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Free Convective Flowing Upon a Plate Embedded in a Penetrating Environment by Symmetry Groups

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Abstract – *An analytical approachby Lie's theory is executed to study the naturally flowing of heat inducing liquid upon a region having some inclination which is partly infinite combined with a penetrating environment. Using scaling symmetries, PDE's depicting the physical phenomenon, are reformed into a of ODE's. The system thus evolved is numerically solved by applying IV order RK-scheme with the trajectory shoot method. The results obtained indicate that speed and warmth rise considerably as parameter S rises. Rising values of void fraction result in the rising speed and declining warmth.*

Keywords — *Heat generation, Inclined plate, Lie's theory, Natural convection, Porous medium.*

I. INTRODUCTION

Convective transfer of warmth on regions combined with penetrable materials nowadays has become a subject of interest for researchers due to its huge practical utilizations, such as geo-thermal systems, petroleum processes, water purification, storing edibles and nuclear wastage handling. Observing generating heat in liquids is potentially significant in certain cases with deal chemical reactions.

This article casts an analysis upon techniques of symmetry incorporated to a boundary layer convective problem. Certain groups formed by continuous transformations called symmetries by which the differential equations do not vary. Because of the fact that these symmetries help in reducing the count of variables, this has become most sought-after method.

Abel *et al*:[1] elucidated transferring of heat of a viscoelastic liquid immersed in a penetrating environment with induced heat and changing viscosity. They inferred that the influence of liquid viscosity results in soaring wall temperature. Molla *et al.* [7] deliberated the effect of free convective flowing upon a spherical region with induced heat and noticed that speed and warmth notably rise whenever induced heat rises. A proper analytical solution to convective transfer of heating in an electrically conducting liquid in the presence of heat induction was derived by Xu [11]. He validated the analytic solution with numerical results. Dominance of porosity on the free convective flowing upon a plate positioned upright subjected to a penetrating environment was considered by Beithou *et al*:[2]. They have depicted that the magniyude of warmth rises due to the elevating porosity.

Kandaswamy *et al*:[6] examined the situation of convectively flowing viscous liquid which conducts electricity and is not compressible in a penetrable base. They have made an observation that growing magnetic strength leads to the retardation of flowing whereas opposite result seen whenever a rise occurs in porosity factor. The behavioral changes in the flowing towards point of stagnating upon a penetrable plate which is vertically kept with energy-mass transferring impacted by Soret-Dufour effect, heat causing and radiating liquid was analyzed by Karthikeyan *et al*:[8]. Shu with Pop [9] studied real convecting wall plumes in a penetrable domain. They have demonstrated that the velocity becomes high and the temperature becomes low with rising angle of tilting.

Sivasankaran *et al*:[10] performed the numerical simulation on convection of non-Newtonian liquid in a penetrable enclosure with non-uniform heating and radiation. Chen [4] studied the natural convection flow over a permeable inclined surface with variable wall temperature and concentration. Chen detected a set back for velocity because of existence of magnetic-force field. Kalpakides with Balassas [5] focussed on free-convective boundary-layer case by applying Lie group analysis. Yurusoy and Pakdemirli [12] studied the boundary-layer equations for Newtonian/non-Newtonian liquids using Lie group method. When convective transfer of warmth upon surface having some slope is concerned, no attempt has been made for using Lie groups. This paper attempts to study this particular category of free convective flowing and warmth transport upon a surface with slope embedding in a penetrating environment with internally generating warmth for certain parameters using Lie groups.

II. MATHEMATICAL ANALYSIS

Here we deal with the free convecting heat transfer by in laminar boundary-layer flow of an incompressible viscous liquid upon a semi-infinite surface of some slope of α ($\alpha < 90^{\circ}$) with respect to the vertical line and imbedded in a liquid-saturated penetrating environment. The environment is kept at warmth T_w which does not vary and is greater than warmth T_∞

of the neighboring liquid. The mathematical equalities concerning mass, momentum and energy for the flow independent of time is modeled as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial U}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)cos\alpha-\frac{\nu}{k'}u, \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho c_p}(T-T_\infty), \quad (3)$$

with the boundary conditions

$$u=v=0,\ T=T_w,\ at\ y=0,$$
$$u=0,\ T=T_\infty \qquad as\ y\to\infty, \quad (4)$$

where u & v: velocity counterparts; x & y: the spacing-coordinates; T:temperature; ν:kinematic viscosity of the liquid; g:gravitational acceleration; β:thermal expansion co-efficient; k:thermal conductivity of liquid; ρ:density of the liquid; c_p:volumetric rate of heat generation and α:angle of inclination. Variables which are free from dimensions are

$$\bar{x}=\frac{xU_\infty}{\nu},\bar{y}=\frac{yU_\infty}{\nu},\bar{u}=\frac{u}{U_\infty},\bar{v}=\frac{v}{U_\infty},\theta=\frac{T-T_\infty}{T_\infty-T_\infty}. \quad (5)$$

Substituting into equations (6)-(8) and dropping the over bars, we obtain,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (6)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\partial^2 u}{\partial y^2}+Gr\theta cos\alpha-\frac{1}{K}u \quad (7)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}+S\theta \quad (8)$$

with the boundary conditions

$$u=v=0, \qquad \theta=1, \qquad at\ y=0,$$
$$u=0, \qquad \theta=0, \qquad as\ y\to\infty, \quad (9)$$

where $Gr=\frac{g\beta(T_W-T_\infty)\nu}{U_\infty^3}$, $Pr=\frac{\rho c_p\nu}{k}$, $K=\frac{k'U_\infty^3}{\nu^3}$ & $S=\frac{Q\nu}{\rho c_p U_\infty^2}$.

III. SYMMETRY GROUPS OF EQUATIONS

The equalities representing symmetry groups for (6)-(8) are formulated by using classical Lie group theoretical approach, see Bluman and Kumei [3]. Lie group of transformations with one-parameter which leave (6)-(8) as same is provided as follows:

$$\begin{aligned} x^* &= x+\epsilon\xi_1(x,y,u,v,\theta)\\ y^* &= y+\epsilon\xi_2(x,y,u,v,\theta)\\ u^* &= u+\epsilon\eta_1(x,y,u,v,\theta)\\ v^* &= v+\epsilon\eta_2(x,y,u,v,\theta)\\ \theta^* &= \theta+\epsilon\eta_3(x,y,u,v,\theta). \end{aligned} \quad (10)$$

By working out laborious algebraic computations, the infinitesimals are derived as

$$\begin{aligned} \xi_1 &= c_1x-c_2\\ \xi_1 &= -\alpha(x)\\ \eta_1 &= c_1u\\ \eta_2 &= u\alpha'(x)\\ \eta_3 &= c_1\theta. \end{aligned} \quad (11)$$

When we consider the constraints from boundaries and incorporating boundary restrictions on infinitesimals, the system (11) takes the form as given below.

$$\begin{aligned} \xi_1 &= c_1x-c_2\\ \xi_1 &= 0\\ \eta_1 &= c_1u\\ \eta_2 &= 0\\ \eta_3 &= c_1\theta, \end{aligned} \quad (12)$$

where c_1:scaling transformation & c_2:translation in the *x* coordinate.

IV. REDUCTION TO ORDINARY DIFFERENTIAL EQUATIONS

Here, in (12), c_1 is considered to be arbitrary and remaining parameters zero. The resulting characteristic equality ratios are

$$\frac{dx}{x} = \frac{dy}{0} = \frac{du}{u} = \frac{dv}{0} = \frac{d\theta}{\theta} \qquad (13)$$

and hence we obtain

$$\eta = y,\ u = xF_1(\eta),\ v = F_2(\eta),\ \theta = xF_3(\eta). \qquad (14)$$

Substituting (14) into equations (6)-(8), we finally obtain the system of nonlinear ordinary differential equations

$$\begin{aligned} F_1{}'' &= F_1{}^2 + F_2F_1{}' - GrF_3cos\alpha + \frac{1}{K}F_1 \\ F_2{}' &= -F_1 \\ F_3{}'' &= \mathrm{Pr}(F_2F_3{}' + F_1F_3 - SF_3). \end{aligned} \qquad (15)$$

The appropriate boundary conditions are expressed as

$$\begin{aligned} &F_1 = F_1 = 0,\ F_1 = 1,\ at\ \eta = 0, \\ &F_1 = 0,\ F_1 = 0,\ as\ \eta \to \infty. \end{aligned} \qquad (16)$$

V. NUMERICAL APPROACH

Because of the fact that the above ODEs are not linear, it is appropriate to deal with numerical treatment. The equations provided in (15) complimenting with their boundary restrictions (16) is solved by numerical approach by incorporating IV order R-K scheme and trajectory shoot method combined withan initial guessing of $F_1'(0)$ and $F_3'(0)$. The iterative process is sustained until we get the desired results which comply with accuracy of 10^{-5}. A MATHEMATICA code is formulated and outcomes are depicted through various graphs.

VI. ANALYSIS OF RESULTS

Solutions through numerical technique are evolved for certain range of values of Pr, Gr, K, S and as α given below.

Parameter	**Range**
Pr	0.1 – 7.0
Gr	0.1 – 1
K	1 – 10
S	0 – 1
α	$0°, 30°, 45°$

The results thus got by numeric computations are given as pictorial graphs to represent the profiles of speed and warmth. The analysis is performed for α = 45°. Results corresponding to α = 0° (vertical plate) and 30°are also discussed.

Figures 1(a,b) display the velocity-temperature profiles for varying values of *S*(= 0, 0.3, 0.6, 1) having *K* = 10, *Gr* = 1 and *Pr* = 0.71 corresponding to liquid&air. Observing Figure 1(a), we infer that the speed is affected considerably and rises for soaring heat. But in the vicinity of surface, it is noted a rise in speed initially, then it slows down and approaches zero at the end. Observing Figure 1(b), we infer that when the values of S rise, a significant rise in temperature happens. Figures 2(a,b) portray the influence of *K* on velocity-temperature for *S* = 0.1, *Gr* = 1 and *Pr* = 0.71. The speed rises up while warmth slips down with rising *K*. The portrayals of velocity-temperature for different Gr values with *S* = 0.1*; K* = 1 and *Pr* = 7 are given in Figures 3 (a,b). It is noted from Figure 3(a) that the speed rises with Gr due to acceleration of the liquid provided by the assisting buoyancy. The warmth diminishes by rising Gr, and as a result of this, the width of thermal boundary-layer shortens. Pr effects on speed and temperature are submitted in the Figures 4 (a,b). The speed as well as temperature reduce when Pr value increases. This is in compliance with well-known phenomenon that the thermal boundary-layer thickness lessens with rise in Pr. Further, a back flow and deficit in temperature are detected for *Pr* = 2.05.

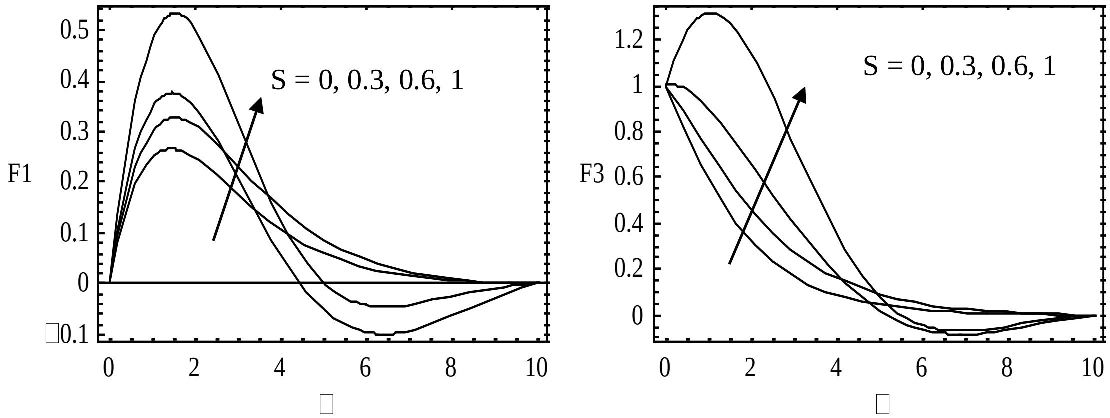

Fig. 1. Portrayal of Velocity-Temperature for S (K= 10,Gr = 1,Pr = 0.71)

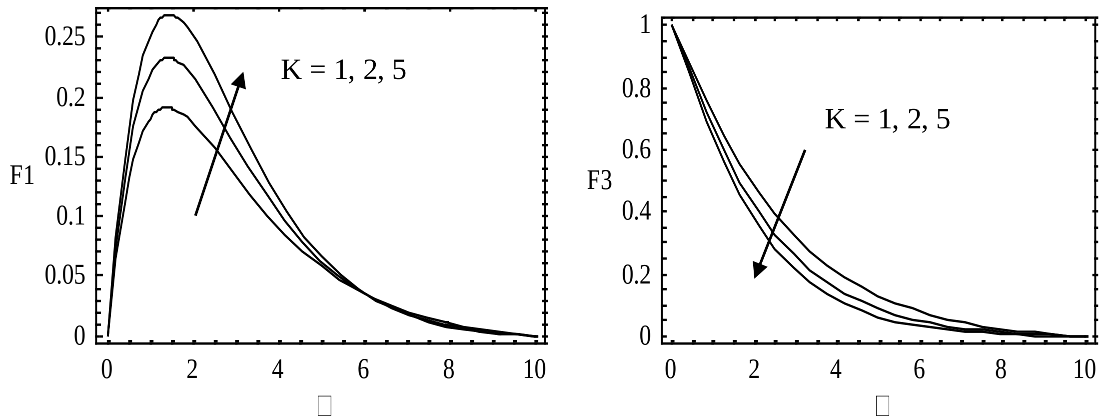

Fig. 2. Portrayal of Velocity-Temperature for K (Pr= 0.71,S = 0.1,Gr = 1)

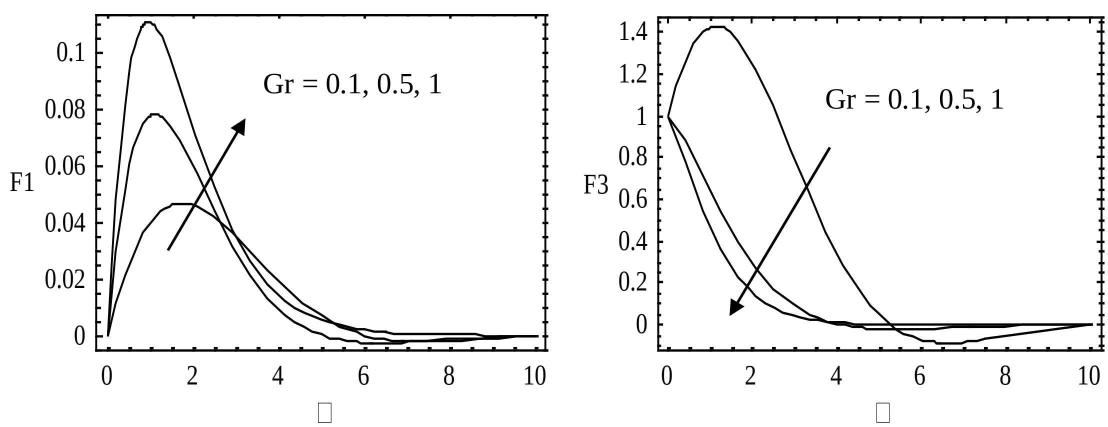

Fig. 3. Portrayal of Velocity-Temperature for Gr (Pr= 7, K= 1,S = 0.1)

The outcomes of different parameters for certain fixed slopes of surface are portrayed in Figures 5&6. The speed rises with increasing S for all slopes, see Figure 5(a). But the liquid experienced a higher speed on the vertical surface than it is inclined. This is because of the fact that the buoyancy force decreases due to gravity components ($g\cos\alpha$) as the surface is on the inclined surface. There is a gain in warmth when S is increased, see Figure 5 (b). The dispersal of warmth is intensified when $\alpha = 30°$. It is realized from Figures 6 (a&b) that a back flow and deficit in temperature are detected in the outer part of boundary-layer for $K = 1$ when keeping the plate vertical ($\alpha = 0°$).

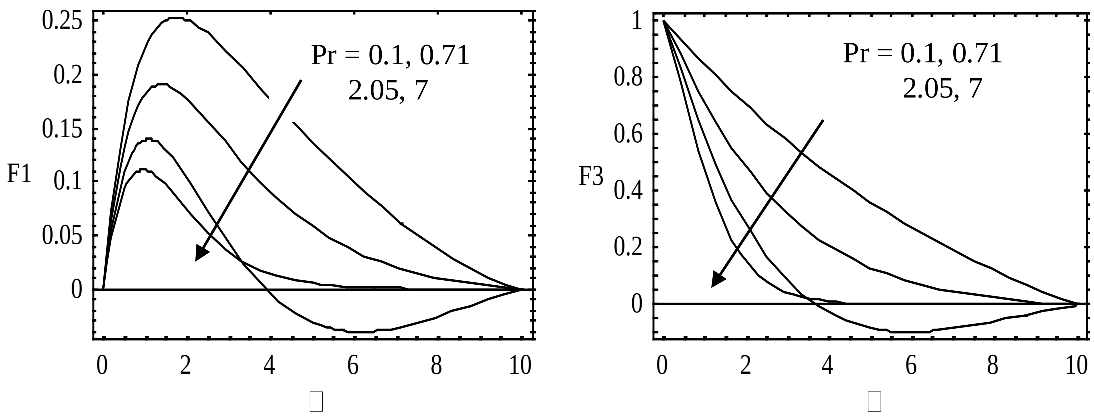

Fig. 4. Portrayal of Velocity-Temperature for Pr (K= 1,Gr=1,S = 0.1)

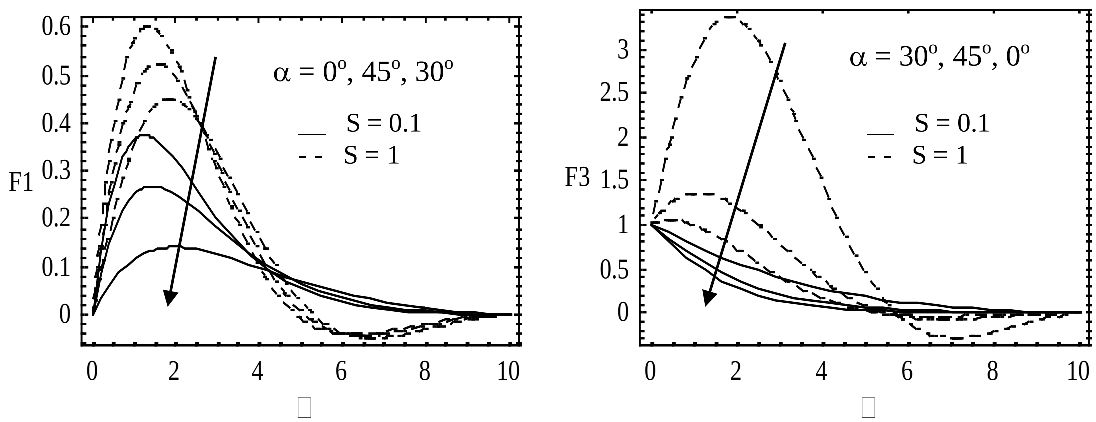

Fig. 5. Portrayal of Velocity-Temperature for S with different α (K= 5, Gr= 1 and Pr = 0.71)

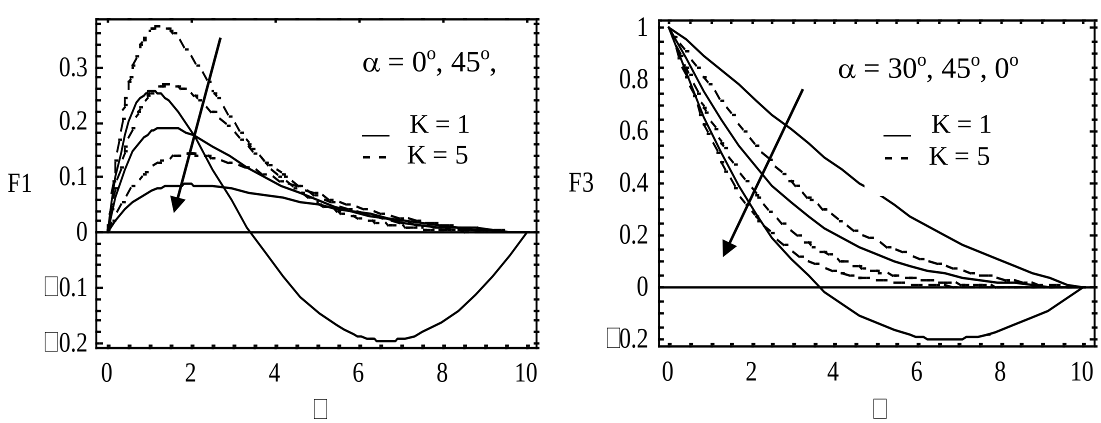

Fig.6. Portrayal of Velocity-Temperature for K with different α (S=0.1,Gr=1,Pr=0.71)

VII. CONCLUSIONS

Symmetries are obtained for the equations which govern the model which are reduced to ODEs by employing scaling group of transformation. Evidently, there has been a notable rise in speed and warmth when the parameter corresponding to heat generation rises. The speed enhances and warmth lowers with a rise in porosity parameter and the Grashof number. The speed and warmth tend to lessen with growing values of Prandtl number.

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