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A Two Parameter New Distribution for Life Time Data

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Abstract - *Here in this study we have introduced a two-parameter new distribution. We have discussed some mathematical and statistical characteristics of the distribution such as the probability density function, cumulative distribution function and hazard rate function, survival function, quantile function, the skewness, and kurtosis measures. The model parameters of the proposed distribution are estimated using three well-accepted estimation techniques which are least-square estimation (LSE), maximum likelihood estimation (MLE), and Cramer-Von-Mises estimation (CVME) methods. of The proposed distribution's goodness of fit is also evaluated by fitting it in comparison with some other existing distributions using a real data set.*

Keywords - *CVM, Estimation, Hazard function, LSE, Survival function*

I. INTRODUCTION

Lifetime models are generally used to study the life span of components of a system, a device, and in general, reliability and survival analysis. In fields like life science, medicine, biology, engineering, insurance, etc. where life-time distributions are frequently used. Many continuous probability distributions such as exponential, Cauchy, Weibull have been frequently applied in statistical literature for analyzing lifetime data. For a few years, most of the researchers are attracted towards the exponential distribution for its potential in modeling lifetime data, and excellent performance has been observed in many applications due to the existence of closed form solutions to many survival analyses. It can easily be justified under the assumption of stable failure rate but in the practice, the failure rates are not always stable. Hence, random use of exponential lifetime model seems to be inappropriate and unrealistic. In recent time, new classes of models have been proposed based on modifications of the existing classical probability models, Marshall and Olkin(2007)[9]. Many attempts are seen, in recent times to generate new distributions to broaden established distributions and at the same time, provide considerable flexibility in the practice of modeling data. By adding extra criteria, many procedures may be used to shape a larger family from an existing established distribution. Thus several groups were introduced in the statistical literature by incorporating one or more parameters to create new models, Rinne(2009) and Pham and Lai (2007)[14][16]. Some of the well-known life time models are

A. Lindley distribution : A random variable T follows Lindley distribution (Lindley, 1958)[8] with parameter θ and its probability density function (pdf) is shown as follows,

$$f(t)=\frac{\theta}{\theta+1}(1+t)e^{-\theta t};\ t>0,\theta>0$$

B. Exponential power: The exponential power distribution has been given by (Srivastava & Kumar, 2011)[19] for evaluating the software reliability data. The PDF of the exponential power distribution is

$$g(x)=\alpha\lambda^{\alpha}x^{\alpha-1}e^{(\lambda x)^{\alpha}}\exp\left[1-e^{(\lambda x)^{\alpha}}\right];\ \alpha,\lambda>0,\ x>0$$

C. Logistic-exponential power : The PDF of the logistic-exponential power distribution introduced by (Joshi et al., 2020)[5] is

$$f(x)=\alpha\beta\lambda\frac{x^{\beta-1}e^{\lambda x^{\beta}}\exp\left(e^{\lambda x^{\beta}}-1\right)\left\{\exp\left(e^{\lambda x^{\beta}}-1\right)-1\right\}^{\alpha-1}}{\left[1+\left\{\exp\left(e^{\lambda x^{\beta}}-1\right)-1\right\}^{\alpha}\right]^{2}};\ (\alpha,\beta,\lambda>0),\ \ x>0$$

D. Lindley exponential power: The Lindley exponential power distribution (Joshi & Kumar, 2020)[3] with three parameters (α, λ, θ) for the random variable $X \sim L-EP(\alpha,\lambda,\theta)$ whose probability density function (PDF) is as follows,

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$$f(x)=\alpha\lambda^{\alpha}\left(\frac{\theta^2}{1+\theta}\right)x^{\alpha-1}e^{(\lambda x)^{\alpha}}\left[1-\left(1-e^{(\lambda x)^{\alpha}}\right)\right]\exp\left[\theta\left(1-e^{(\lambda x)^{\alpha}}\right)\right];\ \alpha,\lambda,\theta>0,\ x>0$$

E. Weibull distribution: The Weibull distribution defined by (Weibull, 1951)[21] having two positive parameters α and β whose density function is

$$f(x)=\alpha\beta x^{\beta-1}\exp(\alpha x^{\beta})\ ;\alpha\beta>0,x>0$$

F. Inverse Weibull: In the literature of probability models and applied statistics, the inverse Weibull distribution has established a degree of recognition. Keller et al. (1982) studies the failure rate and density functions' shapes for the basic inverse model [2]. The inverse Weibull distribution with parameters α (scale parameter) and β (shape parameter) with the PDF of a random variable X is given as follows,

$$g\left(x\right)=\alpha\beta x^{-(\beta+1)}\exp\left(-\alpha x^{-\beta}\right);\ \ x\geq 0,\ \alpha>0,\ \beta>0$$

G. Lindley inverse Weibull (LIW): The PDF of Lindley inverse Weibull (LIW) distribution introduced by (Joshi & Kumar, 2020)[4] can be expressed as,

$$f(x;\alpha,\beta,\theta)=\alpha\beta\left(\frac{\theta^2}{\theta+1}\right)x^{-(\beta+1)}e^{-\alpha x^{-\beta}}\left(1-e^{-\alpha x^{-\beta}}\right)^{\theta-1}\left\{1-\ln\left(1-e^{-\alpha x^{-\beta}}\right)\right\};\ \ x>0$$

Where α = scale parameter, β and θ are shape parameters of the LIW distribution.

H. Exponentiated Weibull: The exponentiated Weibull distribution defined by (Mudholkar et al., 1995)[11] having three positive parameters α, β and λ whose density function is

$$f(x)=\alpha\beta\lambda x^{\lambda-1}\exp(-\alpha e^{\lambda})\left[1-\exp(-\alpha e^{\lambda})\right]^{\beta-1}\ \ ;\alpha\beta\lambda>0,x>0$$

The key objective of this article is to put forward a more flexible model to attain a better fit for the lifetime datasets. The rest of the article shows following structure. In Section 2 we have introduced the new distribution and illustrated its important mathematical and statistical properties. In Section 3 we present methods of parameter estimation. We have made use of some well-known estimation methods, namely the maximum likelihood estimation (MLE), least-square estimation (LSE) and Cramer-Von-Mises estimation (CVME) methods. We have created the asymptotic confidence intervals for model parameters using the observed information matrix through ML estimates. A real data set has been evaluated to investigate the applications and capabilities of the proposed distribution are shown in Section 4. In this section, we present the estimated value of the parameters and log-likelihood, AIC, BIC and AICC criterion for ML, LSE, and CVME methods. The goodness of fit of the proposed distribution is evaluated by fitting it and compare with some other existing distributions using a real data set. Conclusions were presented in Section 5.

II. NEW DISTRIBUTION

A two-parameter new distribution (ND) is introduced in this section. The CDF of ND can be expressed as

$$F(x)=1-\left\{1-\left(1+\frac{\lambda}{x}\right)e^{-\lambda/x}\right\}^{\alpha}\ \ ;\alpha>0,\lambda>0,\ x>0 \tag{2.1}$$

The corresponding PDF can be expressed as,

$$f(x)=\frac{\alpha\lambda^2e^{-\lambda/x}}{x^3}\left\{1-\left(1+\frac{\lambda}{x}\right)e^{-\lambda/x}\right\}^{\alpha-1}\ \ ;\alpha,\lambda>0,\ x>0 \tag{2.2}$$

The survival function is

$$S(x)=\left\{1-\left(1+\frac{\lambda}{x}\right)e^{-\lambda/x}\right\}^{\alpha}\ \ ;\alpha>0,\lambda>0,\ x>0 \tag{2.3}$$

Similarly, the hazard rate function (HRF) is

$$h(x)=\frac{\alpha\lambda^2 e^{-\lambda/x}}{1-\left(1+\frac{\lambda}{x}\right)e^{-\lambda/x}};\alpha,\lambda>0,\ x>0 \tag{2.4}$$

2.1 Quantile and Generating Functions

Quantile Function

The quantile function related to the random variable's probability distribution, in probability statistics, defines the value of the random variable in a way that the probability of the variable being less than or equal to the value is equal to the probability assigned. It is also called the inverse cumulative distribution function or percent-point function. The definition of the pth quantile is following equation's real solution.

$$Q(p)=F^{-1}(p)$$

the quantile function of ND is attained by inverting CDF (2.1) as

$$1-\left(1+\frac{\lambda}{x}\right)e^{-\lambda/x}-(1-p)^{1/\alpha}=0\ ;\ 0<p<1 \tag{2.5}$$

Generation of the random numbers:

For the random numbers generation of the ND distribution, we stimulate values of random variable X with CDF (2.1). Let V represent a uniform random variable in (0, 1), then the simulated values of X are obtained by

$$1-\left(1+\frac{\lambda}{x}\right)e^{-\lambda/x}-(1-v)^{1/\alpha}=0\ ;\ 0<v<1 \tag{2.6}$$

Skewness and Kurtosis:

The coefficient of skewness based on quantile can be calculated as

$$S_k(Bowley)=\frac{Q(3/4)+Q(1/4)-2Q(1/2)}{Q(3/4)-Q(1/4)},\text{ and} \tag{2.7}$$

The coefficient of kurtosis based on octiles was defined by (Moors, 1988)[10] which can be obtained as

$$K_u(M)=\frac{Q(0.875)-Q(0.625)+Q(0.375)-Q(0.125)}{Q(3/4)-Q(1/4)}, \tag{2.8}$$

The various probability density function and hazard rate function's shapes of ND (α, β, λ) with different values of parameters are illustrated in Figure 1.

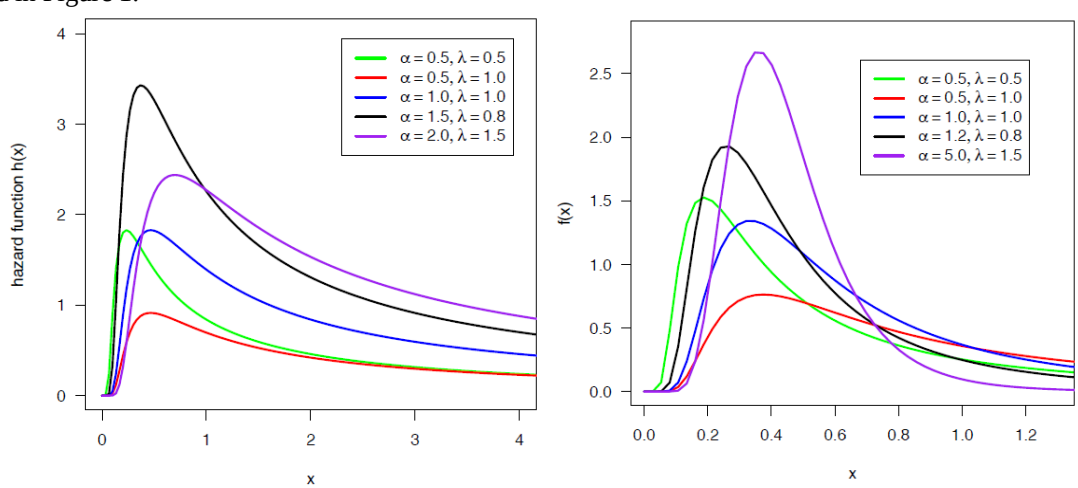

Figure 1. For different values of α, β, and λ, graphs of hazard function (left panel) and density function (right panel)

III. METHODS OF PARAMETER ESTIMATION

3.1 Maximum Likelihood Estimation (MLE) Method

In this portion, we have discussed the MLEs of the ND distribution. Let $\underline{x} = (x_1, \ldots, x_n)$ be a random sample of size 'n' from ND (α, λ) then the log likelihood function l(α, λ / $\underline{x}$) can be written as,

$$l(\alpha, \lambda \mid \underline{x}) = n \ln \alpha + 2n \ln \lambda - 3\sum_{i=1}^{n} \ln x_i - \lambda \sum_{i=1}^{n} (1/x_i) + (\alpha - 1)\sum_{i=1}^{n} \ln\left[1 - \left(1 + (\lambda / x_i)\right) e^{-\lambda / x_i}\right] \quad (3.1.1)$$

By differentiating (3.1.1) with respect to unknown parameters α, and λ, we get

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \ln\left[1 - \left(1 + (\lambda / x_i)\right) e^{-\lambda / x_i}\right]$$

$$\frac{\partial l}{\partial \lambda} = \frac{2n}{\lambda} - \sum_{i=1}^{n} \frac{1}{x_i} + \lambda(\alpha - 1)\sum_{i=1}^{n} \frac{e^{-\lambda / x_i}}{x_i^2 \{1 - \left(1 + (\lambda / x_i)\right) e^{-\lambda / x_i}\}}$$

After equating these non-linear equations to zero and solving for the unknown parameters (α, λ) we will obtain the ML estimators of the ND distribution. Manually, it is difficult to solve hence using appropriate computer software one can solve these equations. Let parameter vector represented by $\underline{\Upsilon} = (\alpha, \lambda)$ and the corresponding MLE of $\underline{\Upsilon}$ as $\underline{\Upsilon} = (\hat{\alpha}, \hat{\lambda})$, then the asymptotic normality results in, $\left(\underline{\Upsilon} - \underline{\Upsilon}\right) \to N_2\left[0, \left(I\left(\underline{\Upsilon}\right)\right)^{-1}\right]$ where $I\left(\underline{\Upsilon}\right)$ is the information matrix of Fisher given by,

$$I\left(\underline{\Upsilon}\right) = -\begin{pmatrix} E\left(\frac{\partial^2 l}{\partial \alpha^2}\right) & E\left(\frac{\partial^2 l}{\partial \alpha \partial \lambda}\right) \\ E\left(\frac{\partial^2 l}{\partial \lambda \partial \alpha}\right) & E\left(\frac{\partial^2 l}{\partial \lambda^2}\right) \end{pmatrix}$$

In practice, we don't know $\underline{\Upsilon}$ hence it is useless that the MLE has an asymptotic variance $\left(I\left(\underline{\Upsilon}\right)\right)^{-1}$. Hence we approximate the asymptotic variance by plugging in the estimated value of the parameters. The observed fisher information matrix $O\left(\underline{\Upsilon}\right)$ is used as an estimate of the information matrix $I\left(\underline{\Upsilon}\right)$ given by

$$O\left(\underline{\Upsilon}\right) = -\begin{pmatrix} \frac{\partial^2 l}{\partial \alpha^2} & \frac{\partial^2 l}{\partial \alpha \partial \lambda} \\ \frac{\partial^2 l}{\partial \lambda \partial \alpha} & \frac{\partial^2 l}{\partial \lambda^2} \end{pmatrix}_{\left|(\hat{\alpha}, \hat{\lambda})\right.} = -H\left(\underline{\Upsilon}\right)_{\left|(\underline{\Upsilon} = \underline{\Upsilon})\right.}$$

where Hessian matrix is denoted by H

To maximize the likelihood, the Newton-Raphson algorithm gives the observed information matrix. Hence, the variance-covariance matrix can be expressed as,

$$\left[-H\left(\underline{\Upsilon}\right)_{\left|(\underline{\Upsilon} = \underline{\Upsilon})\right.}\right]^{-1} = \begin{pmatrix} \mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha}, \hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha}, \hat{\lambda}) & \mathrm{var}(\hat{\lambda}) \end{pmatrix} \quad (3.3)$$

Therefore from the asymptotic normality of Maximul Likelihood Estimates, approximate 100(1-a) % confidence intervals for α, and λ can be built as,

$$\hat{\alpha} \pm Z_{\alpha/2}\sqrt{\mathrm{var}(\hat{\alpha})} \text{ , and } \hat{\lambda} \pm Z_{\alpha/2}\sqrt{\mathrm{var}(\hat{\lambda})} \text{ ,}$$

where upper percentile of standard normal variate is denoted by $Z_{\alpha/2}$.

3.2. Method of Least-Square Estimation (LSE)

Swain et al. (1988) have introduced the weighted least square estimators and ordinary least square estimators for estimating the parameters of Beta distributions[20]. The least-square estimators of the unknown parameters α and λ of ND distribution can be calculated by minimizing

$$Q\left(X;\alpha,\lambda\right)=\sum_{i=1}^{n}\left[F(X_i)-\frac{i}{n+1}\right]^2 \tag{3.2.1}$$

with respect to unknown parameters α and λ.

Consider $F(X_i)$ represent the distribution function of the ordered random variables $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ and $\{X_1, X_2, \ldots, X_n\}$ denote random sample of size n from a distribution function F(.). The least-square estimators of α and λ say $\hat{\alpha}$ *and* $\hat{\lambda}$ respectively, can be attained by minimizing

$$Q\left(X;\alpha,\lambda\right)=\sum_{i=1}^{n}\left[1-\left\{1-\left(1+\frac{\lambda}{x}\right)e^{-\lambda/x}\right\}^{\alpha}-\frac{i}{n+1}\right]^2;x\geq 0,\left(\alpha,\lambda\right)>0. \tag{3.2.2}$$

with respect to α and λ.

Differentiating (3.2.2) with respect to α, β and λ we get,

$$\frac{\partial Q}{\partial\alpha}=-2\sum_{i=1}^{n}\left[1-\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}^{\alpha}-\frac{i}{n+1}\right]\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}^{\alpha}\ln\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}$$

$$\frac{\partial Q}{\partial\lambda}=2\alpha\sum_{i=1}^{n}\left[1-\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}^{\alpha}-\frac{i}{n+1}\right]\frac{\lambda}{x_i}e^{-\lambda/x_i}\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}^{\alpha-1}$$

Similarly the weighted least square estimators can be acquired by minimizing

$$Q\left(X;\alpha,\lambda\right)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right]^2$$

with respect to α and λ. The weights w_i are $w_i=\dfrac{1}{Var(X_{(i)})}=\dfrac{\left(n+1\right)^2\left(n+2\right)}{i\left(n-i+1\right)}$

Hence, the weighted least square estimators of α and λ respectively can be acquired by minimizing,

$$Q\left(X;\alpha,\lambda\right)=\sum_{i=1}^{n}\frac{\left(n+1\right)^2\left(n+2\right)}{i\left(n-i+1\right)}\left[1-\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}^{\alpha}-\frac{i}{n+1}\right]^2 \tag{3.2.3}$$

with respect to α and λ.

3.3. Method of Cramer-Von-Mises estimation (CVME)

The Cramer-Von-Mises estimators of α and λ are acquired by minimizing the function

$$B\left(X;\alpha,\lambda\right)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\alpha,\lambda\right)-\frac{2i-1}{2n}\right]^2$$

$$= \frac{1}{12n} + \sum_{i=1}^{n}\left[1-\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}^{\alpha} - \frac{2i-1}{2n}\right]^2 \tag{3.3.1}$$

Differentiating (3.3.1) with respect to α and λ we get,

$$\frac{\partial B}{\partial \alpha} = -2\sum_{i=1}^{n}\left[1-\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}^{\alpha} - \frac{2i-1}{2n}\right]\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}^{\alpha} \ln\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}$$

$$\frac{\partial B}{\partial \lambda} = 2\alpha\sum_{i=1}^{n}\left[1-\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}^{\alpha} - \frac{2i-1}{2n}\right]\frac{\lambda}{x_i}e^{-\lambda/x_i}\left\{1-\left(1+\frac{\lambda}{x_i}\right)e^{-\lambda/x_i}\right\}^{\alpha-1}$$

By solving $\frac{\partial B}{\partial \alpha} = 0$ *and* $\frac{\partial B}{\partial \lambda} = 0$ simultaneously we will get the CVM estimators.

IV. ILLUSTRATION WITH REAL DATASET

From accelerated life test of 59 conductors, the failure time data in hours with no censored observation provided in this section was derived (Nelson & Doganaksoy, 1995)[13]. Owing to the diffusion of atoms in the coils in the circuit, we may see microcircuit failure; such phenomenon is known as electro-migration.

6.545, 9.289, 7.543, 6.956, 6.492, 5.459, 8.120, 4.706, 8.687, 2.997, 8.591, 6.129, 11.038, 5.381, 6.958, 4.288, 6.522, 4.137, 7.459, 7.495, 6.573, 6.538, 5.589, 6.087, 5.807, 6.725, 8.532, 9.663, 6.369, 7.024, 8.336, 9.218, 7.945, 6.869, 6.352, 4.700, 6.948, 9.254, 5.009, 7.489, 7.398, 6.033, 10.092, 7.496, 4.531, 7.974, 8.799, 7.683, 7.224, 7.365, 6.923, 5.640, 5.434, 7.937, 6.515, 6.476, 6.071, 10.491, 5.923.

We have obtained the estimated values of the parameters using MLE method by utilizing the optim() function in R software (R Core Team, 2020)[15] and (Schmuller, J., 2017)[17] by maximizing the likelihood function (3.1). We have obtained the Log-Likelihood value is l = -111.8084. The contour plot and fitted CDF with empirical distribution function are depicted in figure 2, (Kumar & Ligges, 2011)[6]

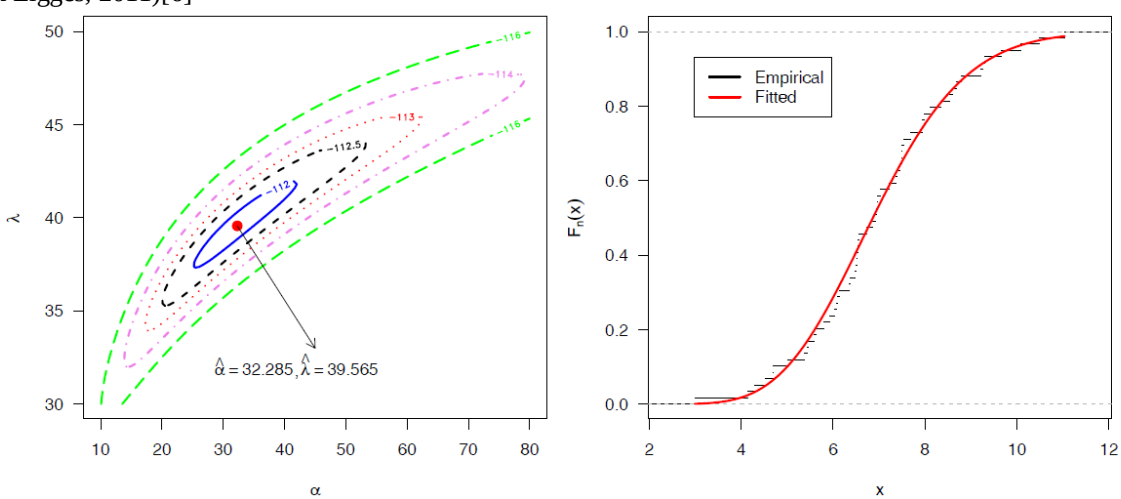

Figure 2. Contour plot (left panel) and fitted CDF with empirical distribution function (right panel)

We have presented the MLE's with their standard errors (SE) and 95% confidence interval for α and λ in Table 1.

Table 1

MLE and SE and 95% confidence interval for α and λ

Parameter	MLE	SE	95% ACI
alpha	32.285	3.824	(24.7900, 39.7800)
lambda	39.565	1.486	(36.6524, 42.4776)

In Figure 3 we have plotted the Q-Q plot and P-P plot and it is seen that the proposed distribution fits the data very well.

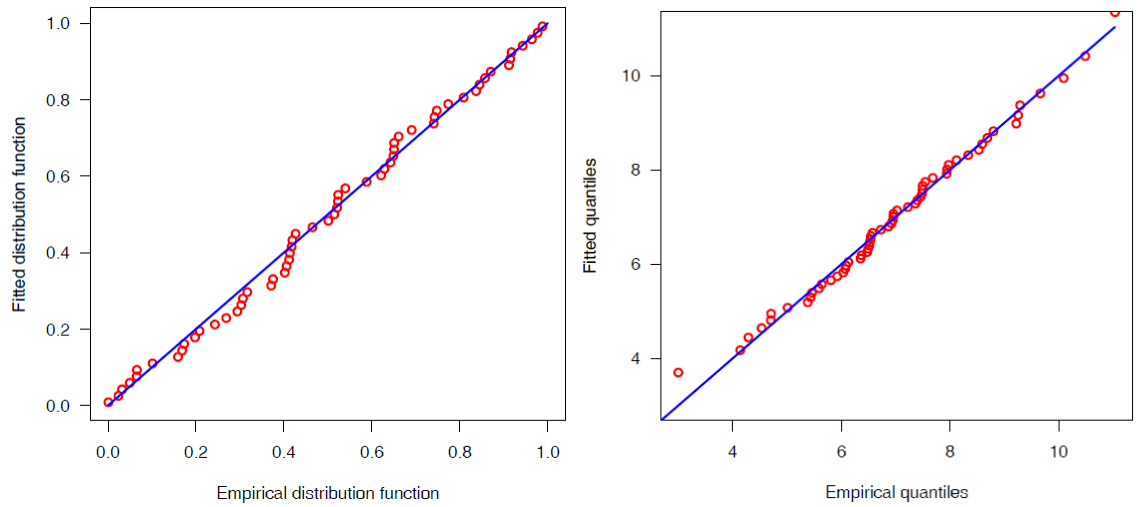

Figure 3. The P-P plot (left panel) and Q-Q plot (right panel) of the ND distribution.

We have displayed the graph of the profile log-likelihood function of α and λ in Figure. 4 and observed that the MLEs are unique.

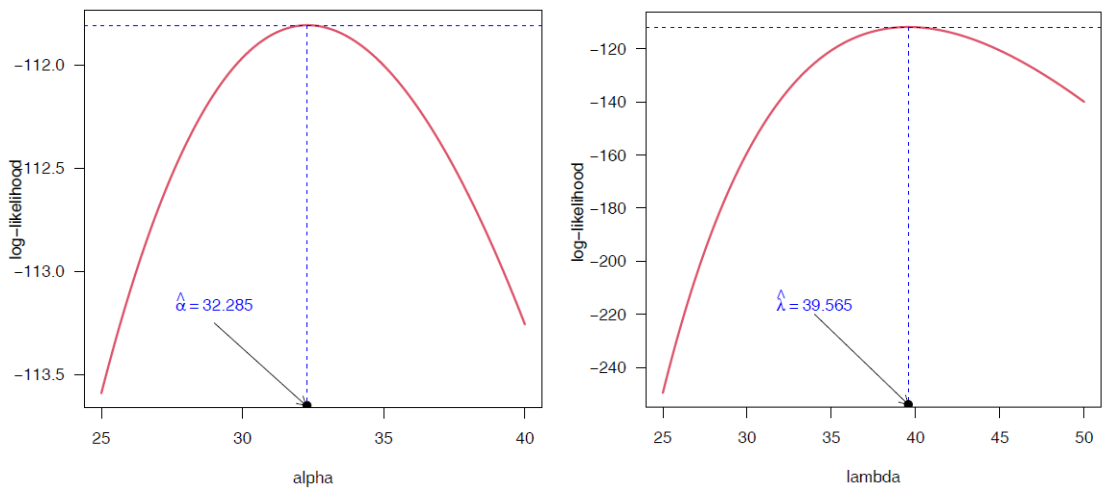

Figure 4. Graph of profile log-likelihood function of α and λ of the ND distribution.

We have presented the estimated value of the parameters of ND distribution using MLE, LSE and CVE method and their corresponding negative log-likelihood, and AIC, BIC and CAIC criterion in Table 2.

Table 2

Estimated parameters, log-likelihood, and AIC

Method of Estimation	$\hat{\alpha}$	$\hat{\lambda}$	-LL	AIC	BIC	CAIC
MLE	32.2852	39.5652	111.8084	227.6168	231.7719	227.8311
LSE	37.3710	40.9357	111.8692	227.7384	231.8935	227.9527
CVE	42.3754	41.9483	112.0107	228.0213	232.1764	228.2356

In Table 3 we have presented The KS, W and A^2 statistics with their corresponding p-value of MLE, LSE and CVE estimates.

Table 3

The KS, W and A^2 statistics with a p-value

Method of Estimation	*KS(p-value)*	*W(p-value)*	*A^2(p-value)*
MLE	0.0658(0.9453)	0.0334(0.9649)	0.2150(0.9858)
LSE	0.0545(0.9906)	0.0271(0.9855)	0.2060(0.9887)
CVE	0.0506(0.9963)	0.0259(0.9882)	0.2302(0.9798)

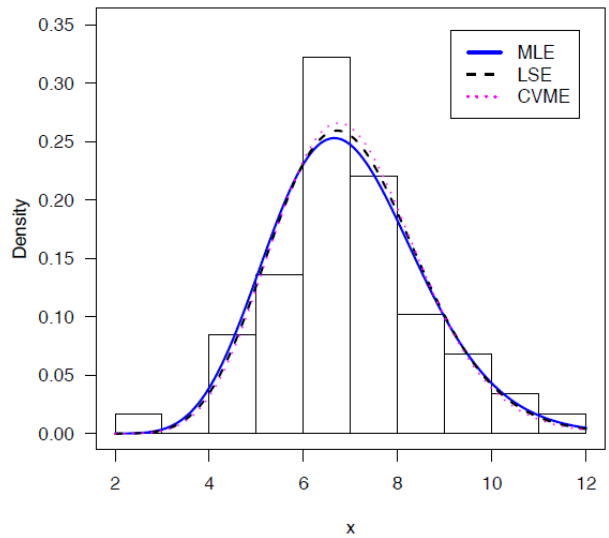

Figure 5. The Histogram and the density function of fitted distributions of estimation methods MLE, LSE and CVM.

To illustrate the goodness of fit of the ND distribution, we have selected some well known distribution for comparison purpose which are listed blew,

A. Modified Weibull (MW)

The MW distribution was given by (Lai et al., 2003)[7] with PDF of

$$f_{MW}(x) = \alpha\left(\lambda + \beta x\right)x^{\lambda-1}\exp(\beta x - \alpha x^{\lambda}e^{\beta x} \quad ;(\alpha\beta\lambda) > 0, x > 0$$

B. Exponential power (EP) distribution:

The PDF of Exponential power (EP) distribution (Smith & Bain, 1975)[18] is

$$f_{EP}(x) = \alpha\,\lambda^{\alpha}\; x^{\alpha-1}\, e^{(\lambda\, x)^{\alpha}}\; \exp\left\{1 - e^{(\lambda\, x)^{\alpha}}\right\} \quad ;(\alpha, \lambda) > 0, \quad x \geq 0.$$

where α and λ are the shape and scale parameters, respectively.

C. Generalized Exponential (GE) distribution

The PDF of generalized exponential distribution (Gupta & Kundu, 1999)[1]

$$f_{GE}\left(x;\alpha,\lambda\right) = \alpha\,\lambda\, e^{-\lambda\, x}\left\{1 - e^{-\lambda\, x}\right\}^{\alpha-1} ;\left(\alpha,\lambda\right) > 0, x > 0.$$

D. Gompertz distribution (GZ)

The PDF of Gompertz distribution (Murthy et al., 2003)[12] with parameters α and θ is

$$f_{GZ}\left(x\right) = \theta\;\; e^{\alpha\, x}\; exp\left\{\frac{\theta}{\alpha}\left(1 - e^{\alpha\, x}\right)\right\} \quad ;x \geq 0, \theta > 0, -\infty < \alpha < \infty.$$

For the assessment of potentiality of the proposed model we have calculated the Bayesian information criterion (BIC), Corrected Akaike information criterion (CAIC), Akaike information criterion (AIC), and Hannan-Quinn information criterion

(HQIC) which are presented in Table 4.

Table 4
Log-likelihood (LL), AIC, BIC, CAIC and HQIC

Distribution	-LL	AIC	BIC	CAIC	HQIC
ND	111.8084	227.6168	231.7719	227.8311	229.2388
MW	112.5218	231.0435	237.2761	231.4799	233.4765
GE	114.9473	233.8946	238.0497	234.1089	235.5166
EP	116.5015	237.0029	241.1580	237.2098	238.6249
GZ	117.1740	238.3480	242.5031	238.5623	239.9700

The Histogram and the density function of fitted distributions and Empirical distribution function with the estimated distribution function of ND distribution and some selected distributions are presented in Figure 5.

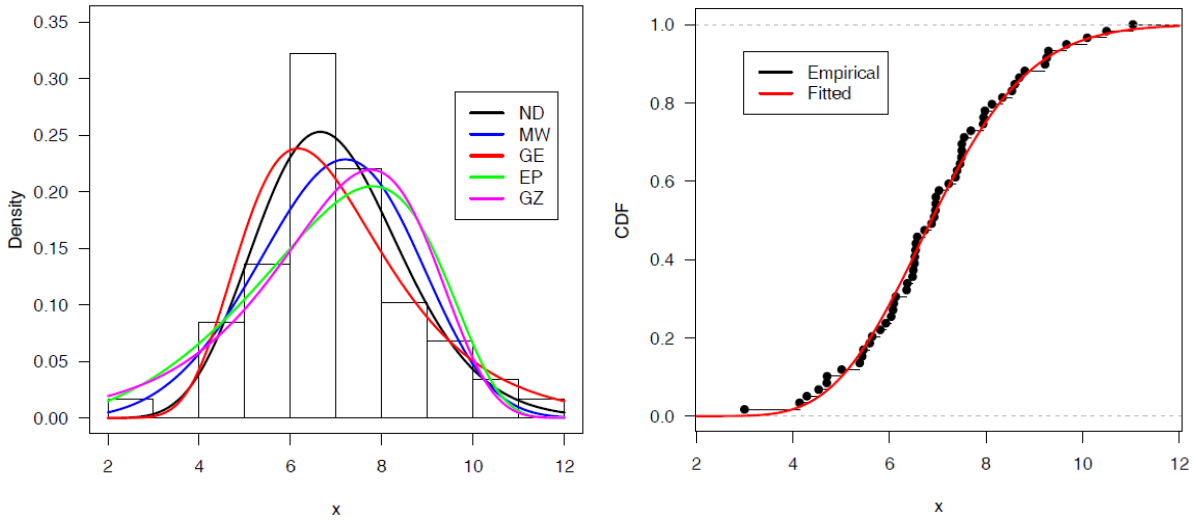

Figure 5. The Histogram and the density function of fitted distributions (left panel) and Empirical distribution function with estimated distribution function (right panel).

For comparison of the goodness-of-fit of the NEEE distribution with well established distributions we have presented the value of Kolmogorov-Simnorov (KS), the Anderson-Darling (W) and the Cramer-Von Mises (A^2) statistics in Table 5. It is observed that the ND distribution has the minimum value of the test statistic and higher *p*-value thus we conclude that the ND distribution gets quite better fit and more consistent and reliable results from others taken for comparison.

Table 5
The goodness-of-fit statistics and their corresponding p-value

Distribution	*KS(p-value)*	*W(p-value)*	*A^2(p-value)*
ND	0.0658(0.9453)	0.0334(0.9649)	0.2150(0.9858)
MW	0.0914(0.6738)	0.0821(0.6816)	0.4839(0.7626)
GE	0.1042(0.5103)	0.1173(0.5079)	0.7368(0.5282)
EP	0.1365(0.2021)	0.2398(0.2021)	1.3735(0.2098)
GZ	0.1306(0.2464)	0.216(0.2387)	1.3143(0.2277)

V. CONCLUSION

This paper introduces a two-parameter continuous distribution named ND distribution. Some important statistical properties of the ND distribution are illustrated such as the shapes of the probability density, cumulative density and hazard rate functions, survival function, reverse hazard rate function. Further quantile function, the skewness, and kurtosis measures are derived and established. We have employed three well-known estimation methods which are maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises estimation (CVME) methods to estimate the model parameters and we

concluded that the MLEs are quite better than LSE, and CVM estimators. A real dataset is considered to explore the applicability of the ND distribution and found that it is quite better than other lifetime models taken into consideration. We expect this distribution may be an alternative in the field of reliability analysis, probability theory and applied statistics.

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