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Quasi B - Pre-Open Functions

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ABSTRACT

In 2011, Navalagi introduced and studied the concepts of Quasi-β-closed, strongly β-closed and weakly β-irresolute functions. In this chapter, we define and study new classes of functions called Quasi-β-pre-open functions and Quasi-β-pre-closed functions using β–open sets and pre-open sets. The basic characterizations of the set functions are studied.

Keywords and Phrases**:** *pre-open sets, pre-closed sets,* ***β*** *–open sets.* ***β*** *-irresoluteness, M-pre-open functions, pre-continuous functions*

I. INTRODUCTION

In 1991, M.L. Thivagar [16] introduced the weak and strong forms of open functions called Quasi-*β*-open and Strongly β-open functions using β-sets due to O. Njastad [12]. In 2011, Navalagi [10] introduced and studied the concepts of Quasi-β-closed functions, strongly *β*–closed functions and weakly β-irresolute functions.

In this chapter, we introduce and study the concepts of Quasi *β*-pre open functions and Quasi-β-pre-closed functions in topological spaces using *β*-closed sets due to A.S. Mashhour et al [6]. We also study the relationships between Quasi-*β*-open, weakly open, and *β*-irresolute functions. On the other hand, in [2] G.L. Faro defined and studied the strongly *β*-irresolute functions in topological space.

II. PRELIMINARIES

Throughout the chapter, (X,τ) and (Y,σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of X. The closure of A and the interior of A are denoted by ClA and IntA, respectively.

DEFINITION 2.1: A subset A is said to be:

(i) β -open [12], if $A \subset$ Int Cl IntA

(ii) preopen [5], if $A \subset$ Int ClA

The complement of an *β*-open (resp. preopen) set is called ***β*** -closed [6] (resp. preclosed [1]). The family of all ***β***–open (resp. pre-open) set s of X is denoted by ***β*** O(X) (resp. PO(X)). Also, it is known that ***β*** O(X) is topology for X stronger than the usual topology on X [12]].

DEFINITION 2.2[1]: The pre-closure of A⊂X is pClA and is defined as the intersection of all pre closed sets of X containing A. Similarly, we have β Cl(A) [6,9] is defined.

DEFINITION 2.3: A function f: X→ Y is said to be:

(i) Continuous [6], if the inverse image of each open set of Y is β –open in X.

(ii) pre-continuous [15], if the inverse image of each open set of Y is preopen in X.

DEFINITION 2.4: A function f: X → Y is said to be:

(i) β -irresolute [4], if the inverse image of each β –open set of Y is β open in X.

(ii) pre-irresolute [15] (or M-pre-continuous [13,14]) if the inverse image of each pre-open set of Y is pre-open in X.

DEFINITION 2.5: A function f: X→ Y is said to be:

(i) β -open [6], if the image of each open set of X is β –open in Y.

(ii) pre-open [7], if the image of each open set of X is pre-open in Y.

(iii) M-pre-open [7] if the image of each pre-open set of X is pre-open in Y.

(iv) M-pre-closed [7,13] if the image of each pre-closed set of X is pre-closed in Y.

(v) Pre-β -open [3] if the image of each β –open set of X is β –open in Y.

(vi) Pre-β -closed [11] if the image of each β –closed set of X is β closed in Y

DEFINITION 2.6: A function f: X→ Y is said to be:

(i) β -closed [6] if the image of each closed set of X is β –closed in Y.

(ii) Pre-closed [1] if the image of each closed set of X is pre-closed in Y.

DEFINITION 2.7: A function f: X→ Y is said to be:

(i) quasi-β -open [16] if the image of each β -open set of X is open in Y.

(ii) quasi-β -closed [10] if the image of each β –closed set of X is closed in Y.

Were call the following,

DEFINITION 3.1[9]: The β -interior of A, denoted by β Int(A) is defined as the union of all β-open sets contained in A.

DEFINITION 3.2[8]: The pre-interior of A, denoted by pInt(A) is defined as the union of all pre-open sets contained in A.

We, define the following.

DEFINITION 3.3: A function f: X→Y is said to be quasi-β-pre-open if the image of every β -open set in X is pre-open in Y. Clearly, every quasi β -pre-open function is pre-open as well as β -open.

We, prove some characterizations of quasi-$\boldsymbol{\beta}$ –pre-open functions in the following.

THEOREM 3.4: A function f: $X \to Y$ is quasi-$\boldsymbol{\beta}$-preopen if and only if or every subset U of X,

$$f(\alpha \text{ Int } U) \subset p \text{ Int}(f(U)).$$

PROOF: Let f be quasi $\propto$-pre-open function. Now, we have pIntU $\subset$ U and αInt(U) is an $\boldsymbol{\beta}$-open set. Hence we obtain that f(αIntU)$\subset$ f(U). As f(αIntU) is pre-open then, f(αIntU)$\subset$ pInt(f(U)).

Conversely, assume the given condition holds. If U be an $\boldsymbol{\beta}$ -open set in X. Then f(U)=f(αIntU) $\subset$pInt(f(U)), but usually pInt(f(U))$\subset$ f(U).

Consequently, f(U)=pInt(f(U)) and hence f is quasi-$\boldsymbol{\beta}$ -pre-open.

LEMMA 3.5: A function f: $X \to Y$ is quasi-$\boldsymbol{\beta}$-pre-open then $\boldsymbol{\beta}$ Int (f^{-1}(G)) $\subset$ f^{-1}(pIntG) for every subset G of Y.

PROOF: Let G be an y arbitrary subset of Y. Then, αInt(f^{-1}(G)) is an $\boldsymbol{\beta}$ –open set in X and f is quasi $\boldsymbol{\beta}$ -pre-open, then f(αInt(f^{-1}(G)))$\subset$ p Intf(f^{1}(G)) $\subset$ pIntG. Thus, $\boldsymbol{\beta}$ Int(f^{-1}(G)) $\subset$ f^{-1}(pIntG).

DEFINITION 3.6[9]: A subset S is called a $\boldsymbol{\beta}$ -neighbourhood of a point x of X if there exists an $\boldsymbol{\beta}$ -open set U such that x$\in$ U$\subset$ S. It is briefly, denoted by $\boldsymbol{\beta}$ -nbd.

THEOREM 3.7: Let X and Y be two spaces and f:$X \to Y$ be a function. Then the following are equivalent:

(i) f is quasi $\boldsymbol{\beta}$ –pre-open.

(ii) For each subset U of X, f ($\boldsymbol{\beta}$ IntU) $\subset$ pInt f(U).

(iii) For each x$\in$ X and each $\boldsymbol{\beta}$ -nbd. U of x in X, there exists a pre-nbd. V off(x) in Y such that V $\subset$ f(U)

PROOF: (i) $\Rightarrow$ (ii): It follows from Th.3.2 above.

(ii)$\Rightarrow$ (iii): Assume (ii) holds.

Let x$\in$ X and U be an arbitrary, β –nbd of x in X. Then there exists an $\boldsymbol{\beta}$-open set V in X such that x$\in$V$\subset$ U. Then by(ii), we f(V)=f ($\boldsymbol{\beta}$ IntV) $\subset$ pInt(f(V)) and hence f(V)=pInt(f(V)). Therefore, it follows that f(V)=W, say, is pre-open in Y such that f(x) $\in$ W$\subset$ f(U). Thus (iii)$\Rightarrow$ (i). Assume(iii) holds, and let U be an arbitrary $\boldsymbol{\beta}$ –openset in X.

Then for each y$\in$ f(U), by(iii) there exists a pre-nbd. V_y of y in Y such that $V_y\subset$ f(U). As V_y is a pre-nbd. of y, there exists a pre-open set W_y in Y such that y$\in$ $W_y\subset V_y$. Thus, f(U)=$\cup$ {W_y|y$\in$f(U)} which is a pre-open set in Y. This implies that f is quasi $\boldsymbol{\beta}$ -pre-open function.

THEOREM 3.8: A function f: X→ Y is quasi β -pre-open if and only if or any subset B of Y and for any β –closed set F of X containing $f^{-1}(B)$, there exists a pre-closed set G of Y containing B such that

$f^{-1}(G)\subset F$.

PROOF: Suppose f is quasi β -pre-open. Let $B\subset Y$ and F be an β -closed set of X containing $f^{-1}(B)$. Now, put G=Y-f(X-F). It is clear that as $f^{-1}(B)\subset F$ implies $B\subset G$. Since f is quasi β-pre-open, we obtain G as a pre-closed set of Y. Then, we have $f^{-1}(G) \subset F$.

Conversely, let U be an β -open set of X and put B=Y-f(U). Then, X-U is an β -closed set in X containing $f^{-1}(B)$. By hypothesis, there exists a pre-closed set F of Y such that $B\subset F$ and $f^{-1}(F)\subset X-U$. Hence, we obtain $f(U)\subset Y-F$. On the other hand, it follows that $B \subset F$, $Y-F\subset Y-B=f(U)$.

Thus, we obtain f(U)=Y-F which is pre-open set and hence f is quasi $\propto$-pre-open function.

THEOREM 3.9: A function f: X→Y is quasi β–pre-open iff f (pClB) $\subset \beta$ Cl(f(B)) for every subset B of Y.

PROOF: Suppose that f is quasi β-pre-open. For any subset B of Y, $f^{-1}(B) \subset \beta\, Cl(f^{-1}(B))$. Therefore, by above Th.3.6, there exists a pre-closed set F in Y such that $B \subset F$ and $f^{-1}(F)\subset \beta\, Cl(f^{-1}(B))$. Therefore, we obtain $f^{-1}(Cl(B))\subset f^{-1}(F)\subset \beta\, Cl(f^{-1}(B))$.

Conversely, let $B\subset Y$ and F be a β -closed set of X containing $f^{-1}(B)$. Put W= $\beta\, Cl_Y B$, then we have $B\subset W$ and W is pre-closed set and $f^{-1}(W) \subset \beta\, Cl_X(f^{-1}(B))\subset F$. Then by Th.3.6 that the function f is quasi β–pre- open.

LEMMA 3.10: Let X, Y and Z be three spaces, and f:X→ Y and g: Y→ Z be two functions and gof: X→ Z is quasi $\propto$-pre-open. Then

(i) If f is an β -continuous, pre-open surjective, then g is quasi β –pre-open.

(ii) If g is continuous and injective, f is quasi β -pre-open.

PROOF:

(a) Let U be an β -open set in Y. Then $f^{-1}(U)$ is an β –open-set in X as f is β –continuous and pre-open function. Since gof is quasi β -pre-open and f is surjective, $g(U)=gof[f^{-1}(U)]$ is pre-open in Z. This shows that g is quasi β pre-open.

(b) Let W be an β-open set in X, then gof(W) is pre-open in Z since gof is quasi β-pre-open. Again g is pre-irresolute and injective, $f(W)=g^{-1}(gof(W))$ is pre-open in Y. This shows that f is quasi $\boldsymbol{\beta}$ –pre-open.

In this section, we define the quasi $\boldsymbol{\beta}$ -pre-closed functions.

DEFINITION 4.1: A mapping f: X→ Y is said to be quasi $\boldsymbol{\beta}$-pre-closed if the image of each $\boldsymbol{\beta}$-closed set in X is pre-closed set in Y.

Clearly, every quasi $\boldsymbol{\beta}$-pre-closed function is pre-closed as well as $\boldsymbol{\beta}$-closed. We, prove some characterizations of quasi-α-pre-closed functions in the following.

LEMMA 4.2: A mapping f:X →Y is quasi $\boldsymbol{\beta}$ –closed iff $f^{-1}(pIntB)\subset \alpha Int(f^{1}(B))$ for every subset B of Y.

Proof is similar to the proof of the Lemma 3.3above.

THEOREM 4.3: A mapping f:X→ Y is quasi $\boldsymbol{\beta}$ -pre-closed iff for any subset B of Y and for any $\boldsymbol{\beta}$-open set Gof X containing $f^{-1}(B)$, there exists an pre-open set U of Y containing B such that $f^{-1}(U)\subset G$.

Proof is similar to that of Th.3.6.

LEMMA 4.4: Let X, Y and Z be threes paces, and f:X→ Y and g: Y→Z be two functions and gof: X→ Z is quasi $\boldsymbol{\beta}$ –pre-closed. Then

(i) If f is an $\boldsymbol{\beta}$-continuous, pre-open surjective, then g is quasi $\boldsymbol{\beta}$-pre-open.

(ii) If g is continuous and injective, f is quasi $\boldsymbol{\beta}$ -pre-open.

PROOF:

(i) Let U be an $\boldsymbol{\beta}$ closed set in Y. Then $f^{-1}(U)$ is an $\boldsymbol{\beta}$ –closed set in X as f is $\boldsymbol{\beta}$ -irresolute function. Since gof is quasi $\boldsymbol{\beta}$-pre-closed and f is surjective, $g(U)=gof[f^{-1}(U)]$ is pre-closed in Z. This shows that g is quasi $\boldsymbol{\beta}$ -pre-closed.

(ii) Let W be an $\boldsymbol{\beta}$ closed set in X, then gof(W) is pre-closed set in Z since gof is quasi $\boldsymbol{\beta}$ pre-closed. Again g is pre-irresolute and injective, $f(W)=g^{-1}(gof(W))$ is pre-closed in Y. This shows that f is quasi $\boldsymbol{\beta}$ –pre-closed.

Next, we prove some decompositions for quasi-$\boldsymbol{\beta}$ –pre-closed functions in the following.

THEOREM 4.5: L e t f: X → Y and g: Y→Z be two functions

Then the following statements are valid.

1. If a function f: XIY is β -open and g: YIZ is pre- β -open, then the composition function gof: XIZ is β -open
2. If a function f: XIY is pre-$\boldsymbol{\beta}$ -open and g: YIZ is Quasi $\boldsymbol{\beta}$-open, then the composition function gof: XIZ

is Quasi- β -open.

3. If a function f: XIY is Quasi- β-open and g: YIZ is pre-open, then the composition function gof: XIZ is Quasi- β –pre-open.
4. If a function f: XIY is Quasi- β-pre-open and g: YIZ is M-pre-open, then the composition function gof: XIZ is Quasi- β -preopen.
5. If a function f: XIY is β –open and g: YIZ is Quasi- β-open, then the composition function gof: XIZ is an open. Easy proof of the Theorem is omitted.

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