International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373 /doi:10.14445/22315373/IJMTT-V66I12P505
Volume 66 Issue 12, 28-36, December, 2020

Alternative Arithmetic of Pentagonal Fuzzy Numbers

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Abstract — *In this paper, we introduce the definition of positive and negative fuzzy numbers based on the concept of area of fuzzy numbers on the right side of r-axis and the left side of r-axis (in the first quadrant and the second quadrant). From the concept of positive and negative fuzzy numbers, an alternative arithmetic for pentagonal fuzzy numbers is constructed. Then the multiplication form of pentagonal fuzzy numbers can be obtained in some cases. Finally, from the multiplication operations, it can later be applied to determine the multiplication identity and inverse of pentagonal fuzzy numbers.*

Keywords — *Fuzzy Number, Arithmetic Fuzzy Number, Pentagonal Fuzzy Number*.

I. INTRODUCTION

The concept of the fuzzy set was first introduced by Lotfi A. Zadeh in 1965 [19, 20]. As science develops, many articles discuss and develop fuzzy algebra. Some articles have discussed pentagonal fuzzy numbers [3, 4, 5, 6, 10, 11, 12, 13, 15, 18].

In [12, 13] the basic concepts of arithmetic of pentagonal fuzzy numbers are discussed. Furthermore, in [6, 10, 14] pentagonal fuzzy numbers are discussed using the α-cut method and provided the same arithmetic operations on pentagonal fuzzy numbers. Then [6] also introduces arithmetic operations in parametric and inverse of fuzzy number in the formof $\tilde{u}^{-1} = 1/\tilde{u}$, provided that none of its components has a value of 0. In [15] pentagonal fuzzy numbers are discussed using the method of incenter of centroid and introduced arithmetic operations with different parametric forms using the α-cut method. Then in [18] pentagonal fuzzy numbers are discussed using the Cholesky decomposition method and the singular value decomposition to solve a fully fuzzy linear system.

Each of these discussions provides various forms of arithmetic operations. However, they only discuss arithmetic operations for positive fuzzy numbers so that most of the solutions obtained are not compatible. Based on the above conditions, the authors feel the need to define fuzzy numbers in positive and negative form which are explained by comparing the area of the area on the positive *r*-axis and the negative *r*-axis. From the positive and negative concepts of pentagonal fuzzy numbers, an alternative arithmetic for pentagonal fuzzy numbers is constructed consisting of the operations of addition, subtraction, scalar multiplication and multiplication of two fuzzy numbers. From the arithmetic alternatives given, it can later be used to determine the multiplication identity and inverse of pentagonal fuzzy numbers.

II. PRELIMINARIES

In fuzzy terms there are fuzzy numbers and pentagonal fuzzy numbers. Some definitions of fuzzy numbers and pentagonal fuzzy numbers have been discussed by some authors [1, 2, 6, 7, 8, 9, 14, 16, 17].

Definition2.1(Fuzzy Number) A fuzzy number f in the real line $\mathbb{R}$ is a fuzzy set$f : \mathbb{R} \to [0, 1]$ that satisfies the following properties:
a. f is piecewise continuous..
b. There exists an $x \in \mathbb{R}$such that $f(x) = 1$.
c. f is convex, if $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0, 1]$ then $f(\lambda x_1 + (1 + \lambda)x_2) > f(x_1) \wedge f(x_2)$.

Definition2. A fuzzy number $\tilde{u}: \mathbb{R} \to [0, 1]$with $\tilde{u} = (a_1, a_2, a_3, a_4, a_5)$ satisfies
a. $\tilde{u}(x)$ is upper semi continuous.
b. $\tilde{u}(x) = 0$ outside the interval $[a_1, a_5]$
c. There exist real number *x* in interval $[a_1, a_5]$ such that,
   i. $\tilde{u}(x)$monotonic increasing in interval $[a_1, a_2]$ and $[a_2, a_3]$
   ii. $\tilde{u}(x)$monotonic decreasing in interval $[a_3, a_4]$ and $[a_4, a_5]$
d. $\tilde{u}(x)$ =1 for $x = a_3$

A pentagonal fuzzy number $\tilde{u} = (a_1, a_2, a_3, a_4, a_5)$with a_1, a_2, a_3, a_4dan $a_5$are real number and $a_1 \le a_2 \le a_3 \le a_4 \le a_5$ has been discussed in [4, 6, 12, 13, 15, 18]. Membership function of pentagonal fuzzy number $\tilde{u}$ is given as follows:

$$\mu_{\tilde{u}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{(x - a_1)}{a_2 - a_1} & a_1 \le x \le a_2 \\ \dfrac{x - a_2}{a_3 - a_2} & a_2 \le x \le a_3 \\ 1 & x = a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & a_3 \le x \le a_4 \\ \dfrac{a_5 - x}{a_5 - a_4} & a_5 \le x \le a_4 \\ 0 & a_5 \le x \end{cases}$$

The parametric form of pentagonal fuzzy number $\tilde{u} = [\underline{u}_1(r), \underline{u}_2(r), \overline{u}_2(r), \overline{u}_1(r)]$ is given as follows:

$$\begin{aligned} \underline{u}_1(r) &= 2r(a_2 - a_1) + a_1 \\ \underline{u}_2(r) &= 2r(a_3 - a_2) + 2a_2 - a_3 \\ \overline{u}_2(r) &= 2r(a_4 - a_3) - a_4 + 2a_3 \\ \overline{u}_1(r) &= -2r(a_5 - a_4) + a_5 \end{aligned}$$

In this paper, we discuss pentagonal fuzzy number in the form of $\tilde{u} = (a, \alpha, \beta, \gamma, \delta)$ with a is centroid point ,α is the distance of left spread from center$a$to points $(a - \alpha)$ and β is the distance of left spread from the point $(a - \alpha)$ to point $(a - \alpha - \beta)$. Furthermore, γ is the distance of right spread from center a to point $(a + \gamma)$ and δ is the distance of right spread from center $(a + \gamma)$ to point $(a + \gamma + \delta)$. Pentagonal fuzzy number $\tilde{u} = (a, \alpha, \beta, \gamma, \delta)$can be illustrated as in Figure 1.

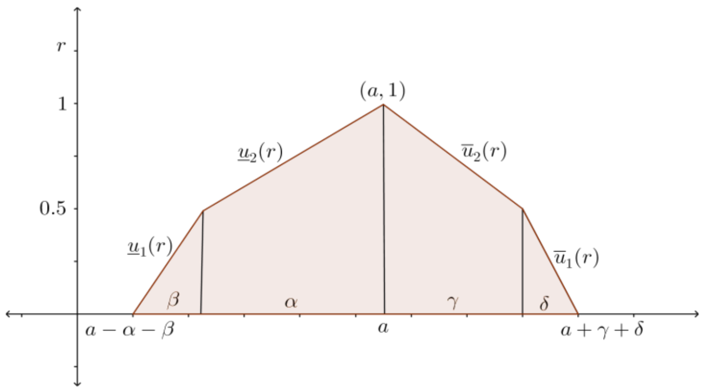

Figure 1: Pentagonal Fuzzy Number $\tilde{u} = (a, \alpha, \beta, \gamma, \delta)$

Definition 2.3A fuzzy number $\tilde{u}: \mathbb{R} \to [0, 1]$ with $\tilde{u} = (a, \alpha, \beta, \gamma, \delta)$ satisfies the following properties:

a. $\tilde{u}(x)$ is upper semi continous.

b. $\tilde{u}(x) = 0$ outside the interval $[a - \alpha - \beta, a + \gamma + \delta]$

c. There exist a real number x in interval $[a - \alpha - \beta, a + \gamma + \delta]$ such that
   i. $\tilde{u}(x)$ monotonic increasing in interval $[a - \alpha - \beta, a - \alpha]$ and $[a - \alpha, a]$
   ii. $\tilde{u}(x)$ monotonic decreasing in interval $[a, a + \gamma]$ and $[a + \gamma, a + \gamma + \delta]$

d. $\tilde{u}(x) = 1$ for $x = a$

Membership function of pentagonal fuzzy number $\tilde{u} = (a, \alpha, \beta, \gamma, \delta)$ is given as follows:

$$\mu_{\tilde{u}}(x)=\begin{cases} \frac{1}{2}+\frac{1}{2}\left(\frac{(x-a+\alpha)}{\beta}\right); & a-\alpha-\beta\le x\le a-\alpha \\ 1+\frac{1}{2}\left(\frac{(x-a)}{\alpha}\right) ; & a-\alpha\le x\le a \\ 1, ; & x=a \\ 1-\frac{1}{2}\left(\frac{(x-a)}{\gamma}\right) ; & a\le x\le a+\gamma \\ \frac{1}{2}-\frac{1}{2}\left(\frac{(x-a+\gamma)}{\alpha}\right): & a+\gamma\le x\le a+\gamma+\alpha \\ 0, & others \end{cases}$$

The parametric form of pentagonal fuzzy number $\tilde{u}=[\underline{u}_1(r),\underline{u}_2(r),\overline{u}_2(r),\overline{u}_1(r)]$ is given as follows:

$$\underline{u}_1(r)=a-(2-2r)\beta-(\alpha-\beta)$$
$$\underline{u}_2(r)=a-(2-2r)\alpha$$
$$\overline{u}_2(r)=a+(2-2r)\gamma$$
$$\overline{u}_1(r)=a+(2-2r)\delta+(\gamma-\delta)$$

A fuzzy number $\tilde{u}$ in $\mathbb{R}$ defined as a function $\tilde{u}=[\underline{u}_1(r),\underline{u}_2(r),\overline{u}_2(r),\overline{u}_1(r)]$has the following properties:
a. $\underline{u}_1(r)$ is a bounded left continuous nondecreasing function over [0, 0.5]
b. $\underline{u}_2(r)$ is a bounded left continuous nondecreasing function over [0.5, 1]
c. $\overline{u}_2(r)$is a bounded right continuous nonincreasing function over [1,0.5]
d. $\overline{u}_1(r)$ is a bounded right continuous nonincreasing function over [0.5,1]

III. POSITIVE AND NEGATIVEPENTAGONAL FUZZY NUMBERS

The following the definition of positive or negative pentagonal fuzzy numbers is given using the ratio of area in several cases:

a. If $a-\alpha-\beta>0$,then the pentagonal fuzzy number $\tilde{u}=(a,\alpha,\beta,\gamma,\delta)$ is said to be a positive pentagonal fuzzy number, as shown in Figure 2.

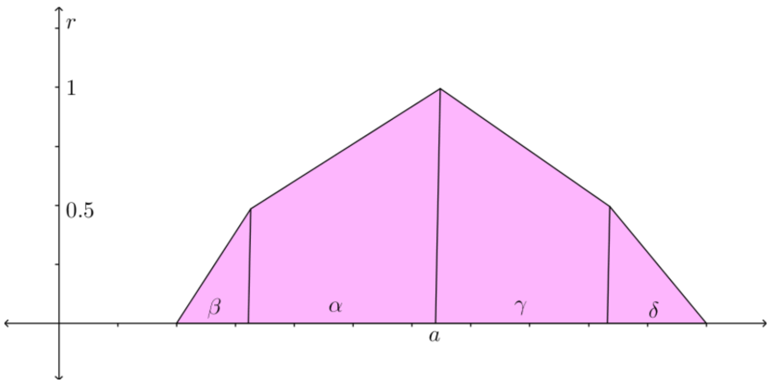

Figure 2: Positive Pentagonal Fuzzy Number $\tilde{u}=(a,\alpha,\beta,\gamma,\delta)$

b. If $a+\gamma+\delta<0$, then the pentagonal fuzzy number $\tilde{u}=(a,\alpha,\beta,\gamma,\delta)$ is said to be a negative pentagonal fuzzy number, as shown in Figure 3.

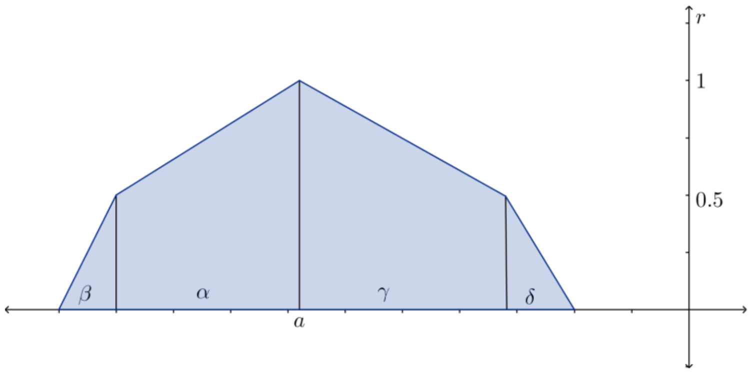

Figure 3: Negative Pentagonal Fuzzy Number $\widetilde{u} = (a, \alpha, \beta, \gamma, \delta)$

c. If $a > 0$ and $a - \alpha < 0$, it is illustrated in Figure 4.

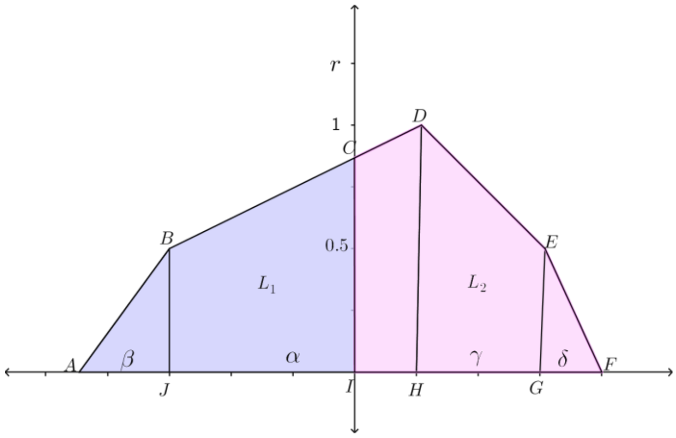

Figure 4: Pentagonal Fuzzy Number for $a > 0$ and $a - \alpha < 0$

Figure 4 shows that there are two areas which are formed from the positive *r*-axis (L_2 area) and negative *r*-axis (L_1 area). Furthermore, to determine the area in Figure 4, the following definitions are used:

$$L_1 = L\Delta ABJ + L\square JBCI$$

$$L_2 = L\square ICDH + L\square HDGE + L\Delta GEF$$

The areas of L_1 and L_2 are obtained as follows:

$$L_1 = -a + \frac{3\alpha}{4} + \frac{\beta}{4} + \frac{a^2}{4\alpha}$$

$$L_2 = a + \frac{\delta}{4} + \frac{3\gamma}{4} - \frac{a^2}{4\alpha}$$

Furthermore, the difference of the area of the positive *r*-axis (L_2 area) and negative *r*-axis (L_1 area) is denoted by *C*, to determine the type of positive or negative fuzzy numbers is as follows:

$$C = L_2 - L_1$$

$$C = \left(a + \frac{\delta}{4} + \frac{3\gamma}{4} - \frac{a^2}{4\alpha}\right) - \left(a + \frac{3\alpha}{4} + \frac{\beta}{4} + \frac{a^2}{4\alpha}\right)$$

$$C = \left(2a + \frac{\delta}{4} - \frac{\beta}{4} + \frac{3\gamma}{4} - \frac{3\alpha}{4} - \frac{a^2}{2\alpha}\right)$$

Finally, in case (c) the pentagonal fuzzy number $\tilde{u} = (a, \alpha, \beta, \gamma, \delta)$ is said to be a positive fuzzy number when $2a + \frac{\delta}{4} - \frac{\hat{a}}{4} + \frac{3\gamma}{4} - \frac{3\alpha}{4} - \frac{a^2}{2\alpha} > 0$ and it is said to be a negative fuzzy number when $2a + \frac{\delta}{4} - \frac{\beta}{4} + \frac{3\gamma}{4} - \frac{3\alpha}{4} - \frac{a^2}{2\alpha} < 0$.

d. If $a = 0$, it is illustrated in Figure 5.

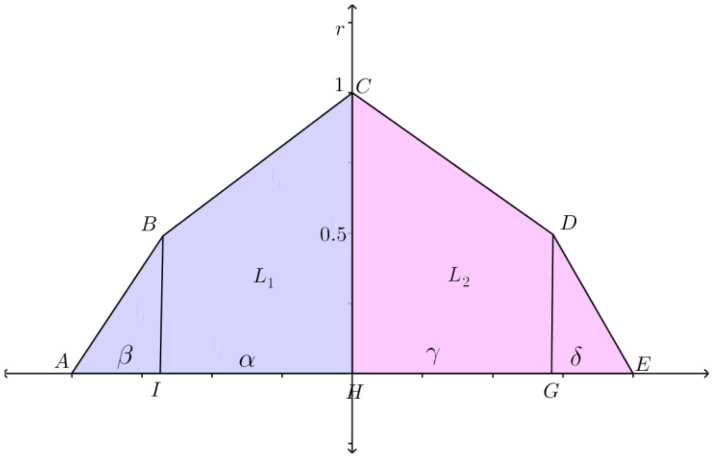

Figure 5: PentagonalFuzzy Number for $a = 0$

Figure 5 shows that there are two areas which are formed from the positive *r*-axis (L_2 area) and negative *r*-axis (L_1 area). Furthermore, to determine the area of the area in Figure 5, the following definitions are used:

$$L_1 = L\Delta ABI \ + \ L\square IBCH$$

$$L_2 = \ L\square HCDG \ + L\Delta GDE$$

The areas of L_1 and L_2 are obtained as follows:

$$L_1 = \frac{\beta}{4} + \frac{3\alpha}{4}$$

$$L_2 = \frac{3\gamma}{4} + \frac{\delta}{4}$$

Furthermore, the difference of the area of the positive *r*-axis (L_2 area) and negative *r*-axis (L_1 area) denoted by *C*, to determine the type of positive or negative fuzzy numbers as follows:

$$C = L_2 - L_1$$

$$C = \left(\frac{3\gamma}{4} + \frac{\delta}{4}\right) - \left(\frac{\beta}{4} + \frac{3\alpha}{4}\right)$$

$$C = \left(\frac{3\gamma}{4} - \frac{3\alpha}{4} + \frac{\delta}{4} - \frac{\beta}{4}\right)$$

Finally, in case (d) the pentagonal fuzzy number $\tilde{u} = (a, \alpha, \beta, \gamma, \delta)$ is said to be a positive fuzzy number when $\frac{3\gamma}{4} - \frac{3\alpha}{4} + \frac{\delta}{4} - \frac{\beta}{4} > 0$ and it is said to be a negative fuzzy number when $\frac{3\gamma}{4} - \frac{3\alpha}{4} + \frac{\delta}{4} - \frac{\beta}{4} < 0$.

e. If $a < 0$ and $a + \gamma < 0$, it is illustrated in Figure 6.

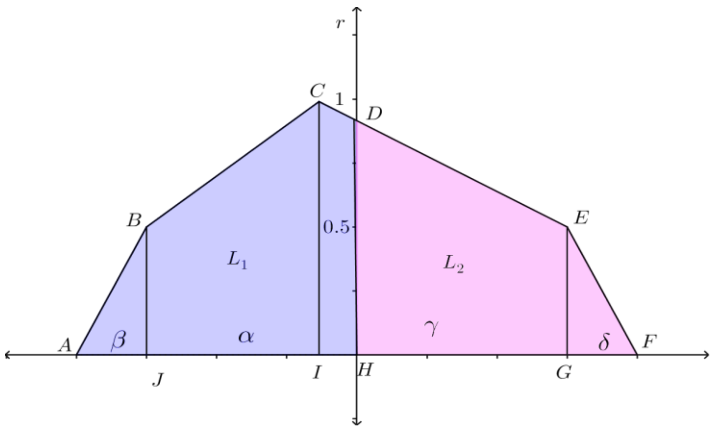

Figure 6: PentagonalFuzzy Number for$a < 0$ and$a + \gamma > 0$

Figure 6 shows that there are two areas which are formed from the positive *r*-axis (L_2 area) and negative *r*-axis (L_1 area). Furthermore, to determine the area of the area in Figure 6, the following definitions are used:

$$L_1 = L\Delta ABJ \ + \ L\square JBCI + L\square ICDH$$

$$L_2 = \ L\square HDEG + \ L\Delta GEF$$

The areas of L_1 and L_2 are obtained as follows:

$$L_1 = a + \frac{3\alpha}{4} + \frac{\beta}{4} + \frac{a^2}{4\gamma}$$

$$L_2 = -\frac{2a}{4} + \frac{\delta}{4} + \frac{3\gamma}{4} - \frac{a^2}{4\gamma}$$

Furthermore, the difference of the area of the positive *r*-axis (L_2 area) and negative *r*-axis (L_1area) is denoted by *C*, to determine the type of positive or negative fuzzy numbers as follows:

$$C = L_2 - L_1$$

$$C = \left(-\frac{2a}{4} + \frac{\delta}{4} + \frac{3\gamma}{4} - \frac{a^2}{4}\right) - \left(a + \frac{3\alpha}{4} + \frac{\beta}{4} + \frac{a^2}{4\alpha}\right)$$

$$C = \left(-\frac{3a}{2} + \frac{\delta}{4} - \frac{\beta}{4} + \frac{3\gamma}{4} - \frac{3\alpha}{4} - \frac{a^2}{2\alpha}\right)$$

Finally, in case (d) the pentagonal fuzzy number $\tilde{u} = (a, \alpha, \beta, \gamma, \delta)$ is said to be a positive fuzzy number when $-\frac{3a}{2} + \frac{\delta}{4} - \frac{\beta}{4} + \frac{3\gamma}{4} - \frac{3\alpha}{4} - \frac{a^2}{2\alpha} > 0$ and it is said to be a negative fuzzy number when $-\frac{3a}{2} + \frac{\delta}{4} - \frac{\beta}{4} + \frac{3\gamma}{4} - \frac{3\alpha}{4} - \frac{a^2}{2} < 0$.

IV. ALTERNATIVE ARITHMETIC PENTAGONAL FUZZY NUMBER

This section discusses arithmetic for pentagonal fuzzy numbers consisting of addition, subtraction, multiplication of two fuzzy numbers, and scalar multiplication. Suppose there are two different pentagonal fuzzy numbers, with$\tilde{u} = (a, \alpha_1, \beta_1, \gamma_1, \delta_1)$ and $\tilde{v} = (b, \alpha_2, \beta_2, \gamma_2, \delta_2)$ with the parametric forms $\tilde{u} = [\underline{u}_1(r), \underline{u}_2(r), \overline{u}_2(r), \overline{u}_1(r)]$ and $\tilde{v} = [\underline{v}_1(r), v_2(r), v_2(r), \overline{v}_1(r)]$ as follows:

$$\begin{matrix} \underline{u}_1(r) = a - (2-2r)\beta_1 - (\alpha_1 - \beta_1) \\ \underline{u}_2(r) = a - (2-2r)\alpha_1 \end{matrix} \quad , \quad \begin{matrix} \overline{u}_2(r) = a + (2-2r)\gamma_1 \\ \overline{u}_1(r) = a + (2-2r)\delta_1 + (\gamma_1 - \delta_1) \end{matrix}$$

and

$$\underline{v}_1(r) = b-(2-2r)\beta_2-(\alpha_2-\beta_2), \quad \overline{v}_2(r) = b+(2-2r)\gamma_2$$
$$\underline{v}_2(r) = b-(2-2r)\alpha_2, \quad \overline{v}_1(r) = b+(2-2r)\delta_2+(\gamma_2-\delta_2)$$

a. Addition of two pentagonal fuzzy numbers.

For $r \in [0,0.5]$

$$\tilde{u} \oplus \tilde{v} = [\underline{u}_1(r)+\underline{v}_1(r), \overline{u}_1(r)+\overline{v}_1(r)]$$
$$= \begin{bmatrix} a+b-(2-2r)(\beta_1+\beta_2)-(\alpha_1+\alpha_2)+(\beta_1+\beta_2), \\ a+b+(2-2r)(\delta_1+\delta_2)-(\gamma_1+\gamma_2)+(\delta_1+\delta_2) \end{bmatrix}$$

For $r \in [0.5,1]$

$$\tilde{u} \oplus \tilde{v} = [\underline{u}_2(r)+\underline{v}_2(r), \overline{u}_2(r)+\overline{v}_2(r)]$$
$$= \begin{bmatrix} a+b-(2-2r)(\alpha_1+\alpha_2), \\ a+b+(2-2r)(\gamma_1+\gamma_2) \end{bmatrix}$$

The parametric form of the addition operation is transformed into the pentagonal fuzzy number, then it is obtained

$$\tilde{u} \oplus \tilde{v} = (a+b, \alpha_1+\alpha_2, \beta_1+\beta_2, \gamma_1+\gamma_2, \delta_1+\delta_2).$$

b. Subtractionof two pentagonal fuzzy numbers.

For $r \in [0,0.5]$

$$\tilde{u} \ominus \tilde{v} = [\underline{u}_1(r)-\overline{v}_1(r), \overline{u}_1(r)-\underline{v}_1(r)]$$
$$= \begin{bmatrix} a-b-(2-2r)(\beta_1+\delta_2)-(\alpha_1-\beta_1)-(\gamma_2-\delta_2), \\ a-b+(2-2r)(\delta_1+\beta_2)-(\gamma_1+\delta_1)+(\alpha_2-\beta_2) \end{bmatrix}$$

For $r \in [0.5,1]$

$$\tilde{u} \ominus \tilde{v} = [\underline{u}_2(r)-\overline{v}_2(r), \overline{u}_2(r)-\underline{v}_2(r)]$$
$$= \begin{bmatrix} a-b-(2-2r)(\alpha_1+\gamma_2), \\ a-b+(2-2r)(\gamma_1+\alpha_2) \end{bmatrix}$$

The parametric form of the subtraction operation is transformed into the pentagonal fuzzy number, then it yields

$$\tilde{u} \ominus \tilde{v} = (a-b, \alpha_1+\gamma_2, \beta_1+\delta_2, \gamma_1+\alpha_2, \delta_1+\beta_2)$$

c. Scalar product
   i. For $\lambda > 0$

$$\lambda\tilde{u} = [\lambda a-(2-2r)\lambda\beta-\lambda(\alpha-\beta), \lambda a-(2-2r)\lambda\alpha, \lambda a+(2-2r)\lambda\delta+\lambda(\gamma-\delta), \lambda a+(2-2r)\lambda\gamma]$$

   The parametric form of the scalar product $\lambda > 0$ is transformed into the pentagonal fuzzy number, then it gives

$$\lambda\tilde{u} = (\lambda a, \lambda\alpha, \lambda\beta, \lambda\gamma, \lambda\delta).$$

   ii. For $\lambda < 0$

$$\lambda\tilde{u} = [-\lambda a-(2-2r)\lambda\beta-\lambda(\alpha-\beta), -\lambda a-(2-2r)\lambda\alpha, -\lambda a+(2-2r)\lambda\delta+\lambda(\gamma-\delta), -\lambda a+(2-2r)\lambda\gamma]$$

   The parametric form of the scalar product $\lambda < 0$ is transformed into the pentagonal fuzzy number, then it is obtained that

$$\lambda\tilde{u} = (\lambda a, -\lambda\alpha, -\lambda\beta, -\lambda\gamma, -\lambda\delta).$$

d. Multiplication of two pentagonal fuzzy numbers

If $\tilde{u} = [\underline{u}_1(r), \underline{u}_2(r), \overline{u}_2(r), \overline{u}_1(r)]$ and $\tilde{v} = [\underline{v}_1(r), v_2(r), v_2(r), \overline{v}_1(r)]$ are two pentagonal fuzzy numbers, then the multiplication operation that applies is $\tilde{w} = \tilde{u} \otimes \tilde{v}$ for $r \in [0,1]$. The following shows some cases for the multiplication operation of pentagonal fuzzy number.

   i. If $\tilde{u}$ positive and $\tilde{v}$ positive, then

   For $r \in [0,0.5]$

$$\underline{w}(r) = \underline{u}_1(r)\underline{v}_1(0.5)+\underline{u}_1(0.5)\underline{v}_1(r)-\underline{u}_1(0.5)\underline{v}_1(0.5)$$
$$= ab-(2-2r)(a\beta_2-\alpha_1\beta_2-b\beta_1-\alpha_2\beta_1)-a\alpha_2+\alpha_1\alpha_2+b\beta_1-b\alpha_1+a\beta_2-\alpha_1\beta_2$$

$$\overline{w}(r) = \overline{u}_1(r)\overline{v}_1(0.5)+\overline{u}_1(0.5)\overline{v}_1(r)-\overline{u}_1(0.5)\overline{v}_1(0.5)$$

$$= ab + (2-2r)(b\delta_1 + \delta_1\gamma_2 + a\delta_2 + \delta_2\gamma_1) + a\gamma_2 + \gamma_1\gamma\beta_2 - b\delta_1 + b\gamma_1 + a\delta_2 - \gamma_1\delta_2$$

For $r \in [0.5,1]$

$\underline{w}(r) = \underline{u}_2(r)\underline{v}_2(1) + \underline{u}_2(1)\underline{v}_2(r) - \underline{u}_2(1)\underline{v}_2(1)$

$= ab - (2-2r)(\alpha_1 b + \alpha_2 a)$

$\overline{w}(r) = \overline{u}_2(r)\overline{v}_2(1) + \overline{u}_2(1)\overline{v}_2(r) - \overline{u}_2(1)\overline{v}_2(1)$

$= ab + (2-2r)(\gamma_1 b + \gamma_2 a)$

The parametric form of the multiplication operation is transformed into the pentagonal fuzzy number, then it is obtained that

$$\tilde{u} \otimes \tilde{v} = \left(ab, (\alpha_1 b + \alpha_2 a), (a\beta_2 - \alpha_1\beta_2 + b\beta_1 - \beta_1\alpha_2), (\gamma_1 b + \gamma_2 a), (b\delta_1 + \gamma_1\delta_2 + a\delta_2 + \gamma_2\delta_1)\right).$$

ii. If $\tilde{u}$ positive and $\tilde{v}$ negative, then

For $r \in [0,0.5]$

$\underline{w}(r) = \overline{u}_1(r)\underline{v}_1(0.5) + \overline{u}_1(0.5)\underline{v}_1(r) - \overline{u}_1(0.5)\underline{v}_1(0.5)$

$= ab - (2-2r)(-b\delta_1 + \alpha_2\delta_1 + a\beta_2 - \gamma_1\beta_2) + b\gamma_1 - \gamma_1\alpha_2 - b\delta_1 + \delta_1\alpha_2 - a\alpha_2 + a\beta_2 + \gamma_1\beta_2$

$\overline{w}(r) = \underline{u}_1(r)\overline{v}_1(0.5) + \underline{u}_1(0.5)\overline{v}_1(r) - \underline{u}_1(0.5)\overline{v}_1(0.5)$

$= ab + (2-2r)(-b\beta_1 - \beta_1\gamma_2 + a\delta_2 - \delta_2\alpha_2) - b\alpha_1 - \gamma_1\alpha_1 + b\beta_1 + \beta_1\gamma_2 + a\gamma_2 - a\delta_2 + \alpha_1\delta_2$

For $r \in [0.5,1]$

$\underline{w}(r) = \overline{u}_2(r)\underline{v}_2(1) + \overline{u}_2(1)\underline{v}_2(r) - \overline{u}_2(1)\underline{v}_2(1)$

$= ab - (2-2r)(\alpha_2 a - \gamma_1 b)$

$\overline{w}(r) = \underline{u}_2(r)\overline{v}_2(1) + \underline{u}_2(1)\overline{v}_2(r) - \underline{u}_2(1)\overline{v}_2(1)$

$= ab + (2-2r)(\gamma_2 a - \alpha_1 b)$

The parametric form of the multiplication operation is transformed into the pentagonal fuzzy number, then it yields

$$\tilde{u} \otimes \tilde{v} = \left(ab, (\alpha_2 a - b\gamma_1), (-b\delta_1 + \alpha_2\delta_1 + a\beta_2 + \gamma_1\beta_2), (\gamma_2 a - \alpha_1 b), (-b\beta_1 - \beta_1\gamma_2 - a\delta_2 + \alpha_2\delta_2)\right).$$

iii. if $\tilde{u}$ negative and $\tilde{v}$ positive, then

For $r \in [0,0.5]$

$\underline{w}(r) = \underline{u}_1(r)\overline{v}_1(0.5) + \underline{u}_1(0.5)\overline{v}_1(r) - \underline{u}_1(0.5)\overline{v}_1(0.5)$

$= ab - (2-2r)(b\beta_1 + \beta_1\gamma_2 - a\delta_2 + \delta_2\alpha_2) - b\alpha_1 - \gamma_1\alpha_1 + b\beta_1 + \beta_1\gamma_2 + a\gamma_2 - a\delta_2 + \alpha_1\delta_2$

$\overline{w}(r) = \overline{u}_1(r)\underline{v}_1(0.5) + \overline{u}_1(0.5)\underline{v}_1(r) - \overline{u}_1(0.5)\underline{v}_1(0.5)$

$= ab + (2-2r)(b\delta_1 - \alpha_2\delta_1 - a\beta_2 - \gamma_1\beta_2) + b\gamma_1 - \gamma_1\alpha_2 - b\delta_1 + \delta_1\alpha_2 - a\alpha_2 + a\beta_2 + \gamma_1\beta_2$

For $r \in [0.5,1]$

$\underline{w}(r) = \underline{u}_2(r)\overline{v}_2(1) + \underline{u}_2(1)\overline{v}_2(r) - \underline{u}_2(1)\overline{v}_2(1)$

$= ab - (2-2r)(-\alpha_1 b + \gamma_2 a)$

$\overline{w}(r) = \overline{u}_2(r)\underline{v}_2(1) + \overline{u}_2(1)\underline{v}_2(r) - \overline{u}_2(1)\underline{v}_2(1)$

$= ab + (2-2r)(\gamma_1 b - \alpha_2 a)$

The parametric form of the multiplication operation is transformed into the pentagonal fuzzy number, then it is earned

$$\tilde{u} \otimes \tilde{v} = \left(ab, (\gamma_2 a - \alpha_1 b), (b\beta_1 + \beta_1\gamma_2 - a\delta_2 + \alpha_2\delta_2), (b\gamma_1 - \alpha_2 a), (b\delta_1 - \alpha_2\delta_1 - a\beta_2 - \gamma_1\beta_2)\right)$$

iv. If $\tilde{u}$ negative and $\tilde{v}$ negative,then

For $r \in [0,0.5]$

$\underline{w}(r) = \overline{u}_1(r)\overline{v}_1(0.5) + \overline{u}_1(0.5)\overline{v}_1(r) - \overline{u}_1(0.5)\overline{v}_1(0.5)$

$$= ab - (2-2r)(-b\delta_1 - \delta_1\gamma_2 - a\delta_2 - \delta_2\gamma_1) + a\gamma_2 + \gamma_1\gamma\beta_2 - b\delta_1 + b\gamma_1 + a\delta_2 - \gamma_1\delta_2$$

$$\overline{w}(r) = \underline{u}_1(r)\underline{v}_1(0.5) + \underline{u}_1(0.5)\underline{v}_1(r) - \underline{u}_1(0.5)\underline{v}_1(0.5)$$
$$= ab + (2-2r)(-a\beta_2 - \alpha_1\beta_2 - b\beta_1 + \alpha_2\beta_1) - a\alpha_2 + \alpha_1\alpha_2 + b\beta_1 - b\alpha_1 + a\beta_2 - \alpha_1\beta_2$$

For $r \in [0.5,1]$

$$\underline{w}(r) = \overline{u}_2(r)\overline{v}_2(1) + \overline{u}_2(1)\overline{v}_2(r) - \overline{u}_2(1)\overline{v}_2(1)$$
$$= ab - (2-2r)(-\gamma_1 b - \gamma_2 a)$$

$$\overline{w}(r) = \underline{u}_2(r)\underline{v}_2(1) + \underline{u}_2(1)\underline{v}_2(r) - \underline{u}_2(1)\underline{v}_2(1)$$
$$= ab + (2-2r)(-\alpha_1 b - \alpha_2 a)$$

The parametric form of the multiplication operation is transformed into the pentagonal fuzzy number, then it provides

$$\tilde{u} \otimes \tilde{v} = \left((ab), -(\gamma_1 b + \gamma_2 a), -(b\delta_1 + \gamma_1\delta_2 + a\delta_2 + \gamma_2\delta_1), -(b\alpha_1 + \alpha_2 a), -(a\beta_2 - \alpha_1\beta_2 - b\beta_1 - \beta_1\alpha_2)\right).$$

V. CONCLUSION

In this paper, we obtain a new definition of positive fuzzy numbers and negative fuzzy numbers based on comparisons of the area. In addition, an alternative arithmetic pentagonal fuzzy number is also obtained which is applied to the multiplication operation of any two fuzzy numbers (positive or negative), so that from the multiplication operation, the multiplication identity and inverse of the pentagonal fuzzy number can be formed.

REFERENCES

[1] A. S. Abidin, Mashadi, and S. Gemawati, Algebraic modification of trapezoidal fuzzy numbers to complete fully fuzzy linear equations system using gauss-jacobi method, International Journal of Management and Fuzzy Systems,5 (2019), 40-46.

[2] Z. Desmita and Mashadi, Alternative multiplying triangular fuzzy number and applied in fully fuzzy linear system, American Scientific Research Journal for Engineering, Technology and Science, 56 (2019), 113-123.

[3] D. S. Dinagar and M. M. Jeyavuthin, Distinct methods for solving fully fuzzy linear programming problems with pentagonal fuzzy numbers, Journalof Computer and Mathematical Sciences, 10 (2019), 1253-1260.

[4] S. S. Geethaand K. Selvakumari, A new method for solving fuzzy transportation problem using pentagonal fuzzy numbers, Journal of Critical Reviews,7 (2020), 171-174.

[5] R. Helen and G. Uma, A operations and rangking on petagonal fuzzy numbers,International Journal of Mathematical Science and Application, 5(2015), 341-346.

[6] A.J. Kamble, Some notes on pentagonal fuzzy numbers, International Journal Fuzzy Mathematical Archive, 13 (2017), 113-121.

[7] H. Kholida and Mashadi, Alternative fuzzy algebra for fuzzy linear system using cramers rules on fuzzy trapozoidal Number, International Journal of Innovative Science and Research Technology, 4 (2019), 494-504.

[8] S. I. Marni, Mashadi, and S. Gemawati, Solving dual fully fuzzy linear system by use factorizations of the coefficient matrix for trapezoidal fuzzy number, Bulletin of Mathematics, 10 (2018), 145-56.

[9] Mashadi, A new method for dual fully fuzzy linear system by Use LU factorizations of The coefficient matrix, Jurnal Matematika dan Sains, 15(2010), 101-106.

[10] S. P. Mondaand M. Mandal, Pentagonal fuzzy number, its properties andapplication in fuzzy equation, Future Computing and Informatics Journal,2 (2017), 110-117.

[11] A. Panda and M. Pal, A study on pentagonal fuzzy number and its corresponding matrices, Pacific Science Review B: Humanities and Social Sciences,1 (2015), 131-139.

[12] T. Pathinathan and K. Ponnivalavan, Pentagonal fuzzy number, International Journal of Computing Algorithm, 3 (2014), 1003-1005.

[13] T. Pathinathan and E. A. Dolorosa ,Symmetric periodic fourier series using pentagonal fuzzy number, Journal of Computer and Mathematical Sciences, 10 (2019), 510-518.

[14] S. Ramliand S. H. Jaaman, Optimal solution of fuzzy optimization using pentagonal fuzzy numbers, American Institute of Physics Conference Proceedings,1974 (2018), 1-8.

[15] P. Selvam, A. Rajkumarand J. S. Easwari, Ranking of pentagonal fuzzy numbers applying incentre of centroids, International Journal of Pure and Applied Mathematics, 117 (2017), 165-174.

[16] Y. SafitriandMashadi, Alternative fuzzy algebra to solve dual fully fuzzy linear system using st decomposition method, The Internasional Organization of Scientific Research -Journal of Mathematics, 15 (2019), 32-38.

[17] D. R. A. Sari and Mashadi, New arithmetic triangular fuzzy numberfor solving fully fuzzy linear system using inverse matrix, International Journal of Science: Basic and Applied Research, 46 (2019),169-180.

[18] V. Vijayalakshmi and A. Karpagam, Pentagonal fuzzy number by Cholesky decomposition and singular value decomposition, International Journal ofMathematics Research, 11 (2019), 19-28.

[19] L. A. Zadeh,Fuzzy Sets, Information and Control, 8 (1965), 338-353.

[20] L. A. Zadeh, The Concept of a linguistic variable and its application toapproximate reasoning-I, Information Sciences, 8 (1975), 199-249.