rho}^m + 3(1+\omega^m)\rho^m H + \dot{\rho}^{de} + 3(1+\omega_{eff}^{de})\rho^{de} H = 0. \tag{14}$$

The Raychaudhuri equation for given distribution is found to be [34, 36]

$$\frac{\ddot{a}}{a} = \frac{1}{2}\xi\theta - \frac{1}{6}\left(\rho^{de} + 3p^{de}\right) - \frac{1}{6}\left(\rho^m + 3p^m\right) - \frac{2}{3}\sigma^2, \tag{15}$$

where $\sigma^2$ is the shear scalar which is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij},\ \sigma_{ij} = u_{i;j} + \frac{1}{2}(u_{i;k}u^k u_j + u_{j;k}u^k u_i) + \frac{\theta}{3}(g_{ij} + u_i u_j),$$

where $\theta = 3H$ is the expansion scalar and $u^i = (1,0,0,0)$ is the four-velocity considered to satisfy

$$g_{ij}u^i u^j = u^i u_i = -1$$

Here, we consider that two-fluid minimally interact with each other. So, the general form of conservation equation (14) can be written in the following form for barotropic matter and viscous dark energy as

$$\dot{\rho}^m + 3\left(1+\omega^m\right)\rho^m H = 0, \tag{16}$$

$$\dot{\rho}^{de} + 3\left(1+\omega_{eff}^{de}\right)\rho^{de} H = 0. \tag{17}$$

Integrating equation (12), we obtain

$$A^2 = lBC. \tag{18}$$

where $l$ is the constant of integration, consider $l = 1$.

Subtracting equation (8) from equation (9), equation (8) from equation (10) and equation (9) from equation (10), taking second integral of each, we obtain

$$\frac{A}{B} = d_1 \exp\left(k_1 \int \frac{dt}{V}\right), \tag{19}$$

$$\frac{A}{C} = d_2 \exp\left(k_2 \int \frac{dt}{V}\right), \tag{20}$$

$$\frac{B}{C} = d_3 \exp\left(k_3 \int \frac{dt}{V}\right), \tag{21}$$

where $d_1$, $d_2$, $d_3$ , $k_1$, $k_2$ and $k_3$ are the constants of integration.
From equations (19), (20) and (21); the scale factors can be explicitly written as

$$A(t) = V^{1/3}, \tag{22}$$

$$B(t) = DV^{1/3} \exp\left(K \int \frac{dt}{V}\right), \tag{23}$$

$$C(t) = D^{-1} V^{1/3} \exp\left(-K \int \frac{dt}{V}\right), \tag{24}$$

where $D = \sqrt[3]{d_2 d_3}$, $K = \frac{k_2 + k_3}{3}$, $d_2 = d_1^{-1}$, $k_2 = -k_1$.

## III. Cosmological solutions

The field equations (8)-(13) are a system of six linearly independent equations with eight unknown parameters as $A$, $B$, $C$, $\beta$, $\Pi$, $\rho^m$, $\rho^{de}$, $\omega_{eff}^{de}$. Therefore, we consider additional condition relating these parameters for obtaining solutions of the system of equations.
Here, we consider the average scale factor $a$ as a combination of power-law and exponential law proposed by Akarsu et al. [69]

$$V = a^3 = (a_0 t^\gamma e^{\delta t})^3 \tag{25}$$

where $a_0$, $\gamma$ and $\delta$ are the non-negative constants. The relation (22) gives the exponential law when $\gamma = 0$ and the power-law when $\delta = 0$. This is the combination of exponential and power- law, which is commonly known as the hybrid expansion law (HEL).
Now, from equations (22)-(24), with the help of equation (25), we obtain scale factors as

$$A = a_0 t^\gamma e^{\delta t}, \tag{26}$$

$$B = D(a_0 t^\gamma e^{\delta t}) \exp\left(K \int (a_0 t^\gamma e^{\delta t})^{-3} dt\right), \tag{27}$$

$$C = D^{-1}(a_0 t^\gamma e^{\delta t}) \exp\left(-K \int (a_0 t^\gamma e^{\delta t})^{-3} dt\right). \tag{28}$$

The solution of equation (13), using equations (26), (27) and (28) is given by

$$\beta = \frac{c_1}{(a_0 t^\gamma e^{\delta t})^3}, \tag{29}$$

where $c_1$ is the constant of integration.

Integrating equation (16) and using equation (25) leads to the density of barotropic matter is given as

$$\rho^m = \rho_0{}^m V^{-(1+\omega^m)} = \rho_0{}^m \left(a_0 t^\gamma e^{\delta t}\right)^{-3(1+\omega^m)} . \tag{30}$$

From equation(11) and using equation (30), the density of viscous dark energy is given as

$$\rho^{de} = 3\left(\frac{\gamma}{t} + \delta\right)^2 - \frac{K^2 + 3/4\, c_1{}^2}{\left(a_0 t^\gamma e^{\delta t}\right)^6} - \frac{3\alpha^2}{\left(a_0 t^\gamma e^{\delta t}\right)^2} - \frac{\rho_0{}^m}{\left(a_0 t^\gamma e^{\delta t}\right)^{3(1+\omega^m)}} \tag{31}$$

By using equations (25) and (31), the viscosity coefficient is given by

$$\Pi = -3H\xi_0(\rho^{de})^\tau = -3\xi_0\left(\frac{\gamma}{t}+\delta\right)\left(3\left(\frac{\gamma}{t}+\delta\right)^2 - \frac{K^2+3/4\,c_1{}^2}{\left(a_0t^\gamma e^{\delta t}\right)^6} - \frac{3\alpha^2}{\left(a_0t^\gamma e^{\delta t}\right)^2} - \frac{\rho_0{}^m}{\left(a_0t^\gamma e^{\delta t}\right)^{3(1+\omega^m)}}\right)^\tau \tag{32}$$

The EoS parameter of viscous dark energy is obtained from the field equation (8) by using equations (26)-(32) as

$$\omega_{eff}^{de} = -\frac{3\left(\frac{\gamma}{t}+\delta\right)^2 - \frac{2\gamma}{t^2} + \frac{K^2+3/4\,c_1{}^2}{\left(a_0t^\gamma e^{\delta t}\right)^6} - \frac{\alpha^2}{\left(a_0t^\gamma e^{\delta t}\right)^2} + \frac{\omega^m\rho_0{}^m}{\left(a_0t^\gamma e^{\delta t}\right)^{3(1+\omega^m)}} + 3\xi_0\left(\frac{\gamma}{t}+\delta\right)\left(3\left(\frac{\gamma}{t}+\delta\right)^2 - \frac{K^2+3/4\,c_1{}^2}{\left(a_0t^\gamma e^{\delta t}\right)^6} - \frac{3\alpha^2}{\left(a_0t^\gamma e^{\delta t}\right)^2} - \frac{\rho_0{}^m}{\left(a_0t^\gamma e^{\delta t}\right)^{3(1+\omega^m)}}\right)^\tau}{3\left(\frac{\gamma}{t}+\delta\right)^2 - \frac{K^2+3/4\,c_1{}^2}{\left(a_0t^\gamma e^{\delta t}\right)^6} - \frac{3\alpha^2}{\left(a_0t^\gamma e^{\delta t}\right)^2} - \frac{\rho_0{}^m}{\left(a_0t^\gamma e^{\delta t}\right)^{3(1+\omega_m)}}} \tag{33}$$

## IV. Cosmological parameters

The mean Hubble parameter is defined and obtained as,

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\gamma}{t} + \delta \,. \tag{34}$$

The deceleration parameter is defined and obtained as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 + \frac{\gamma}{(\gamma+\delta t)^2} \,. \tag{35}$$

The anisotropic parameter of the expansion is defined and obtained as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2t^2K^2(a_0t^\gamma e^{\delta t})^{-6}}{3(\gamma+\delta t)^2} \,. \tag{36}$$

## V. Statefinder diagnostic

For the purpose of describing the current cosmic acceleration, several models of dark energy have been suggested by many authors. It is important to differentiate these various dark energy models. So, a new pair called as statefinder diagnostic pair $\{r, s\}$ has been introduced by Sahni et al. [70]. The parameters $r$ and $s$ depend on $a$ (scale factor) and are dimensionless, so $\{r, s\}$ is a geometric diagnostic. The Statefinder parameter $\{r, s\}$ is defined as

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3(q-1/2)} \,. \tag{37}$$

Sahni [70] and his co-workers are in large to allow curved universe models with the help of statefinder parameter, can be distinguish between various form of DE. For example phantom, chaplygin gas and quintessence models approach to $\Lambda CDM$ $\{r,s\}_{\Lambda CDM} = \{1,0\}$. Trajectories of chaplygin gas models lie in the region $s < 0$ , $r > 1$ whereas trajectories of phantom and quintessence model lie in the region $s > 0$ , $r < 1$.
The *s-r* plane containing trajectories which corresponds to other cosmological models show qualitatively different behaviors. A fixed point $\{r,s\}\big|_{\Lambda CDM} = \{1,0\}$ shown in the fig. 6 corresponds to flat $\Lambda CDM$ model. A good way which is provided for demonstration of 'distance' of this model from $\Lambda CDM$ , depart a given dark energy model from the fixed point [71]. For differentiation amongst a large variety of the DE models including quintessence, cosmological constant, the Chaplygin gas, interacting dark energy models and brane world models, the state finder may prove to be successful tool [72]-[74].
Here we obtain the state finder parameter as

$$r = \frac{\frac{-3\gamma}{t^2} + \frac{2\gamma}{t^2(\gamma+\delta t)} + \left(\frac{\gamma}{t}+\delta\right)^2}{\left(\frac{\gamma}{t}+\delta\right)^2} \quad \text{and} \quad s = \frac{\frac{-3\gamma}{t^2} + \frac{2\gamma}{t^2(\gamma+\delta t)}}{\frac{3\gamma}{t^2} - 4.5\left(\frac{\gamma}{t}+\delta\right)^2} \tag{38}$$

## VI. Discussion

**(i)*The deceleration parameter* ( $q$ ):** The sign of $q$ denotes whether the model inflates or not. A positive sign of $q$ indicates the decelerating model whereas the negative sign of $q$ denotes accelerating model. Recent observational data suggest that the universe is in phase of accelerating expansion. The evolution of the decelerating parameter is as shown in fig.1It clearly indicates that our model accelerates and attains the value $q = -1$.

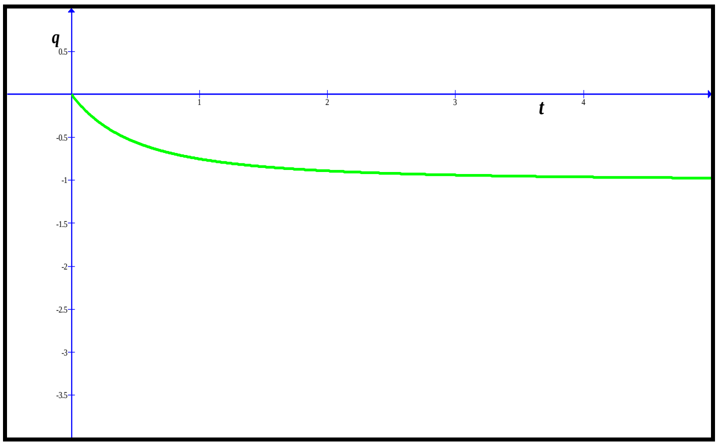


**Fig. 1:** ***The deceleration parameter*** $q$ ***versus time*** $t$ ***for*** $\gamma = 1, \delta = 1$**.**

**(ii)*****The anisotropy parameter of the expansion*****(**$\Delta$**):** The dynamical behavior of the anisotropic parameter of the expansion is as shown in fig. 2.It is observed that the anisotropy parameter $\Delta \to 0$ i.e. the model reduces to isotropy after some finite time, which is consistent with present day observation as the universe at present is isotropic on large scale.

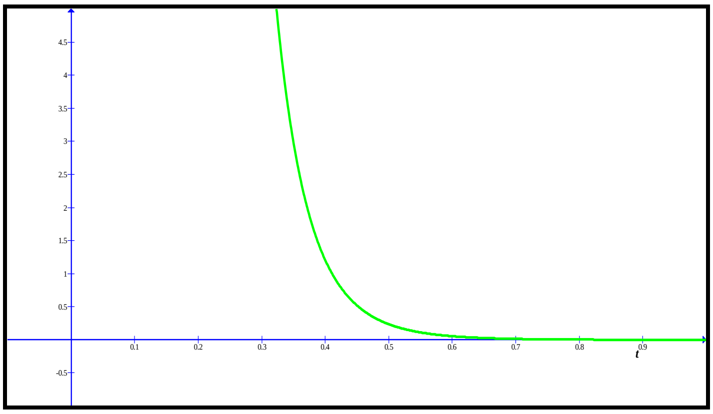


**Fig. 2**:***The anisotropic parameter of the expansion*** **(** $\Delta$ **)** ***versus time*** **(** $t$ **)** ***for*** $\gamma = 1, \delta = 1$**,** $a_0 = 1, K = 1$

**(iii)** ***The barotropic matter density*** **(** $\rho^m$ ): The evolution of the barotropic matter density ( $\rho^m$ ) is as shown in fig.3. It is observed that as $t \to 0$, $V \to 0$, $p^m \to \infty$ and $\rho^m \to \infty$, which shows that there is Big-Bang type of initial singularity and as $t \to \infty$, $\rho^m \to 0$ which indicates that our universe ends with vacuum universe.

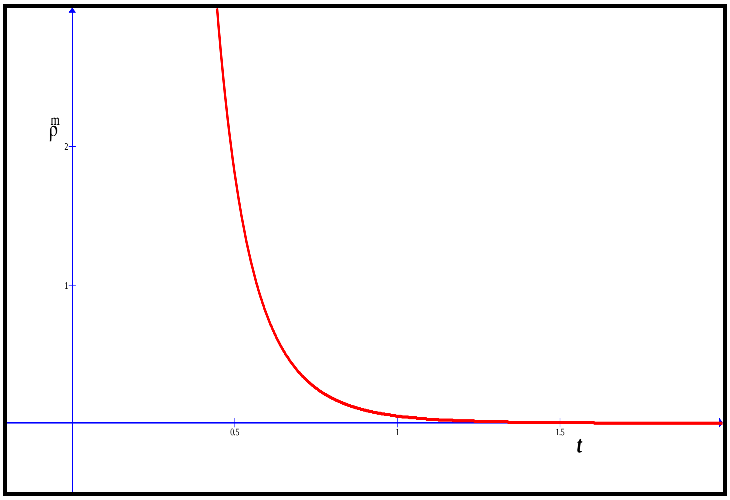


**Fig 3**:***The barotropic matter density (*** $\rho^m$ ***) versus time (*** $t$ ***) for*** $\rho_0^m = 1,\ a_0 = 1,$ $\gamma = 1, \delta = 1,\ \omega^m = 0$.

**(iv)** ***The EoS parameter of DE*** **(** $\omega^{de}$ **) :**The evolution of EoS parameter of dark energy is as shown in fig. 4.1 and fig. 4.2. Fig. 4.1 shows the evolution of EoS parameter of dark energy with viscosity ( $\omega_{eff}^{de}$ ) and without viscosity ($\omega^{de}$) indicated by red line and doted green line respectively. It is observed that as $t \to 0, \omega_{eff}^{de} \to \infty$. As time increases $\omega_{eff}^{de}$ starts from phantom region. After some finite time, it approaches to $\omega_{eff}^{de} = -1$(i.e. cosmological constant $\Lambda$ ), which denotes that our model reduces to $\Lambda CDM$ . Then, it enters and remains into quintessence region ( $-1 < \omega_{eff}^{de} < -1/3$ ) for further time $t$ , which matches with the present-day cosmological observation [75]. According to the plank observation [69], the value of $\omega^{de}$ is $-1.13_{-0.10}^{+0.13}$. Same results are obtained in the evolution of EoS parameter of dark energy without viscosity ($\omega^{de}$). It is observed that due to the presence of viscosity the $\omega_{eff}^{de}$ tends to $-1$ more quickly than without viscosity.

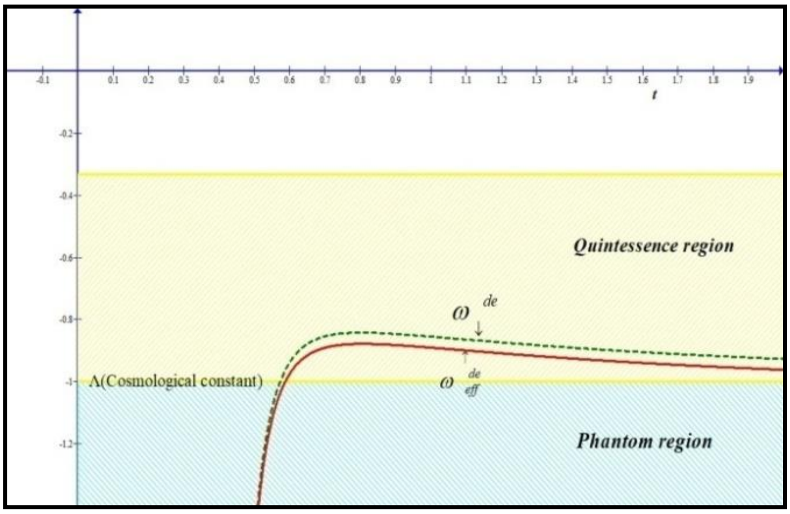


**Fig. 4.1**: ***The EoS parameter of dark energy (*** $\omega^{de}$ ***) v/s time (*** $t$ ***) for*** $\rho_0^m = 1, K = 1, c_1 = 1, a_0 = 1, \gamma = 1, \delta = 1,$ $\omega^m = 0, \xi = 0.02,\ \tau = 0.5$.

The evolution of EoS parameter of viscous dark energy ( $\omega_{eff}^{de}$ ) with displacement vector ( $\beta$ ) ( i.e. in Lyra Geometry) indicated by red line and without displacement vector ( $\beta$ ) (i.e. in Riemann Geometry of GR ) indicated by doted green line are shown in figure 4.2. It is observed that at initial time $\omega_{eff}^{de}$ diverges but as time increases at some finite time it crossed phantom divide line ( $\omega_{eff}^{de} = -1$) and it enter into the quintessence region and remain in quintessence region for later time. It is observed that due to presence of $\beta$ the $\omega_{eff}^{de}$ tends to $-1$ more quickly than without $\beta$.

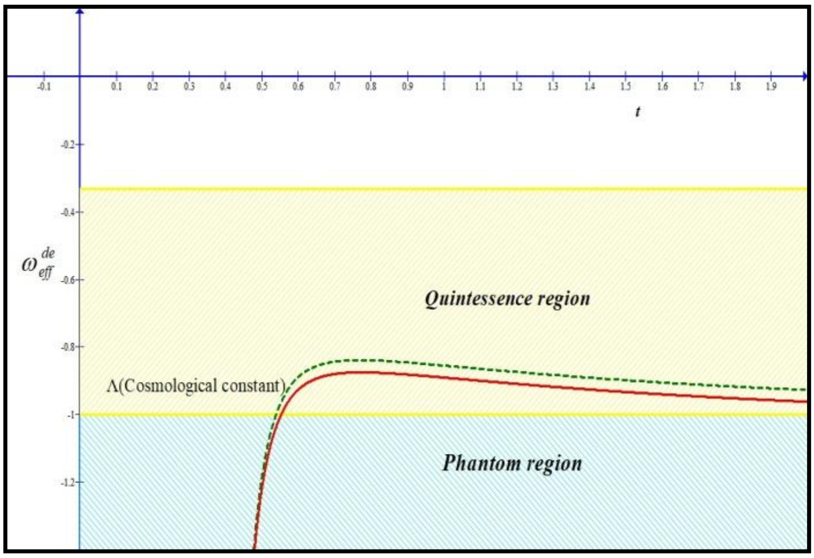


**Fig. 4.2:** ***The EoS parameter ( $\omega_{eff}^{de}$ ) versus time ( $t$ ) for*** $\rho_0^m = 1, K = 1,\ c_1 = 1, a_0 = 1,\ \alpha = 1, \gamma = 1, \delta = 1,$ $\omega^m = 0, \xi = 0.02,\ \tau = 0.5.$

**(v)*The displacement vector* ( $\beta$ ):**The evolution of displacement vector ( $\beta$ ) is as shown in fig. 5. It is observed that as $t \to 0$, $\beta \to \infty$ and after some finite time it becomes zero. So, our model reduces to GR after some finite time.

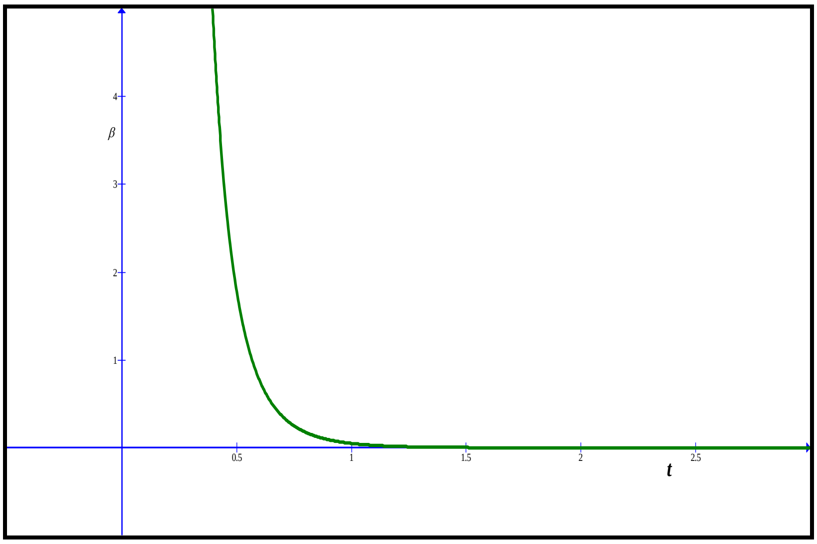


**Fig. 5:** ***The displacement vector* ( $\beta$ ) *versus time* ( $t$ ) for** $\gamma = 1, \delta = 1, c_1 = 1.$

**(vi)** ***The statefinder parameter***: The available observation in cosmology, especially the three year WMAP data [7], the SNeIa data [76],[77] and the SDSS data [78] pointed out that the $\Lambda CDM$ model or the model reducible to $\Lambda CDM$ be present as a standard model in cosmology, which is similar to present universe. The fig. 6 shows that the *s-r* plane having evolving trajectory for corresponding model is different from other dark energy models.

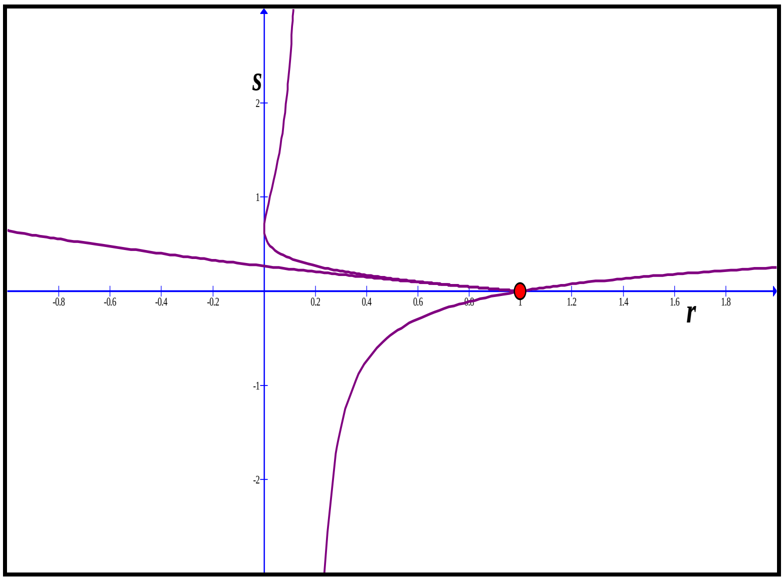


**Fig. 6:*The statefinder parameter $r$ versus $s$***

## VII. Conclusion

In the present paper, we have studied a spatially homogeneous and anisotropic Bianchi type-V universe filled with barotropic matter and viscous dark energy in Lyra geometry. We have used hybrid expansion law for average scale factor in order to obtain the solutions of the field equations. It is observed that there is Big-Bang type of initial singularity in the model. The decelerating parameter ( $q$ ) is dynamical and attain value $-1$,which indicates that the universe is accelerating. The model reduces to isotropy after some finite time as the anisotropy parameter of expansion tends to zero which is consistent with the current cosmological observations. It is found that EoS parameter of viscous dark energy starts from phantom region and remains in quintessence region for late time. The effect of viscosity and displacement vector ( $\beta$ ) on the EoS parameter of dark energy is also discussed. Also it is observed that our model of Lyra geometry reduces to Riemann geometry of GR after some finite time (as $\beta \to 0$ after some finite time). The Statefinder parameter pair $\{r, s\}$ has been calculated in order to differentiate our model with all other models of DE.

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