International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373 /doi:10.14445/22315373/IJMTT-V66I12P520

Volume 66 Issue 12, 148-156, December, 2020

Acceptance Sampling Plan for Life Testing under Generalized Exponential-Poisson Distribution

V Kaviyarasu[1] and S. Sivasankari[2]
[1]*Assistant Professor*, [2]*Ph D Research Scholar*
Department of Statistics, Bharathiar University, Coimbatore- 641 046, Tamil Nadu, India

Abstract — *In this article, a new compound distribution named as Generalized Exponential-Poisson (GEP) distribution is studied the truncated life test of a sampling plan. Their probability of acceptance for the single sampling is considered along with its associated decision rule are given to obtain the smallest sample size for the three parameter distribution. Under this study, the specified percentile life time is calculated and the design parameter such as sample sizes, acceptance number are determined to satisfying the specified quality levels. OC curve and the minimum ratio values for the specified producer's risk are obtained and tabulated for the easy selection of the plan parameters. Further, the suitable illustration for the sampling plan are given to study the plan parameters with an example.*

Keywords: *Generalized Exponential-Poisson (GEP) Distribution, Percentiles, Truncated acceptance sampling plan.*

1. INTRODUCTION

Quality that every product possesses is a number of factors jointly describes what the user or consumer thinks. Sometimes quality becomes totally a consumer decision factor. Hence it is inevitable, the producer must improve or maintain the quality characteristic to satisfy the consumer. Statistical Quality Control deals various techniques which leads to improve the quality of the product. Acceptance Sampling plays a vital role in quality improvement while making a decision on lot based on the inspection of the samples. It provides the rule in which a product is sampled, tested and evaluated. The information gathered from the data is used to control and improve the manufacturing process. Among the techniques like Process Control and Product Control, Reliability is one of the main dimensions even though the quality of a product evaluated in several ways.

Reliability is the discipline of ensuring that a product will be reliable when it is operated in a specified manner. The objective of the reliability studies are to maximize the reliability and minimize the effects of failures by minimizing cost and time. The major objective of the reliability life test plan or the truncated life test plan is to obtain the minimum sample size that will assure whether a product is reliable or not for the given life time data with the minimum period and cost. Both the producer and customer will satisfy if the manufactured product is reliable. To satisfy both of them, testing the product should be reliable. In case of the destructive item, 100% inspection is meaningless at the same time while testing the time truncated which is well known as truncated acceptance sampling plan. Many authors have been given their valuable contribution towards Acceptance sampling plan of truncated life test for instance Epstein(1954) [3], Sobel and Tischendrof (1959)[16], Goode and Kao (1961)[7], Gupta and Groll (1961)[5], Gupta (1962)[6], Fertig and Mann (1980)[4], Kantam and Rosaiah (1998)[8], Kantam *et al.* (2001)[9], Baklizi (2003)[2], Wu and Tsai (2005) [20]and Tsai and Wu (2006)[17].

In the production Industry, it is necessary to bring out the information that how long a product operated successfully with a precise measure. That measure should be a good representation for a particular distribution. Even though there are numerous ways to measures the behaviour of the distribution, the Mean life time or the Median life time of the product split the whole distribution into two equal parts. When the distribution is asymmetrical, it does not provide the accurate information. While the percentiles divides the distribution into 100 equal parts which discusses more information about its position than the measures of central tendencies. Percentile is an alternative one and an important measure to study the lifetime distribution than any other measures available in distribution function. Furthermore, Acceptance Sampling plan based on mean lifetime may not satisfy the need of engineers like strength or breaking stress of products. Hence for the symmetrical distribution, mean and median are equal. Considering this manipulation of the Truncated acceptance sampling under life test plan based on percentile lifetime is a generalization of acceptance sampling plans than the Mean or Median life time.

Accordingly many authors proposed their acceptance sampling plans under percentile for life time studies under their life time distribution are given in brief. Lio et al.(2009) [12] discussed that a small decrease in the mean life from a lot could be accepted after inspection. But the material strengths of products are deteriorated significantly and may not meet the consumer's expectation such problem will not occur in percentiles. Hence the engineers pay more attention to the percentiles life time than the mean life time. Rao and Kantam (2010)[14] Acceptance sampling plans from truncated life tests based on the Log-Logistic distribution for percentiles. Rao et al. (2012)[15] proposed truncated life sampling plan for

Inverse Rayleigh Distribution for percentiles. Aslam et al (2010)[1] acceptance sampling plans for Burr type XII distribution percentiles under the truncated life test. Kaviyarasu and Fawaz (2017)[10] developed Acceptance Sampling Plans for Percentiles Based on the Modified Weibull Distribution and Weibull-Poisson distribution.

This paper further explains about, the Generalised Exponential–Poisson distribution and its application. Here, the minimum sample size, OC values and the Producer's minimum ratio tables are constructed for the confidence levels and their specified percentiles values are given. The table values are useful for the selection of attribute sampling plan with ready-to-use. The real life time examples are provided with the numerical illustrations for easy understanding of this method. The final part made with the conclusions.

2. GENERALIZED EXPONENTIAL-POISSON(GEP) DISTRIBUTION

The exponential distribution may be appropriate for modelling random failures. Random failures are usually caused by external shocks such as an unexpected change in load. External shocks usually can be modelled using the Poisson process. If every shock causes a failure, the product life can be approximated using the exponential distribution. The exponential distribution is widely studied is especially popular in modelling the life of the product for instant when some electronic components and systems are manufactured. For example, Murphy et al. (2002)[13] indicate that the exponential distribution adequately fits the failure data of a wide variety of systems, such as radar, aircraft and spacecraft electronics, satellite constellations, communication equipment, and computer networks. If this kind of situation arises, the GEP distribution can be well suited distribution and it is good for such kind of situations occurs.

Wagner Barreto-Souza and Francisco Cribari-Neto(2009)[18] introduces the Generalized Exponential–Poisson distribution(also known as Exponentiated Exponential-Poisson distribution) which is by compounding an Exponential distribution and a Poisson distribution with three parameters. The failure rate of the distribution can be decreasing or increasing and also upside-down bathtub shaped model.

The Cumulative Distribution Function of the Generalized EP is given

$$F(X;\theta) = \left(\frac{1-e^{-\lambda+\lambda \exp(-\beta x)}}{1-e^{-\lambda}}\right)^{\alpha} \quad ; \ x>0 \qquad (1)$$

The Probability density function of Generalized EP is

$$f(x;\theta) = \frac{\alpha\lambda\beta}{\left(1-e^{-\lambda}\right)^{\alpha}} \ \left\{1-e^{-\lambda+\lambda\exp(-\beta x}\right\}^{\alpha-1} \ e^{-\lambda-\beta x+\lambda\exp(-\beta x)} \qquad (2)$$

Where θ (>0) = (α, β, λ) , α is the shape paramaeter , β is the scale parameter of the Exponential distribution and λ is the Poisson parameter. When α = 1 , Generalized Exponential Poisson reduces to Exponential Poisson distribution. When α = 1 and λ→ 0, Exponential poisson reduces Exponential distribution with parameter β. The Percentile denoted, tq is the time by which a specified fraction q of the population fails.

$$\text{ie., } P(T\leq t_q) = q$$

The 100qth percentile or the q[th] quantile of the GEP distribution is given by,

$$t_q = -\beta^{-1} \ log\left(1+\lambda^{-1} \ log\left(1-q^{1/\alpha} \ \left(1-e^{-\lambda}\right)\right)\right)$$

tq and q are directly proportional, let

$$\eta = - \ log\left(1+\lambda^{-1} \ log\left(1-q^{1/\alpha} \ \left(1-e^{-\lambda}\right)\right)\right) \qquad (3)$$

$$\Rightarrow t_q = {}^{\eta}/_{\beta}$$

$$\Rightarrow \ \beta = {}^{\eta}/_{t_q} \qquad (4)$$

Hence the cumulative distribution function becomes by replacing the scale parameter by (4)

$$F(t) = \left(1 - e^{-\lambda + \lambda\, exp\left(t/t_q\ .\eta\right)}\right)^{\alpha} \ \left((1-e^{-\lambda})^{-1}\right)^{\alpha} \quad t > 0, \eta > 0$$

Let $\delta = t/t_q$

$$F(t;\delta) = \left(1 - e^{-\lambda + \lambda\, exp(\delta\ .\eta)}\right)^{\alpha} \ \left((1-e^{-\lambda})^{-1}\right)^{\alpha} \ , t > 0,\ \delta > 0 \quad (5)$$

3. GEP FOR PERCENTILE LIFE TIME

The truncated life test of the acceptance single sampling plan characterized by the triplet $(n, c, t/t_q^0)$. In this life testing , the noted criteria is to check whether the life time of the product meet the consumer's expectation or not. The product will meet the consumer's expectation when the true percentile life time of the products exceeds the specified one. when $t_q \geq t_q^0$, the lot will be accepted otherwise it is rejected. A truncated life test may be conducted to determine the minimum sample size that will ensure specified percentile life time of the products at a pre-assigned time. To attain the minimum sample size, consider the consumer's risk such as the probability of accepting a bad lot should not exceed 1-p*. For given p*, by using Binomial distribution, one can obtain the minimum sample size.

3.1 Designing Truncated Acceptance Sampling Plan

The Single Sampling Plan (SSP) is one of the well-known and most widely used sampling plan among the acceptance sampling techniques in the Production Industry. In truncated sampling using SSP, one can utilize the sampling plan parameters which follows the Binomial distribution since the life time of the products are categorised as success or failure. The parameters of the Binomial distribution (n,c,p).

where n = The sample size and c = The acceptance number and Here p is assumed to follow GEP for the life time random variable with parameter t/t_q^0.

Where t = Pre-assigned test time

t_q^0 = Specified 100q[th] percentile life time

The sample size n is obtained by asserting the hypothesis $tq > t_q^0$ with the given values of p* and c must satisfy the inequality

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \ \leq \ 1 - p^*$$

Since the probability p* is the confidence level, chance of rejecting a bad lot with $tq < t_q^0$ is atleast equal to p*. Obviously the consumer's risk is the probability of accepting a bad lot should not exceed 1-p*.

Functionally p= $F(t,\ \delta_0)$ depends on $\delta_0 = t/t_q^0$. Hence it is sufficient to specify δ_0.

3.2 OC Function

The Operating Characteristic Function of the sampling plan is assumed to follow B (n, c, p) gives the Probability of Acceptance ie. L(p) corresponding to their product quality.

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$$

Where p= $F\ (t\ ,\ \delta)$, Here $F\ (t\ ,\ \delta)$ is a function of $\delta = {}^{t}/_{t_q}$.

Therefore $p = F\left(\frac{t}{t_q^0}\ \frac{1}{d_q}\right)$ Where $d_q = \frac{t_q}{t_q^0}$. Using the above OC function, one can obtain OC values and OC curve for the given sampling plan $\left(n, c, {}^{t}/_{t_q^0}\right)$. The OC values presented in the Table (2) for the sampling plan $\left(n, c = 2, {}^{t}/_{t_{0.50}^0}\right)$. Figure 1 shows the OC curve for the values given Table (4) with $p^* = 0.90$.

Aslam et. al (2009) stated that when one wants to implement an acceptance sampling plan to study the percentile lifetime in a truncated life test experiments it is to determine the time in terms of the specified percentile lifetime. Let α be the producer's risk and β the consumer's risk. Here the producer always requires the lot acceptance probability should be larger than 1- α at the various level of percentile ratio and the consumer wants it should be smaller than β. Therefore any plan studied in acceptance sampling plan should support both the producer and consumer simultaneously.

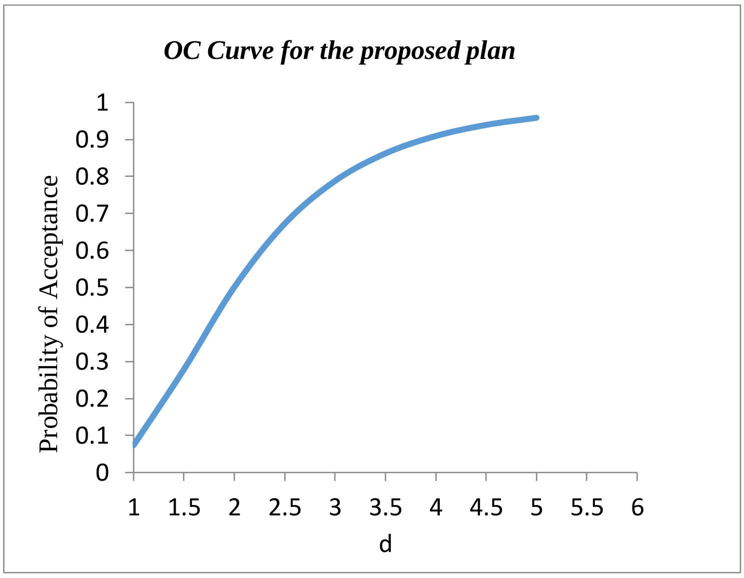

Fig. 1: Shows the OC curve for the sampling plan (n=5, c=2, $t/t^o_{q0.50}$) with P^*=0.90.

3.3 Producer's Risk

The Producer's risk is known as the chance of rejecting a lot when $t_q > t_q^0$. For a given value of α , the value of d_q to be calculated which ensures the producer's risk is less than or equal to α. Hence the chance of acceptance exceeds 1- α, by using this condition one can obtain the minimum ratio d_q with the specified confidence level p*.

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq 1 - \alpha$$

To the development of sampling plan, $d_{0.50}$ can be calculated when α=0.05.

3.4 Operating Procedure

Step 1: To find the value of η , fix α=2 and λ=2 , substitute in the equation (3)
We get η = 0.639826.

Step 2: Substitute $\beta = {}^{\eta}/_{t_q}$ in the equation (4), Find p= F(t , δ)

Step 3: Find the minimum sample size n by satisfying the inequality,

$$\sum_{i=0}^{c}\binom{n}{i}p^{i}(1-p)^{n-i} \leq 1-p^{*}$$

Step 4: Find the OC values by using the equation,

$$L(p)=\sum_{i=0}^{c}\binom{n}{i}p^{i}(1-p)^{n-i}$$

Step 5: Find the Minimum Ratio when we fix the producer's risk α=0.5 by satisfying the inequality,

$$\sum_{i=0}^{c}\binom{n}{i}p^{i}(1-p)^{n-i} \geq 1-\alpha$$

Illustrative example

Here the data refers to the software releases in hours presented by Wood(1996)[19]. The data are 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, 5218 and 5823. We assume that the data set followed by the Generalized Exponential-Poisson distribution. Suppose the tester wants to utilize the proposed plan on the software releases and wish to establish the unknown 50th percentile to be at least 500 hours and life test to be end with 1000 hours which leads to the ratio $t/t^0_{0.50} = 2.0$. Hence with c =2 and p*=0.90, the tester may take the sample size from the Table (1) as 5. Thus the sampling plan according to the above data is

$$\left(n, c, t/t^0_{0.50}\right) = (5, 2, 2.0)$$

Since there is two items with a failure time less than or equal to 1000 hrs in the given sample of 10 observations, the lot is accepted as the result indicates that the 50th percentile life time $t_{0.5}$ is at least 500 hrs with a confidence level of p*=0.90. The probability of acceptance of the sampling plan (5, 2 , 2.0) under GEP for p*=0.90 is given in the Table (2).

Table 4: OC values of (5, 2 , 2.0) under GEP for p*=0.90.

t/t^0_q	1	1.5	2	2.5	3	3.5	4	4.5	5
L(p)	0.0739	0.2779	0.5000	0.6714	0.7872	0.8617	0.9089	0.9390	0.9585

This shows that if the true 50th percentile is equal to the required 50th percentile $(t/t^0_{0.50} = 1)$ the producer's risk is approximately equal to 0.9261 which is (1-0.0739).
The producer's risk is approximately equal to 0.05 when the true 50th percentile is greater than or equal to 5 times the specified 50th percentile.
From table(3), one can easily find out the values of $d_{0.50}$ for the different given values of c and $t/t^0_{0.50}$ in order to assert that the producer's risk with less than 0.05. For the sample of c=2 and $t/t^0_{0.50}$ =2.0 and p*=0.90. one can easily find the value of $d_{0.50}$= 4.7691. It means that the product will have a 50th percentile life of 4.7691 times the required 50th percentile lifetime in order that under the proposed acceptance sampling plan the product is accepted with the probability of at least 0.95. Thus the acceptance sampling plan based on Generalized Exponential-Poisson distribution could have a better chance to report less failure than the other methods based on the percentiles.

4. SUMMARY AND CONCLUSION

This article provides a new sampling distribution to test the quality of products when acceptance sampling for life test is studied when it follows the Generalized Exponential-Poisson distribution. Table are developed to obtain the minimum sample size, OC values and the minimum ratio of the producer's risk when the life data is assumed to follow this new compound distribution. This paper also dealt to study about the percentile life time which protects both the producer and the consumer with more precision than the specified average life. To ensure the life quality of the products exceeds a specified one in terms of the percentile life is studied. Suitable illustrations are given for ready to reference for the industries while taking a decision of the acceptance/rejection of the lot.

Acknowledgement

The authors express their gratitude for the unknown referees and thanks to the authorities of their parent university for providing necessary facilities in the Department of Statistics through DST-FIST and UGC-SAP Programmes.

REFERENCES

[1] Aslam .M, Mahmood.Y, Lio.YL, Tsai .TR and Khan.MA. Acceptance sampling plans from truncated life tests based on the Burr type XII percentiles. *Journal of the Chinese Institute of Industrial Engineers*, 27; (2010)270-280.

[2] Baklizi, A. and El Masri, A. E. K. Acceptance Sampling based on Truncated Life tests in Birnbaum–Saunders model. *Risk Analysis* 24; (2004)1453–1457.

[3] Epstein, B. Truncated Life Tests in the Exponential Case. *Annals of Mathematical Statistics*, 25; (1954)555-564.

[4] F.W. Fertig and N. R. Mann. Life-test sampling plans for two parameter Weibull populations. *Technometrics*, 22; (1980)165–177.

[5] Gupta, S.S. and Groll, P. A.. Gamma Distribution in Acceptance Sampling based on Life Tests. *Journal of the American Statistical Association*, 56; (1961) 942-970.

[6] Gupta SS. Life test sampling plans for normal and lognormal distribution. *Technometrics*. 4; (1962)151-175.

[7] Goode, H.P. and Kao, J.H.K. Sampling Plans based on the Weibull Distribution. *Proceedings of Seventh National Symposium on Reliability and Quality Control*, Philadelphia, (1961)24-40.

[8] Kantam RRL, Rosaiah K, Rao GS. Half logistic distribution in acceptance sampling based on life tests. *IAPQR Transaction*. 1998; 23: (1998)117-125.

[9] Kantam, R.R.L., Rosaiah, K. and Rao, G.S. Acceptance Sampling based on Life Tests: Log-Logistic Mode. *Journal of Applied Statistics*, 28; (2001) 121-128.

[10] Kaviyarasu V, Fawaz P. Certain Studies on Acceptance Sampling Plans for Percentiles Based on the Modified Weibull Distribution. *International Journal of Statistics and Systems*. 12(2); (2017)343-354.

[11] Kaviyarasu V, Fawaz P. A Reliability Sampling Plan to ensure Percentiles through Weibull Poisson Distribution. *International Journal of Pure and Applied Mathematics*. 117; (2017)155-163.

[12] Lio YL, Tsai TR, Wu SJ. Acceptance sampling plans from truncated life tests based on the Birnbaum-Saunders distribution for percentiles". *Communications in Statistics -Simulation and Computation*. 39(1): (2009)119-136.

[13] Murphy, K. E., Carter, C. M., and Brown, S. O. The exponential distribution: the good, the bad and the ugly: a practical guide to its implementation, *Proc. Reliability and Maintainability Symposium*, (2002)pp.550–555.

[14] Srinivasa Rao. G and R. R. L. Kantam. Acceptance sampling plans from truncated life tests based on log-logistic distribution for percentiles, *Economic Quality Control*. 25;2; (2010)153–167.

[15] Rao GS, Kantam RRL, Rosaiah K, Reddy JP. Acceptance sampling plans for percentiles based on the Inverse Rayleigh Distribution. *Electronic Journal of Applied Statistical Analysis*.5(2): (2012)164-177.

[16] Sobel, M. and Tischendrof, J. A. Acceptance sampling with sew life test objective. *Proceedings of Fifth National Symposium on Reliability and Quality Control*, Philadelphia, (1959)pp. 108-118.

[17] Tsai, T.-R. and Wu, S.-J. Acceptance sampling based on truncated life tests for generalized Rayleigh distribution. *Journal of Applied Statistics*, 33; (2006)595-600.

[18] W. Barreto-Souza and F. Cribari-Neto. *A generalization of the exponential-Poisson distribution*, Statist. Probab. Lett.79 , (2009) pp. 2493–2500.

[19] Wood.A. Predicting software reliability," *IEEE Transactions on Software Engineering*, 22; , (1996) 69–77.

[20] Wu.S.J and Tsai.T.R. Acceptance sampling plans for Birnbaum-saunders distribution under truncated life tests, *International Journal of Reliability, Quality and Safety Engineering*, vol. 12, no. 6, (2005)pp. 507–519**.**

TABLE 1: Minimum sample sizes for 50th percentile values for the GEP distribution.

p*	c	$t/t_q{}^0{}_{0.50}$									
		0.25	0.5	0.75	1	1.5	2	2.5	3	3.5	4
0.75	0	16	6	3	3	2	2	1	1	1	1
0.75	1	31	11	7	5	4	3	3	2	2	2
0.75	2	44	16	10	8	5	5	5	5	3	3
0.75	3	58	21	13	10	7	7	7	7	5	5
0.75	4	71	26	16	12	8	8	8	8	7	6
0.75	5	85	31	19	15	11	9	9	9	9	8
0.75	6	98	35	22	16	13	10	10	10	10	9
0.75	7	109	40	25	18	14	12	12	11	11	11
0.75	8	122	45	27	22	15	14	13	13	12	12
0.75	9	134	50	31	24	17	15	14	14	14	13
0.75	10	148	54	33	26	18	16	15	15	15	13
0.90	0	26	10	5	5	4	3	2	2	1	1
0.90	1	44	16	10	8	5	4	4	4	3	3
0.90	2	60	21	13	9	7	5	5	5	5	5
0.90	3	74	27	16	12	8	7	7	7	7	7
0.90	4	91	32	20	15	10	8	8	7	7	7
0.90	5	104	38	23	17	12	10	9	8	8	8
0.90	6	119	42	25	19	14	12	11	9	9	9
0.90	7	133	47	29	21	15	14	13	11	11	11
0.90	8	146	53	32	24	17	15	14	13	13	12
0.90	9	159	58	35	27	18	16	15	14	14	14
0.90	10	173	62	38	28	20	17	16	15	15	15
0.95	0	33	12	7	5	4	4	3	2	2	2
0.95	1	53	18	10	8	7	5	5	4	4	3
0.95	2	69	25	15	11	8	7	7	6	6	4
0.95	3	87	31	18	14	9	8	8	7	6	5
0.95	4	102	36	22	17	11	9	9	7	7	7
0.95	5	117	42	25	18	13	10	9	8	8	7
0.95	6	132	46	29	21	15	12	10	10	9	9
0.95	7	147	53	33	23	16	14	12	12	11	11
0.95	8	163	59	35	26	18	15	14	13	13	12
0.95	9	177	63	40	29	21	18	15	14	14	13
0.95	10	191	69	42	30	23	20	16	15	15	15
0.99	0	52	17	11	8	5	4	4	3	3	3
0.99	1	74	25	15	10	7	6	5	5	4	4
0.99	2	94	33	19	14	9	8	7	6	5	5
0.99	3	113	39	23	17	12	9	8	8	7	7
0.99	4	128	45	27	19	14	10	9	8	8	7
0.99	5	144	52	31	22	15	12	10	9	9	8
0.99	6	162	58	34	25	18	14	12	11	10	9
0.99	7	180	62	37	27	19	15	14	13	12	11
0.99	8	196	68	41	31	21	18	15	14	13	12
0.99	9	212	75	45	32	23	19	16	15	14	14
0.99	10	225	80	48	35	25	21	17	16	15	15

Table 2: OC values for (n, c =2, $t/tq^0_{0.50}$) for a given P* under GEP distribution.

p*	n	$t/t^0_{q0.50}$	$t_{0.50}/t^0_{q0.50}$								
			1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.75	44	0.25	0.2468	0.6839	0.8830	0.9542	0.9804	0.9908	0.9954	0.9975	0.9986
	16	0.50	0.2293	0.6257	0.8401	0.9300	0.9674	0.9838	0.9914	0.9952	0.9972
	10	0.75	0.2007	0.5625	0.7900	0.8991	0.9495	0.9734	0.9853	0.9915	0.9948
	8	1.00	0.1445	0.4634	0.7087	0.8459	0.9169	0.9536	0.9731	0.9838	0.9899
	5	1.50	0.2019	0.5000	0.7156	0.8405	0.9089	0.9465	0.9675	0.9797	0.9869
	5	2.00	0.0739	0.2789	0.5011	0.6724	0.7880	0.8622	0.9093	0.9393	0.9587
0.90	60	0.25	0.0942	0.4917	0.7771	0.9040	0.9564	0.9789	0.9891	0.9940	0.9966
	21	0.50	0.0934	0.4411	0.7221	0.8668	0.9343	0.9660	0.9815	0.9895	0.9937
	13	0.75	0.0760	0.3706	0.6486	0.8143	0.9010	0.9456	0.9689	0.9816	0.9887
	9	1.00	0.0898	0.3724	0.6348	0.7978	0.8876	0.9359	0.9622	0.9770	0.9856
	7	1.50	0.0422	0.2266	0.4629	0.6533	0.7813	0.8619	0.9116	0.9424	0.9617
	5	2.00	0.0739	0.2779	0.5000	0.6714	0.7872	0.8617	0.9089	0.9390	0.9585
0.95	69	0.25	0.0523	0.3966	0.7113	0.8690	0.9387	0.9697	0.9841	0.9912	0.9949
	25	0.50	0.0428	0.3196	0.6231	0.8065	0.9002	0.9469	0.9705	0.9829	0.9898
	15	0.75	0.0378	0.2707	0.5550	0.7503	0.8615	0.9217	0.9544	0.9726	0.9830
	11	1.00	0.0327	0.2300	0.4936	0.6946	0.8198	0.8928	0.9350	0.9595	0.9741
	8	1.50	0.0180	0.1445	0.3573	0.5587	0.7087	0.8096	0.8750	0.9169	0.9439
	7	2.00	0.0071	0.0754	0.2266	0.4060	0.5662	0.6907	0.7813	0.8452	0.8897
0.99	94	0.25	0.0091	0.2013	0.5284	0.7556	0.8755	0.9350	0.9646	0.9799	0.9882
	33	0.50	0.0080	0.1546	0.4385	0.6731	0.8168	0.8966	0.9403	0.9644	0.9782
	19	0.75	0.0086	0.1354	0.3881	0.6171	0.7708	0.8634	0.9174	0.9489	0.9676
	14	1.00	0.0065	0.1026	0.3177	0.5391	0.7048	0.8137	0.8819	0.9240	0.9502
	9	1.50	0.0075	0.0898	0.2702	0.4701	0.6348	0.7535	0.8341	0.8876	0.9229
	8	2.00	0.0071	0.0754	0.2266	0.4060	0.5662	0.6907	0.7813	0.8452	0.8897

Table 3: Minimum ratio for $d_{0.50}$ for the GEP Distribution with producer's risk of 0.05.

p*	c	$t/t^0_{q0.50}$									
		0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00	3.50	4.00
0.75	0	6.2706	7.5063	7.7860	10.2953	12.4705	16.6126	14.0481	16.7431	19.6835	22.4134
0.75	1	3.1955	3.5642	4.0832	4.3521	5.6302	6.0338	7.5473	6.3027	7.3522	8.4030
0.75	2	2.4490	2.7130	3.0207	3.4501	3.5779	4.7730	5.9669	7.1002	4.8328	5.5232
0.75	3	2.1491	2.3471	2.5620	2.8120	3.1292	4.1723	5.2154	6.2372	5.1450	5.8800
0.75	4	1.9607	2.1350	2.3005	2.4570	2.5636	3.4182	4.2727	5.1256	5.1450	4.8613
0.75	5	1.8536	2.0031	2.1300	2.3637	2.6977	2.9301	3.6627	4.3950	5.1168	5.0592
0.75	6	1.7635	1.8652	2.0157	2.0764	2.5769	2.5824	3.2280	3.8735	4.5053	4.5027
0.75	7	1.6792	1.7977	1.9264	1.9637	2.3204	2.5985	3.2482	3.4933	4.0758	4.6472
0.75	8	1.6272	1.7442	1.8049	2.0335	2.1113	2.5985	2.9708	3.5609	3.7334	4.2664
0.75	9	1.5802	1.6980	1.8049	1.9465	2.0902	2.4042	2.7472	3.2966	3.8383	3.9516
0.90	0	8.0548	9.8189	10.2105	13.6538	18.0990	20.6523	20.6831	24.7910	19.6133	22.3780
0.90	1	3.8681	4.4354	5.0621	5.8797	6.5589	7.4911	9.3799	11.2548	10.5766	12.0938
0.90	2	2.9077	3.2115	3.5764	3.7466	4.6770	4.7691	5.9628	7.1553	8.3548	9.5491
0.90	3	2.4644	2.7499	2.9489	3.2188	3.5145	4.1722	5.2153	6.2584	7.3017	8.3448
0.90	4	2.2573	2.4321	2.6885	2.9195	3.1686	3.4182	4.2727	4.4333	5.1722	5.9111
0.90	5	2.0774	2.2847	2.4462	2.6133	2.9281	3.2756	3.6627	3.8129	4.4484	5.0839
0.90	6	1.9724	2.1038	2.2131	2.3956	2.7675	3.1608	3.6072	3.3640	3.9247	4.4854
0.90	7	1.8835	2.0030	2.1467	2.2353	2.4871	3.0808	3.5711	3.4833	4.0639	4.6444
0.90	8	1.8075	1.9456	2.0562	2.1864	2.4064	2.8155	3.2531	3.5600	4.1533	4.2663
0.90	9	1.7466	1.8801	1.9786	2.1485	2.2197	2.5988	3.0054	3.2966	3.8460	4.3866
0.95	0	9.1315	10.7879	12.1356	13.6647	18.1450	24.2372	25.7665	24.7910	28.8955	32.9403
0.95	1	4.2833	4.7524	5.0426	5.8796	8.1593	8.7428	10.8802	11.2548	13.1296	12.0713
0.95	2	3.1394	3.5482	3.9090	4.2875	5.1762	6.2362	7.7977	8.3291	9.7173	7.7365
0.95	3	2.7002	2.9847	3.1926	3.5861	3.8721	4.6860	5.8576	6.2532	6.3065	5.8810
0.95	4	2.4032	2.6222	2.8754	3.1916	3.4236	3.8367	4.7958	4.4336	5.1743	5.8810
0.95	5	2.2198	2.4273	2.5920	2.7350	3.1443	3.2831	3.6502	3.8130	4.4486	4.2061
0.95	6	2.0885	2.2305	2.4628	2.5932	2.9502	3.1594	3.2197	3.8722	3.9246	4.4905
0.95	7	1.9943	2.1620	2.3621	2.4058	2.6458	3.0810	3.2519	3.8978	4.0639	4.6453
0.95	8	1.9245	2.0867	2.1950	2.2630	2.5453	2.8155	3.2519	3.5650	4.1533	4.2663
0.95	9	1.8581	1.9880	2.1818	2.2786	2.5839	2.9555	3.0054	3.2967	3.8460	3.9516
0.99	0	11.4790	12.8748	15.5078	17.4854	20.3990	24.0537	30.1638	30.8739	36.2488	41.4014
0.99	1	5.0964	5.6997	6.3854	6.7233	8.1132	9.8092	10.9315	13.0562	13.1075	14.9805
0.99	2	3.7272	4.1860	4.5285	4.9927	5.5981	6.8700	7.7951	8.3226	8.3458	9.5381
0.99	3	3.1128	3.4298	3.7320	4.0965	4.8264	5.1625	5.8575	7.0568	7.3014	8.3445
0.99	4	2.7277	3.0032	3.2812	3.4602	4.1631	4.2237	4.7958	5.1362	5.9818	5.9111
0.99	5	2.4928	2.7690	2.9975	3.1608	3.5454	3.9041	4.1039	4.3963	5.1277	5.0839
0.99	6	2.3432	2.5770	2.7450	2.9506	3.4296	3.6901	3.9492	4.3286	4.5191	4.4854
0.99	7	2.2362	2.3885	2.5571	2.7223	3.0823	3.3162	3.8512	4.2853	4.5474	4.6444
0.99	8	2.1386	2.2857	2.4556	2.6737	2.9341	3.3891	3.5194	3.9037	4.1591	4.2663
0.99	9	2.0596	2.2198	2.3731	2.4653	2.8117	3.1286	3.2485	3.6065	3.8461	4.3866