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# A Presentation of Rectifier Circuits using Semiconductor Diodes: Mathematical Models approach

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***Abstract:*** *This paper introduce the mathematical model for rectifier circuits. Here the characteristics of rectifier circuits using semiconductor diodes presented by mathematical model using a canonical system. According on the result of this study, we can propose an optimal process for an assembly line of rectifiers in electrical engineering.*

**Keywords:** *Rectifier circuit, differential models, differential inclusions.*

**MSC 2010: 34B6**

## I. INTRODUCTION

As we known that, mathematics is the tool for describing changes in each domain as dynamic systems, through which one can indicate their characteristics. One of the problems that attracts attention is to study by mathematical modelling an operation of rectifier circuits [1]-[5],[7][10]. Most electrical installations use direct current, but the power source is alternating current. Therefore, rectifiers are very important, indispensable and widely used in the electrical industry. A rectifier is an electric circuit consisting of electrical components used to convert alternating current to direct current. The positive elements in the rectifier circuit are semiconductor diodes. A semiconductor diode or diode is a type of semiconductor device that allows the current to flow through it in a single direction: from the anode to the cathode and without reversedirection [8],[9],[11].

In this paper, we will study the appropriate mathematical model for some common rectifier circuits in the science of electrical engineering, these are halfwave rectifying circuit and full-wave rectifying circuit. The mathematical model will be written by the canonical system which has the following form:

$$\begin{cases} \frac{dX}{dt} + \Im X + Y = F(t), \\ X \in E, \\ X \in E^*, \\ (X, Y) = 0. \end{cases} \tag{1}$$

where, a set $E$ and $E^*$ are conjiugate cone in the space $\mathbb{R}^n, X(t), Y(t)$ are unknown function whose values belong to $\mathbb{R}^n$ at moment $t$, $\Im$ is a known constant square matrix of order $n$, $F(t)$ is a known continuous vector function with its values in $\mathbb{R}^n$. There have been many studies showing that differential models are equivalent to some differential equations with discontinuous right hand sides, such as in [1]. These studies help to find solutions of differential models. Here, the solution of the system (1) is understood as a locally absolutely function which satisfies (1) almost everywhere.

The main content of this paper shows that, given a mathematical model for rectifier circuits, the model is presented by of the form (1).

## II. THE MATHEMATICAL MODEL FOR RECTIFIER CIRCUITS

As we known that, if in the rectifier circuit, there is only one semiconductor diode, it is the halfwave rectifier circuit. In this type of circuit, there is only half a cycle (positive or negative) that can pass the diode, however, the rest half cycle will be blocked. It depends on the direction of installation of the diode. And as, only half a cycle is rectified, the half-wave rectifier circuit achieves a very low efficiency of power transmission. The half-wave rectifier circuit includes a diode that is connected to a source with voltage $E(t)$, resistance $R$ and bobbin $L$. Suppose that $I$ and $U$ are in turn the intensity of the current and the voltage which passes from the positive pole to the negetive pole of the diode. Then

$$L\frac{dI}{dt} + RI + U = E(t). \tag{2}$$

If we denote $X = I, A = \frac{R}{L}, \mathcal{E}(t) = \frac{E(t)}{L}$ end $Y = \frac{U}{L}$ then the equation (2) can be written as

$$\dot{X} + AX + Y = \mathcal{E}(t). \tag{3}$$

According to the principle of the operation of the diodes (see [5]), we see that the vector $Y$ defined on the closed convex set $Q^* = (-\infty, 0] \subset \mathbb{R}$, can be written explicitly as following form:

$$\begin{cases} Y \in \{0\}, & if \quad X > 0 \\ Y = (-\infty, 0] & if \quad X = 0. \end{cases} \tag{4}$$

Therefore, the operation of the half-wave rectifier circuit will be written in the form of the cononical system of (1), that means

$$\begin{cases} \dot{X} + AX + Y = \mathcal{E}(t), \\ X \in Q, \\ X \in Q^*, \\ (X, Y) = 0, \end{cases} \tag{5}$$

where $Q = [0, \infty)$.

Let us consider an electrical circuit having a circuit diagram $S$ and including resistances, inductances and a diode converter $D$. Such that, the diode converter $D$ contains $m$ diodes. In each diode, positive current readily goes from the anode to the cathode. We denote the current and the voltage across the $j - th$ diode by $x_j, y_j$ $(j = 1, \dots, m)$ respectively. Assume that, diodes are ideal, that is, their currents $x_j$ and voltages $y_j$ are satisfied

$$\begin{cases} x_j \geq 0 \\ y_j \leq 0 \quad ; j = 1, \dots, m. \\ x_j y_j = 0 \end{cases} \tag{6}$$

Note that, from (4), for $x = (x_1, x_2, \dots, x_m)$ and $y = (y_1, y_2, \dots, y_m)$, it follows

$$x \in \mathbb{R}^n_+, y \in \mathbb{R}^n_- \text{ and } (x, y) = 0.$$

We see that, in the circuit diagram $S$ all nodes are numbered in some order from $0$ to $n$. We denote the current passing the $k - th$ node, the voltage between the node $k$ and node $0$ by $i_k, u_k$, $k = 0, \dots, n$ respectively. After that, we are interested in vectors $i_D = (i_1, i_2, \dots, i_n)$ and $u_D = (u_1, u_2, \dots, u_n)$ (the vectors $i_0$ and $u_0$ are not interested, because they are presented through, other currents, other voltages, respectively). In order to show this, denoting $\mathbb{T}_1$ is a tree consists of all nodes. By the first Kirchhoff's law, we have

$$\sum_{j=1}^{m} a_{kj} x_j = i_k, k = 1, \dots, n.$$

Consequently

$$Ax = i_D, \tag{6}$$

where $A = (a_{kj})_{n \times m}$ is a matrix whose elements receive values $1, -1$ and $0$, respectively, if $j - th$ diode's anode is connected with the $k - th$ node, $j - th$ diode's cathode is connected with the $k - th$ node and in other cases:

$$a_{kj} = \begin{cases} 1 \\ -1, (k = 1, \dots, n; j = 1, \dots, m). \\ 0 \end{cases} \tag{7}$$

First hand, we can see that $A$ is a matrix of one linear operator $(.): \mathbb{R}^n \longrightarrow \mathbb{R}^n$ satisfies (6). We note by $A^{-1}$ a matrix of an inverse operator $(.)^{-1}$.

On the other hand, using (3) we obtain

$$A^T u_D = y. \tag{8}$$

Now, we denote by $\mathbb{T}_2$ for a tree containing resistances $R$, inductances $L$ and a supply source. Therefore, branches complementing the tree $\mathbb{T}_2$ to the original circuit diagram $S$ are included resistances $r$, inductances $l$ and the diode converter $D$. From the second Kirchhoff's law, we have

$$U_R = M_1 u_r, \tag{9}$$

$$U_L = M_2 u_r + M_3 u_l + M_4 u_D + E(t), \tag{10}$$

where $E(t)$ depends on the voltage $e(t)$ of the supply source.

From (9) and (10), it implies that

$$\begin{pmatrix} U_R \\ U_L \end{pmatrix} = \begin{pmatrix} M_1 & 0 & 0 \\ M_2 & M_3 & M_4 \end{pmatrix} . \begin{pmatrix} u_r \\ u_l \\ u_D \end{pmatrix} + \begin{pmatrix} 0 \\ E(t) \end{pmatrix}.$$

Using the last equation and equation (3), we obtain

$$\begin{pmatrix} i_r \\ i_l \\ i_D \end{pmatrix} = - \begin{pmatrix} M_1^T & M_2^T \\ 0 & M_3^T \\ 0 & M_4^T \end{pmatrix} . \begin{pmatrix} I_R \\ I_L \end{pmatrix}.$$

Therefore

$$i_D = -M_4^T I_L, \tag{11}$$

$$i_l = -M_3^T I_L, \tag{12}$$

$$i_r = -M_2^T I_L - M_1^T I_R. \tag{13}$$

In order to find the mathematical model for rectifier circuits, we use obvious following equations

$$U_R = R I_R, \tag{14}$$

$$u_r = r i_r, \tag{15}$$

$$L \frac{dI_L}{dt} = U_L, \tag{16}$$

$$l \frac{di_l}{dt} = u_l. \tag{17}$$

where $L, I, R$ and $r$ are diagonal matrices whose diagonal elements are positive values. To solve the system (16) - (17), we consider $I_L$ and $u_D$ as the main unknowns. Further more, by (12) and (17) we get

$$M_3 u_l = -M_3 l M_3^T \frac{dI_L}{dt} \ . \tag{18}$$

Moreover, from equations (9), (13), (14) and (15), it implies that

$$u_r = ri_r = -r(M_2^T I_L + M_1^T I_R) = -rM_2^T I_L - rM_1^T R^{-1} M_1 u_r \ .$$

Thus, with $I$ is the identity matrix, we have

$$u_r = -(I + r\, M_1^T R^{-1} M_1)^{-1} r M_2^T I_L \ . \tag{19}$$

Using (10), (16), (18) and (19), characteristics of the research circuit are represented by

$$\Theta \frac{dI_L}{dt} + BI_L - M_4 u_D = E(t) \tag{20}$$

here $\Theta := \mathrm{L} + M_3 l M_3^T$ and $B := M_2(I + r\, M_1^T R^{-1} M_1)^{-1} r M_2^T I_L$ .

Note that

$$X = \Theta^{\frac{1}{2}} I_L; \ Y = \Theta^{-\frac{1}{2}}(-M_4) u_D. \tag{21}$$

And so, the equation (20) can be written as

$$\frac{dX}{dt} + \Im X + Y = F(t), \tag{22}$$

$$\begin{cases} \Im = \Theta^{-\frac{1}{2}} B \Theta^{-\frac{1}{2}}, \\ F(t) = \Theta^{-\frac{1}{2}} E(t). \end{cases} \tag{23}$$

In order to finish the proof of the theorem we will study properties of $X$ and $Y$.

First, by (21), we obtain

$$(X, Y) = \left(\Theta^{\frac{1}{2}} I_L, \Theta^{-\frac{1}{2}}(-M_4) u_D\right) = \left(I_L, \left(\Theta^{\frac{1}{2}}\right)^T \Theta^{-\frac{1}{2}}(-M_4) u_D\right).$$

Since the matrix $\Theta$ is diagonal, we can see that: $(X, Y) = (I_L, -M_4 u_D)$.

Furthermore, using (11), (6) and (8), we have

$$(X, Y) = (-M_4^T I_L, u_D) = (i_D, u_D) = (Ax, u_D) = (i_D, A^T u_D) = (x, y).$$

And, by (5) we also obtain

$$(X, Y) = 0. \tag{24}$$

Thus, using (22), (23) and (24), we obtain

$$\begin{cases} \dfrac{dX}{dt} + \Im X + Y = F(t), \\ X \in E, \\ X \in E^*, \\ (X, Y) = 0. \end{cases}$$

where $F(t), \Im$ are defined by (23).

Therefore, we have the following theorem:

**Theorem 2.1.** *The mathematical model for rectifier circuits is presented by differential models of the form* (1) *in which a function $F(t)$and a matrix $\Im$ are defined in (23).*

To illustrate the results of the study, we consider a electric circuit of a following figure known as a full wave rectifier; it contains 4 diodes, a source, resistance $R$ and inductance $L$. In a supply circuit there is a source including a voltage $e(t)$, resistance $r$ and inductance $l$. This case is used to show the mathematical model of the form (1) for the considered rectifier circuit. For this, the choice of positive voltage is marked by indicators and on the other all nodes are numbered by 0, 1, 2, 3 as the figure (see Figure 1).

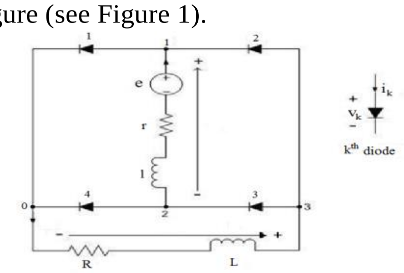


**Figure 1: The full wave rectifier**

For this rectifier circuit, we note the current and the voltage (from positive to negative) of the diodes $j$ ($j = 1, \dots, 4$) are $x_j$ and $y_j$ repectively. Then, by the operating principle of the diodes, we have

$$x_j \geq 0, y_j \leq 0, (x_j, y_j) = 0, j = 1, \dots, 4. \tag{25}$$

In addition, all the nodes through which the diodes can be connected to other circuits are numbered 0, 1, 2, 3 as shown in the figure. The intensity of the current passing the $k$-node $k = 0,1,2,3$ are denoted by $i_k$. The voltage between node $k$ and node 0 are denoted by $u_k$.

In following, we show the operation of the full wave rectifier circuit for the nonlinear analytical model of (1). Assuming that the direction of the current is marked as in the figure, then according to Kirchhoff's law, the operation of the circuit is represented by the following equations:

$$\begin{cases} l\frac{di_1}{dt} + ri_1 + u_1 - u_2 = e(t) \\ L\frac{di_3}{dt} + Ri_3 + u_3 = 0 \\ i_2 = i_3 - i_1. \end{cases} \tag{26}$$

If we define $I_L = \begin{pmatrix} i_1 \\ i_3 \end{pmatrix}$ then the first equation of the system (26) will be rewritten as the following equations:

$$L\frac{dI_L}{dt} + BI_L + Hu_D = E(t),$$

where

$$u_D = (u_1, u_2, u_3), L = \begin{pmatrix} l & 0 \\ 0 & L \end{pmatrix}, B = \begin{pmatrix} r & 0 \\ 0 & R \end{pmatrix},$$

and the matrix $H, E(t)$ are determined by

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, E(t) = \begin{pmatrix} e(t) \\ 0 \end{pmatrix}.$$

We use the following transform:

$$X = L^{1/2} I_L \text{ and } Y = L^{-\frac{1}{2}}(Hu_D). \tag{27}$$

Therefore, the operation of wave rectifier circuit can be written by the following differential equation:

$$\frac{dX}{dt} + \mathfrak{J}X + Y = \mathcal{E}(t), \tag{28}$$

where $\mathfrak{J} = \begin{pmatrix} \frac{r}{l} & 0 \\ 0 & \frac{R}{L} \end{pmatrix}$ and $\mathcal{E}(t) = \begin{pmatrix} \frac{e(t)}{l^{1/2}} \\ 0 \end{pmatrix}$.

The operation of wave rectifier circuit (27) will have the form of the canonical system (2), if we prove the existence of a conic $K$ in $\mathbb{R}^2$ space such that

$$\begin{cases} X \in K \\ Y \in K^* \\ (X, Y) = 0. \end{cases} \tag{29}$$

Indeed, according to first Kirchhoffs law, we have

$$i_1 = x_1 - x_2, i_2 = -x_3 + x_4, i_3 = x_2 + x_3, i_1 = -i_2. \tag{30}$$

From this, we obtain

$$\begin{cases} i_3 \geq 0, i_3 + i_1 \geq 0, i_3 - i_1 \geq 0, \\ i_3 \geq \|i_1\|. \end{cases} \tag{31}$$

On the other hand, by second Kirchhoffs law, we have

$$\begin{cases} u_1 - u_2 = y_1 - y_4 = y_3 - y_2 \\ u_3 = y_1 + y_2 = y_3 + y_4. \end{cases} \tag{32}$$

From the systhem (32), we have

$$\begin{cases} u_3 + u_1 - u_2 = y_1 + y_3 \leq 0 \\ u_3 - (u_1 - u_2) = y_2 + y_4 \leq 0. \end{cases}$$

Therefore

$$u_3 \leq -|u_1 - u_2| < 0. \tag{33}$$

From (27), (30), (32), we have

$$(X, Y) = \frac{1}{2}(x_2 y_4 + x_1 y_3 + x_3 y_1 + x_4 y_2). \tag{34}$$

By virtue of (25) we can obtain that $(X, Y) \leq 0$. To justify

$$(X, Y) = 0. \tag{35}$$

Now, we have to prove that each term on the right-hand side of (34) is equal to 0. We suppose an absurd, then:

$$x_2 y_4 < 0 \Leftrightarrow \begin{cases} x_2 > 0 \Rightarrow y_2 = 0 \\ y_4 < 0 \Rightarrow x_4 = 0 \Rightarrow x_1 = i_3 \Rightarrow y_1 = 0. \end{cases}$$

Hence, in conjunction with (32), it is easy to deduce $u_3 = y_1 + y_2 = 0$. This contradicts with (33), that mean, $x_2 y_4 = 0$. The remaining terms of the right-hand side of (34) are equal to 0.

Now, we define the cone $K \subset \mathbb{R}^2$ such that $X \in K$. From the transformation (27) and the estimate (31), it is easy to deduce:

$$X \in K = \left\{X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \in \mathbb{R}^2 : X_2 > 0, X_2 \geq \sqrt{\frac{L}{l}} X_1, X_2 \geq -\sqrt{\frac{L}{l}} X_1 \right\}.$$

Under the modification of variables (27) and the evaluation (33), we have:

$$Y \in K^* = \left\{ \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \in \mathbb{R}^2 : Y_2 < 0, Y_2 \leq \sqrt{\frac{l}{L}} Y_1, Y_2 \leq -\sqrt{\frac{l}{L}} Y_1 \right\}.$$

Thus, the characteristics of the full wave rectifier are presented by models of the form (1).

## III. CONCLUSION

Mathematical simulation of engineering systems from which to study in an overview, the nature of their operating principle is one of the most important applications of mathematics. The characteristics of the rectifier using semiconductor diodes have been investigated in this paper by establishing a mathematical model that describes these characteristics and analyzes the mathematical models received. We have also considered a concrete case to illustrate the result of the study.

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