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ii-regular Spaces in Topological Spaces

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Abstract: *The aim of this paper is to introduce and study a new class of spaces, namely ii-regular spaces by using ii-open sets. The relationships among regular, strongly rg-regular, almost regular, softly regular, weakly regular, α-regular, s-regular and ii-regular spaces are investigated. Also we obtain some characterizations of ii-regular spaces, properties of the forms of gii-closed, iig-closed functions and preservation theorems for ii-regular spaces.*

Key words: *ii-open sets; ii-regular, s-regular, almost regular and softly regular spaces; gii-closed and ii-gii-closed functions.*

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1. Introduction

N. Levine [**7**] introduced the concept of semi-open sets in topological spaces. O. Njastad [**10**] introduced and studied the notion of α-open sets. M. K. Singal and S. P. Arya [**12**] introduced two new classes of regular spaces, namely almost regular and weakly regular. S. N. Maheshwari and R. Prasad [**8**] defined a new class of regular spaces called s-regular. S. S. Benchalli [**1**] introduced and studied the notion of α-regular spaces. M. C. Sharma, P. Sharma and M. Singh [**11**] introduced a new class of regular spaces called ξ-regular spaces. Hamant Kumar [**6**] obtained some more characterizations and preservation theorems for ξ-regular spaces. Hamant Kumar and M. C. Sharma [**3**] introduced two new classes of separation axioms, namely softly regular and partly regular spaces which are weaker than regular spaces. Hamant Kumar [**4**] introduced some new types of separation axioms, namely ii-T_0, ii-T_1 and ii-T_2 etc. in topological spaces by using ii-open sets due to A. A. Mohammed and B. S. Abdullah [**9**]. Hamant Kumar [**5**] introduced two new classes of sets called gii-closed and iig-closed and by using these sets, obtained some characterizations of ii-normal spaces and properties of the forms of generalized ii-closed functions.

In this paper, we utilize ii-open sets to define and study a new class of spaces, called ii-regular spaces in topology. The relationships among regular, strongly rg-regular, almost regular, softly regular, weakly regular, α-regular, s-regular and ii-regular spaces are investigated. Also we obtain some characterizations and preservation theorems for ii-regular spaces.

2. Preliminaries

Throughout this paper, spaces (X, τ), (Y, σ), and (Z, γ) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and interior of A are denoted by **cl(A)** and **int(A)** respectively.

2.1 Definition. A subset A of a space (X, ℑ) is said to be:
(1) **semi-open** [**7**] if $A \subset cl(int(A))$.
(2) **α-open** [**10**] if $A \subset int(cl(int(A)))$.
(3) **ii-open** [**9**] set if there exists an open set $G \in ℑ$, such that
(i) $G \neq \phi$, X
(ii) $A \subset cl(A \cap G)$
(iii) int(A) = G.

2.2 Remark. We have the following implications for the properties of subsets:

open → α-open → s-open → ii-open

Where none of the implications is reversible as can be seen from [**9**].

The complement of a s-open (resp. α-open, ii-open,) set is called **s-closed** (resp. **α-closed**, **ii-closed**).

The intersection of all ii-closed sets containing A, is called the **ii-closure** of A and is denoted by **ii-cl(A)**. Dually, the **ii-interior** of A, denoted by **ii-int(A)** is defined to be the union of all ii-open sets contained in A.

The family of all ii-open (resp. ii-closed) sets of a space X is denoted by ii-O(X) (resp. ii-C(X)).

2.3 Definition. A subset A of a space $(X, \Im)$ is said to be
(1) **generalized ii-closed** [**5**] (briefly **gii-closed**) if ii-cl(A) $\subset$ U whenever A $\subset$ U and U $\in \Im$.
(2) **ii-generalized closed** [**5**] (briefly **iig-closed**) if ii-cl(A) $\subset$ U whenever A $\subset$ U and U $\in$ ii-O(X).

The complement of gii-closed (resp. iig-closed) set is said to be **gii-open** (resp. **iig-open**).

2.4 Remark. We have the following implications for the properties of subsets:

closed $\rightarrow$ ii-closed $\rightarrow$ gii-closed $\rightarrow$ iig-closed

Where none of the implications is reversible as can be seen from [**5**]:

2.5 Lemma. Let A be a subset of a space X and x $\in$ X. The following properties hold for ii-cl(A) :
(i) x $\in$ ii-c1(A) if and only if $A \cap U \neq \phi$ for every U $\in$ ii-O(X) containing x.
(ii) A is ii-closed if and only if A = ii-cl(A) .
(iii) ii-c1(A) $\subset$ ii-c1(B) if A $\subset$ B.
(iv) ii-c1(ii-c1(A)) = ii-c1(A).
(v) ii-c1(A) is ii-closed.

2.6 Lemma [5]. A subset A of a space X is gii-open in X if and only if F $\subset$ ii-int(A) whenever F $\subset$ A and F is closed in X.

4. ii-regular spaces.

4.1 Definition. A space X is said to be **ii-regular** (resp. **α-regular** [**1**], **s-regular [8], ξ-regular [11]**) if for each closed set F of X, and each point x $\in$ X − F, there exist disjoint ii-open (resp. α-open, s-open, ξ-open) set U, V such that F $\subset$ U and x $\in$ V.

4.2 Definition. A space X is said to be **softly regular** [**3**] (resp. **almost regular** [**12**], **strongly rg-regular** [**2**]) if for every π-closed (resp. regular closed, rg-closed) set F of X, and a point x $\in$ X − F, there exist disjoint open sets U and V such that F $\subset$ U and x $\in$ V.

4.3 Definition. A space X is said to be **weakly regular** [**12**] if for every point x and every regularly open set U containing x, there is an open set V such that x $\in$ V $\subset$ cl(V) $\subset$ U.

4.4 Theorem. Every regular space is ii-regular.

Proof. Let X be a regular space. Let F be any closed set in X and a point x $\in$ X such that x $\notin$ F. Since X is a regular space, there exists a pair of disjoint sets U and V such that F $\subset$ U and x $\in$ V. Since we know that every open set is ii-open. So U and V are ii-open sets. Hence X is ii-regular.

By the definitions stated above, we have the following diagram:

ξ-regular

$\Uparrow$

strongly rg-regular $\Rightarrow$ regular $\Rightarrow$ α-regular $\Rightarrow$ s-regular $\Rightarrow$ ii-regular

$\Downarrow$

softly regular $\Rightarrow$ almost regular $\Rightarrow$ weakly regular

Where none of the implications is reversible as can be seen from the following examples:

4.5 Example. Let $X = \{a, b, c\}$ and $\Im = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then the space $(X, \Im)$ is weakly regular. But it is neither almost regular nor softly regular.

4.6 Example. Let $X = \{a, b, c, d\}$ and $\Im = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$. Then the space $(X, \Im)$ is almost regular but not strongly rg-regular.

4.7 Example. Let $X = \{a, b, c\}$ and $\Im = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Then the space $(X, \Im)$ is regular.

4.8 Example. Let $X = \{a, b, c\}$ and $\Im = \{\phi, \{a\}, \{b, c\}, X\}$. Then the space $(X, \Im)$ is regular but not strongly rg-regular. Since $F = \{b\}$ is a rg-closed set such that $c \notin \{b\}$. We can not separate c and $\{b\}$ by disjoint open sets.

4.9 Theorem. The following properties are equivalent for a space X:

(a) X is ii-regular.

(b) For each $x \in X$ and each open set U of X containing x, there exists $V \in$ ii-O(X) such that $x \in V \subset$ ii-cl(V) $\subset U$.

(c) For each closed set F of X, $\cap$ {ii-cl(V) : $F \subset V \in$ ii-O(X)} = F.

(d) For each subset A of X and each open set U of X such that $A \cap U \neq \phi$, there exists $V \in$ ii-O(X) such that $A \cap V \neq \phi$ and ii-cl(V) $\subset U$.

(e) For each non empty subset A of X and each closed subset F of X such that $A \cap F = \phi$, there exist V, W $\in$ ii-O(X) such that $A \cap V \neq \phi$, $F \subset W$ and $V \cap W \neq \phi$.

Proof.

(a) $\Rightarrow$ (b). Let U be an open set containing x, then $X - U$ is closed in X and $x \notin X - U$. By (a), there exist W, V $\in$ ii-O(X) such that $x \in V$, $X - U \subset W$ and $V \cap W = \phi$ .By **Lemma 2.5**, we have ii-cl(V) $\cap W = \phi$ and hence $x \in V \subset$ ii-cl(V) $\subset U$.

(b) $\Rightarrow$ (c). Let F be a closed set of X. If $F \subset V$, then by **Lemma 2.5 (iii)**, ii-cl(F) $\subset$ ii-cl(V) which gives $F \subset$ ii-cl(V) as $F \subset$ ii-cl(F). Therefore, $\cap$ {ii-cl(V) : $F \subset V \in$ ii-O(X)} $\supset$ F.

Conversely, let $x \notin F$. Then $X - F$ is an open set containing x. By (b), there exists $U \in$ ii-O(X) such that $x \in U \subset$ ii-cl(U) $\subset X - F$. Put $V = X -$ ii-cl(U). By **Lemma 2.5,** $F \subset V \in$ ii-O(X) and $x \notin$ ii-cl(V). This implies that $\cap$ {ii-cl(V) : $F \subset V \in$ ii-O(X)} $\subset$ F.

Hence $\cap$ {ii-cl(V) : $F \subset V \in$ ii-O(X)} = F.

(c) $\Rightarrow$ (d). Let A be a subset of X and let U be open in X such that $A \cap U \neq \phi$. Let $x \in A \cap U$, then $X - U$ is a closed set not containing x. By (c), there exists $W \in$ ii-O(X) such that $X - U \subset W$ and $x \notin$ ii-cl(W). Put $V = X -$ ii-cl(W). Then $V \subset X - W$. Also $x \in V \cap A$. By using **Lemma 2.5**, we obtain $V \in$ ii-O(X), and ii-cl(V) $\subset$ ii-cl(X $-$ W) = $X - W \subset U$.

(d) $\Rightarrow$ (e). Let A be a subset of X and let F be a closed set in X such that $A \cap F = \phi$, where $A \neq \phi$. Since $X - F$ is open in X and $A \neq \phi$, by (d), there exists $V \in$ ii-O(X) such that $A \cap V \neq \phi$ and ii-c1(V) $\subset X - F$. Put $W = X -$ ii-c1(V), then $F \subset W$. Also, $V \cap W = \phi$. By **Lemma 2.5**, $W \in$ ii-O(X).

(e) $\Rightarrow$ (a). This is obvious.

4.10 Theorem. A topological space $(X, \Im)$ is ii-regular if and only if for each closed set F of $(X, \Im)$ and each $x \in X - F$, there exist ii-open sets U and V of $(X, \Im)$ such that $x \in U$ and $F \subset V$ and ii-cl(U) $\cap$ ii-cl(V) = ϕ.

Proof: Let F be a closed set in $(X, \Im)$ and $x \notin F$. Then there exist ii-open sets U_x and V such that $x \in U_x$, $F \subset V$ and $U_x \cap V = \phi$. This Implies that $U_x \cap$ ii-cl(V) = ϕ. Since ii-cl(V) is ii-closed and $x \notin$ ii-Cl(V). Since $(X, \Im)$ is ii-regular, there exist ii-open sets G and H of $(X, \Im)$ such that $x \in G$, ii-cl(V) $\subset$ H and $G \cap H = \phi$. This implies ii-cl(G) $\cap$ H = ϕ. Take $U = U_x \cap G$. Then U and V are ii-open sets of $(X, \Im)$ such that $x \in U$ and $F \subset V$ and ii-cl(U) $\cap$ ii-cl(V) = ϕ, since ii-cl(U) $\cap$ ii-cl(V) $\subset$ ii-cl(G) $\cap$ H = ϕ. Conversely, suppose for each closed set F of $(X, \Im)$ and each $x \in X - F$, there exist ii-open sets U and V of $(X, \Im)$ such that $x \in U$, $F \subset V$ and and ii-cl(U) $\cap$ ii-cl(V) = ϕ. Now $U \cap V \subset$ ii-cl(U) $\cap$ ii-cl(V) = ϕ. Therefore $U \cap V = \phi$. Thus $(X, \Im)$ is ii-regular.

4.11 Definition. A space X is said to be **ii-T_3 space** if it is ii-regular as well as ii-T_1 space.

4.12 Theorem. Every ii-T_3 space is an ii-T_2 space.

Proof. Let X be ii-T_3 , so it is both ii-T_1 and ii-regular. Also X is ii-T_1 $\Rightarrow$ every singleton subset {x} of X is an ii-closed. Let {x} be an ii-closed subset of X and $y \in X - \{x\}$. Then we have $x \neq y$ since X is ii-regular, there exist disjoint ii-open sets U and V such that $\{x\} \subset U$, $y \in V$, and such that $U \cap V = \phi$ (or) U and V are disjoint ii-open sets containing x and y respectively. Since x and y are arbitrary, for every pair of distinct points, there exist disjoint ii-open sets. Hence X is ii-T_2 space.

4.13 Theorem. Every subspace of an ii-regular space is ii-regular.

Proof. Let X be an ii-regular space. Let Y be a subspace of X. Let $x \in Y$ and F be a closed set in Y such that $x \notin F$. Then there is a closed set A of X with $F = Y \cap A$ and $x \notin A$. Since X is ii-regular, there exist disjoint ii-open sets U and V such that $x \in U$ and $A \subset V$. Note that $Y \cap U$ and $Y \cap V$ are ii-open sets in Y. Also $x \in U$ and $x \in Y$, which implies $x \in Y \cap U$ and $A \subset V$ implies $Y \cap U \subset Y \cap V$, $F \subset Y \cap V$. Also, $(Y \cap U) \cap (Y \cap V) = \phi$. Hence Y is ii-regular space.

4.14 Theorem. Every ii-compact Hausdorff space is an ii-T_3 space and hence an ii-regular space.

Proof. Let $(X, \Im)$ be a compact Hausdorff space, that is an ii-T_3 space. But every ii-T_2 space is ii-T_1. To prove that it is ii-T_3 space it is sufficient to prove that it is ii-regular. Let F be a closed subset of X, and $x \notin F$. Now $x \in X - F$ so that any point $y \in F$ is a point of X which is different from x. Since $(X, \Im)$ is an ii-T_2 space corresponding to x and y, there exists two ii-open sets H_y and G_y such that $G_y \cap H_y = \phi$ where $x \in H_y$ and $y \in G_y$. Now let $\Im^*$ denote the relative topology for F so that the collection $C^* = \{F \cap H_y : y \in F\}$ is an ii-$\Im^*$ open cover of F. But F is closed and since $(X, \Im)$ is ii-compact $(F, \Im^*)$ is also ii-compact. Hence a finite subcover of F (or) there exist points $y_1, y_2, \ldots y_n$ in F such that $C^* = \{F \cap H_{yi} : i = 1, 2, \ldots n\}$ is a finite sub cover for F. Now $\mathbf{F} = \cup \{F \cap H_{yi} : i = 1, 2, \ldots n\}$ or $F = F \cap \{\cup \{ H_{yi} : i = 1, 2, \ldots n\}\}$
Hence $F \subset \cup \{ H_{yi} : i = 1, 2, \ldots n\}$ or $F \subset H$ where $H = \cup \{H_{yi} : i = 1, 2, \ldots n\}$ is ii-open set containing H, being the union of ii-open sets. Again G_{yi} for $i = 1, 2, 3, \ldots n$ is ii-open set containing x and hence $G = \cap \{G_{yi} : i = 1, 2, \ldots n\}$ is also an ii-open set containing x.
Also $G \cap H = \phi$, otherwise $G_{yi} \cap H_{yi} \neq \phi$ for some i. Hence corresponding to each closed set F and an element x in $X - F$ we have two ii-open sets G and H such that $x \in G$, $F \subset H$ and $G \cap H = \phi$. Hence $(X, \Im)$ is ii-regular. Since it is ii-T_2 so ii-T_1 and hence $(X, \Im)$ ii-T_3.

5. Some related functions with ii-regular spaces

5.1 Definition. A function $f : X \to Y$ is said to be **ii-closed** [**4**] if for each closed set F of X, f(F) is ii-closed in Y**.**

5.2 Definition. A function $f : X \to Y$ is said to be
(i) **generalized ii-closed** [**5**] (briefly **gii-closed**) if for each closed set F of X, f (F) is gii-closed in Y.
(ii) **ii-generalized ii-closed** [**5**] (briefly **ii-gii-closed**) if for each ii-closed set F of X, f (F) is gii-closed in Y.

5.3 Remark. Every closed function is ii-closed but not conversely. Also, every ii-closed function is gii-closed because every ii-cosed set is gii-closed. It is obvious that both ii-closedness and ii-gii-closedness imply gii-closedness.

5.4 Theorem. A surjective function $f : X \to Y$ is gii-closed (resp. ii-gii-closed) if and only if for each subset B of Y and each open (resp. ii-open) set U of X containing $f^{-1}(B)$, there exists a gii-open set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof. Suppose that f is gii-closed (resp. ii-gii-closed). Let B be any subset of Y and U be open (resp. ii-open) set of X containing $f^{-1}(B)$. Put $V = Y - f(X - U)$. Then the complement V^c of V is $V^c = Y - V = f(X - U)$. Since $X - U$ is closed in X and f is gii-closed, $f(X - U) = V^c$ is gii-closed. Therefore, V is gii-open in Y. It is easy to see that $B \subset V$ and $f^{-1}(V) \subset U$.

Conversely, let F be a closed (resp. ii-closed) set of X. Put $B = Y - f(F)$, then we have $f^{-1}(B) \subset X - F$ and $X - F$ is open (resp. ii-open) in X. Then by assumption, there exists a gii-open set V of Y such that $B = Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Now $f^{-1}(V) \subset X - F$ implies $V \subset Y - f(F) = B$. Also $B \subset V$ and so $B = V$. Therefore, we obtain $f(F) = Y - V$ and hence f(F) is gii-closed in Y. This shows that f is gii-closed (resp. ii-gii-closed).

5.5 Remark. We can prove the necessity part of the above theorem by replacing each set to closed set in the form of the proposition given below:

5.6 Proposition. If a surjective function $f : X \to Y$ is gii-closed (resp. ii-gii-closed) then for a closed set F of Y and for any open (resp. ii-open) set U of X containing $f^{-1}(F)$, there exists an ii-open set V of Y such that $F \subset V$ and $f^{-1}(V) \subset U$.

Proof. By **Theorem 5.4**, there exists a gii-open set W of Y such that $F \subset W$ and $f^{-1}(W) \subset U$. Since F is closed, by **Lemma 2.6** we have $F \subset$ ii-int(W). Put V = ii-int(W). Then $V \in$ ii-O(Y), $F \subset V$ and $f^{-1}(V) \subset U$.

5.7 Proposition. If $f : X \to Y$ is continuous ii-gii-closed and A is gii-closed in X, then f(A) is gii-closed in Y .

Proof. Let V be a open set of Y containing f(A). Then $A \subset f^{-1}(V)$. Since f is continuous, $f^{-1}(V)$ is open in X. Since A is gii-closed in X, by a definition, we get ii-c1(A) $\subset f^{-1}(V)$ and hence f(ii-c1(A)) $\subset V$. Since f is ii-gii-closed and ii-c1(A) is ii-closed in X, f(ii-c1(A)) is gii-closed in Y and hence we have ii-c1(f(ii-c1(A))) $\subset V$. By definition of the ii-closure of a set, $A \subset$ ii-c1(A) which implies f(A) $\subset$ f(ii-c1(A)) and using **Lemma 2.5,** ii-c1(f(A)) $\subset$ ii-c1(f(ii-c1(A))) $\subset U$. That is ii-c1(f(A)) $\subset U$. This shows that f (A) is gii-closed in Y .

5.8 Definition. A function f **:** $X \to Y$ is said to be **ii-irresolute** [**4**] if for each $V \in$ ii-O(Y), $f^{-1}(V) \in$ ii-O(X).

5.9 Proposition. If $f : X \to Y$ is an open ii-irresolute bijection and B is gii-closed in Y, then $f^{-1}(B)$ is gii-closed in X .

Proof. Let U be a open set of X containing $f^{-1}(B)$. Then $B \subset f(U)$ and f(U) is open in Y. Since B is gii-closed in Y, ii-c1(B) $\subset$ f(U) and hence we have f^{-1}(ii-c1(B)) $\subset U$. Since f is ii-irresolute, f^{-1}(ii-c1(B)) is ii-closed in X (**Theorem 2.5** (i) and (v)), we have ii-c1(f^{-1}(B)) $\subset f^{-1}$(ii-c1(B) $\subset U$. This shows that $f^{-1}(B)$ is gii-closed in X.

5.10 Theorem. Let $f : X \to Y$ and $h : Y \to Z$ be the two functions, then

(i) If hof : $X \to Z$ is gii-closed and if f : $X \to Y$ is a continuous surjection, then h : $X \to Z$ is gii-closed .
(ii) If f : $X \to Y$ is gii-closed with h : $Y \to Z$ is continuous and ii-gii-closed, then hof : $X \to Z$ is gii-closed.
(iii) If f : $X \to Y$ is closed and h : $Y \to Z$ is gii-closed, then hof : $X \to Z$ is gii-closed.

Proof.
(i) Let F be a closed set of Y. Then $f^{-1}(F)$ is closed in X since f is continuous. By hypothesis (hof) ($f^{-1}(F)$) is gii-closed in Z. Hence h is gii-closed.
(ii)The proof follows from the P**roposition 5.7**.
(iii)The proof is obvious from definitions.

5.11 Theorem. The following properties are equivalent for a space X :
(a) X is ii-regular.
(b) For each closed set F and each point $x \in X - F$, there exists $U \in$ ii-O(X) and a gii-open set V such that $x \in U$ and $F \subset V$ and $U \cap V = \phi$.
(c) For each subset A of X and each closed set F such that $A \cap F = \phi$, there exist $U \in$ ii-O(X) and a gii-open set V such that $A \cap U \neq \phi$, $F \subset V$ and $U \cap V = \phi$.
(d) For each closed set F of X, $F = \cap$ {ii-c1(V) : $F \subset V$ and V is gii-open}.

Proof.
(a) $\Rightarrow$ (b). The proof is obvious since every ii-open set is gii-open.

(b) $\Rightarrow$ (c). Let A be a subset of X and let F be a closed set in X such that $A \cap F = \phi$. For a point $x \in A$, $x \in X - F$ and hence there exists $U \in$ ii-O(X) and a gii-open set V such that $x \in U$ and $F \subset V$ and $U \cap V = \phi$. Also $x \in A$, $x \in U$ implies $x \in A \cap U$. So $A \cap U \neq \phi$.
(c) $\Rightarrow$ (a). Let F be a closed set and let $x \in X - F$. Then, $\{x\} \cap F = \phi$ and there exist $U \in$ ii-O(X) and a gii-open set W such that $x \in U$, $F \subset W$ and $U \cap W = \phi$. Put V = ii-int(W), then by **Lemma 2.6**, we have $F \subset V$, $V \in$ ii-O(X) and $U \cap V = \phi$. Therefore X is ii-regular.

(a) $\Rightarrow$ (d). For a closed set F of X, by **Theorem 4.9**, we obtain

$$F \subset \cap \{\text{ii-c1}(V) : F \subset V \text{ and } V \text{ is gii-open}\}$$
$$\subset \cap \{\text{ii-c1}(V) : F \subset V \text{ and } V \in \text{ii-O}(X)\} = F$$

Therefore, $F = \cap$ {ii-c1(V) : $F \subset V$ and V is gii-open}.

(d) $\Rightarrow$ (a). Let F be a closed set of X and $x \in X - F$. by (d), there exists a gii-open set W of X such that $F \subset W$ and $x \in X$ – ii-c1(W). Since F is closed, $F \subset$ ii-int(W) by **Lemma 2.6**. Put V = ii-int(W), then $F \subset V$ and $V \in$ ii-O(X). Since $x \in X$ – ii-c1(W), $x \in X$ – ii-c1(V). Put U = X – ii-c1(V) then, $x \in U$, $U \in$ ii-O(X) and $U \cap V = \phi$. This shows that X is ii-regular.

5.12 Definition. A function f : $X \to Y$ is said to be **ii-open** if for each open set U of X, $f(U) \in$ ii-O(Y).

5.13 Theorem. If f : $X \to Y$ is a continuous ii-open gii-closed surjection and X is regular, then Y is ii-regular.

Proof. Let $y \in Y$ and let V be an open set of Y containing y. Let x be a point of X such that y = f(x). By the regularity of X, there exists an open set U of X such that $x \in U \subset c1(U) \subset f^{-1}(V)$. We have $y \in f(U) \subset f(c1(U)) \subset V$. since f is ii-open and gii-closed, $f(U) \in$ ii-O(Y) and f(c1(U)) is gii-closed in Y. So, we obtain, $y \in f(U) \subset$ ii-c1(f(U)) $\subset$ ii-cl(f(c1(U))) $\subset V$. It follows from **Theorem 5.11** that Y is ii-regular.

5.14 Definition. A function f : $X \to Y$ is said to be **pre ii-open** [**5**] if for each ii-open set U of X, $f(U) \in$ ii-O(Y).

5.15 Theorem. If f : X $\rightarrow$ Y is a continuous pre ii-open ii-gii-closed surjection and X is ii-regular, then Y is ii-regular.

Proof. Let F be any closed set of Y and y $\in$ Y – F. Then $f^{-1}(Y) \cap f^{-1}(F) = \phi$ and $f^{-1}(F)$ is closed in X. Since X is ii-regular, for a point x $\in$ $f^{-1}(y)$, there exist U, V $\in$ ii-O(X) such that x $\in$ U, $f^{-1}(F) \subset V$ and $U \cap V = \phi$. Since F is closed in Y, by **Proposition 5.6**, there exists W $\in$ ii-O(Y) such that F $\subset$ W and $f^{-1}(W) \subset V$. Since f pre ii-open, we have y = f (x) $\in$ f(U) and f(U) $\in$ ii-O(Y). Since $U \cap V = \phi$, $f^{-1}(W) \cap U = \phi$ and hence $W \cap f(U) = \phi$. This shows that Y is ii-regular.

6. Conclusion

In this paper, we introduce and study a new class of spaces, namely ii-regular spaces by using ii-open sets. The relationships among regular, strongly rg-regular, almost regular, softly regular, weakly regular, α-regular, s-regular and ii- regular spaces are investigated. Also we obtained some characterizations of ii-regular spaces, properties of the forms of gii-closed, iig-closed functions and preservation theorems for ii-regular spaces. Of course, the entire content will be a successful tool for the researchers for finding the way to obtain the results in the context of such types of regular spaces.

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