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Steady State Analysis of M/M/1 Retrial Queueing Network with Catastrophes

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Abstract *- In this paper we analyze the M/M/1 retrial queueing network with catastrophes. Here we derive the steady state probabilities, the stationary probabilities of the giving network. The expectation number of customers in the nodes 1, 2 and 3 are derived. The particular cases are also derived. The numerical examples are given to test the correctness of the result.*

Keywords - *Average queue length, Catastrophes, Steady state probability, Steady state solution*

I. INTRODUCTION

Queueing theory is a branch of applied probability theory. Queue is a waiting line of customers which demands service from a server. In the year of 1909, queueing theory was introduced by *Agner Krarup Erlang* [1].The concepts of queueing theory have been applied invarious fields like phone systems, communication network systems, computer systems, industrial sectors and so on. The techniques adopted in the queueing models produce remarkable solutions.

Queueing network can be described as a group of nodes, where each node represents a service facility. The Queueing networks were first introduced by *James. R. Jackson* [6] in 1957. The most significant contribution in queueing network is *Jackson's* network[7]. Queueing network models have various applications in many areas, such as service centers, computer networks, communication networks, production and flexible manufacturing systems, airport terminals and healthcare systems etc. Queueing networks can be classified as open, closed and mixed networks. Open network receive external arrivals and send them to an external departure. Closed networks have a constant number of customers that moves between the queues but never leaves the system. Mixed networks open for some types of customers and closed for other types.

Retrial queueing models are characterized by the fact that, when the server is busy, the arriving customers do not line up or leave the system immediately forever and the customers go to some virtual place which is called as orbit and try their luck after some random time. This type of queueing problems arise in many communication protocols, local area networks and daily life situations. The concept of retrial queues was first introduced by *Kosten* [9] in 1947.*Cohen* [8] analyzed the basic problems of telephone traffic theory through the influence of repeated calls. A two class retrial system with a single server no waiting room, batch arrivals and classical retrial scheme was introduced and analyzed by *Kulkarni* [15]. *Falin* [5] has analyzed an M/M/1 retrail queue with loss but without feedback.

Catastrophes means sudden calamity occur in the service facility. *Gelenbe* [4] introduced the notion of catastrophes and it has been gaining significant scholarly attention during the last few decades since their applications are widely used in service systems, computer systems, manufacturing systems. The concept of catastrophe has played a vital role in the areas of Science and Technology. It occurs at random leading to extinction of all the units and activate the service facility until a new arrival is not unusual in most of the real life situations. When catastrophe occurs in the system, all the available customers are destroyed immediately and the server gets inactivated. The server is ready for service when a new arrival happens. Moreover, with the occurrence of catastrophic events like power failure, virus attacks, corruption of hard discs of computer systems etc. For example if a job is infected with a virus arrives, it transmit the virus to other processor and inactivate them in computer networks with a virus by queueing networks with catastrophes which has been studied by *Chao* [18]. *Jain and Kumar* [10] introduced the concept of restoration in catastrophic queues.

Krishnakumar and Arivudainambi [3] have presented a transient solution of an M/M/1 queue with catastrophes. *Chandrasekaran et.al* [16] have analyzed the M/M/1 queue with catastrophes, failures and repairs. *Thangaraj et.al* [17] have focused the M/M/1 queue with catastrophes using continued fractions. *Krishnakumar et.al* [2] have presented a transient analysis of a single server queue with catastrophes,failures and repairs. *Sophia* [14] has analyzed the transient analysis of a single server queue with chain sequence rates subject to catastrophes, failures and repairs. *Rakesh kumar et.al* [11] have presented a Multi- server Markovian Feedback Queue with Balking Reneging and Retention of Reneged customers. *Shanmugasundaram and Chitra* [12] have analyzed the time dependent solution of M/M/1 retrial queue and feedback on non retrial customers with catastrophes. *Shanmugasundaram and Vanitha* [13] have focused a M/M/1 queueing network with classical retrial policy.
In this paper we consider a M/M/1 retrial queue with catastrophes.

II DESCRIPTION OF THE MODEL

We consider an open queueing network consisting of three single nodes with retrial and catastrophe. External customers arrive to the system according to a Poisson process with rate λ. If, the server is idle, the external arrival of customers enter into the queue with probability s. If the server is busy, the customers join the retrial queue with probability $1-s$ .It is assumed that only the customer at the head of the queue is allowed to get service in FIFO basis and the capacity is infinite. If the server is busy upon retrial, the customer joins the orbit again. Such a process is repeated until the customer finds the server in node 1 is idle and gets requested and required service at the time of a retrial. If the server is occupied, customers are forced to wait in the orbit of infinite size and repeat their demand after an exponential time. After getting the service in the node 1 they can either go to node 2 with probability p or they go to node 3 with probability $1-p$. After getting the service at node 2 the customers can either leave the system with probability q or they go to node 3 with probability $1-q$. After completing the service at node 3 the customer leave the system. Each node follows an M/M/1 schedule. The service rates for node 1, node 2 and node 3 are exponentially distributed with service rates μ_1, μ_2, and μ_3 respectively. Catastrophes occur from the arrival and service process which follow poisson process with rate γ . All the available customers are destroyed immediately when the catastrophes occurred in the system, the server gets inactivated. The server is ready for service when a new arrival happens. Fig.1 represents the system.

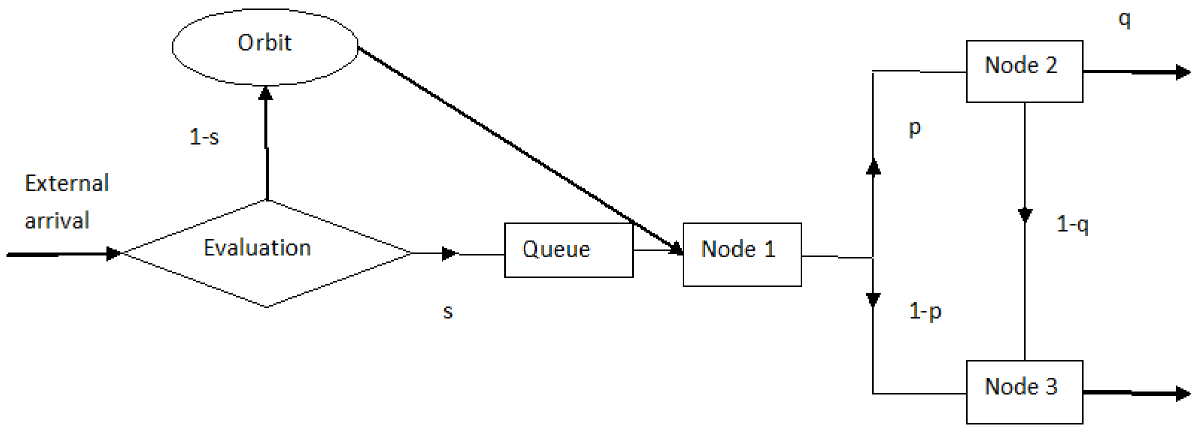

Fig.1 Classical Retrial Queueing System

Let $Q_n(t) = Q(Z(t) = n), n = 0,1,2....$ denote the transient state probabilities of n customers in the system at a time t.

Let $Q(z,t) = \sum_{n=0}^{\infty} Q(t) z^n$ be the probability generating function. Generally it is assumed that there are no customers in the system at a time t = 0.

i.e. $Q_0(0) = 1$ (1)

The system of differential-difference equations for the probability $Q_n(t)$ is

$$\frac{d}{dt}Q_0(t) = -\lambda Q_0(t) + [[(\mu_1(s+p) + \mu_2(s+q)) + (\mu_1(s+p) + \mu_2(s+1-q) + \mu_3 s) + (\mu_1(s+1-p) + \mu_3 s)] + [(\mu_1(1-s+p) + \mu_2(1-s+q)$$
$$+(\mu_1(1-s+p) + \mu_2(1-s+1-q) + \mu_3(1-s)) + (\mu_1(1-s+1-p) + \mu_3(1-s)]]Q_1(t) + \gamma(1-Q_0(t))$$

$$= -\lambda Q_0(t) + \left[\left[\mu_1(1+3s+p) + \mu_2(2s+1) + 2\mu_3 s\right] + \left[\mu_1(4-3s+p) + \mu_2(3-2s) + 2\mu_3(1-s)\right]\right]Q_1(t) + \gamma(1-Q_0(t))$$
$$= -\lambda Q_0(t) + (\mu_1(5+2p) + 4\mu_2 + 2\mu_3)Q_1(t) + \gamma(1-Q_0(t)) \quad (2)$$

For n=1, 2, 3.....

$$\frac{d}{dt}Q_n(t) = \lambda Q_{n-1}(t) - (\lambda + \gamma + \mu_1(5+2p) + 4\mu_2 + 2\mu_3)Q_n(t) + (\mu_1(5+2p) + 4\mu_2 + 2\mu_3)Q_{n+1}(t) \quad (3)$$

In steady state $\lim_{t\to\infty} Q_n(t) = Q_n$ and therefore $\frac{d}{dt}Q_n(t) = 0$ as $t\to\infty$ and hence the steady state equations corresponding to equations (2) and (3) are as follows,

$$0 = -\lambda Q_0 + (\mu_1(5+2p)+4\mu_2+2\mu_3)Q_1 + \gamma(1-Q_0)$$

$$Q_0(\lambda+\gamma) = \gamma + (\mu_1(5+2p)+4\mu_2+2\mu_3)Q_1 \qquad (4)$$

$$0 = \lambda Q_{n-1} - (\lambda+\gamma+\mu_1(5+2p)+4\mu_2+2\mu_3)Q_n + (\mu_1(5+2p)+4\mu_2+2\mu_3)Q_{n+1}$$

$$(\lambda+\gamma+\mu_1(5+2p)+4\mu_2+2\mu_3)Q_n = \lambda Q_{n-1} + (\mu_1(5+2p)+4\mu_2+2\mu_3)Q_{n+1} \qquad (5)$$

From equations (4) and (5) we get,

$$Q_0 = \frac{\gamma}{(\lambda+\gamma) - (\mu_1(5+2p)+4\mu_2+2\mu_3)\frac{Q_1}{Q_0}} \qquad (6)$$

$$\frac{Q_n}{Q_{n-1}} = \frac{\lambda}{(\lambda+\gamma+\mu_1(5+2p)+4\mu_2+2\mu_3) - (\mu_1(5+2p)+4\mu_2+2\mu_3)\frac{Q_{n+1}}{Q_n}} \qquad (7)$$

Substituting the equation (7) in (6) and using continued fraction,

$$Q_0 = \frac{\gamma}{(\lambda+\gamma) - \cfrac{\lambda\beta}{(\lambda+\beta+\gamma) - \cfrac{\beta Q_2}{Q_1}}}$$

Where $\mu_1(5+2p)+4\mu_2+2\mu_3 = \beta$

$$Q_0 = \frac{\gamma}{(\lambda+\gamma) - \cfrac{\lambda\beta}{(\lambda+\beta+\gamma) - \cfrac{\lambda\beta}{(\lambda+\beta+\gamma) - \ldots\ldots}}} \qquad (8)$$

$$= \frac{\gamma}{(\lambda+\gamma) - \psi} \qquad (9)$$

where, , $\psi = \cfrac{\lambda\beta}{(\lambda+\beta+\gamma) - \cfrac{\lambda\beta}{(\lambda+\beta+\gamma) - \ldots\ldots}}$ (10)

$$\psi^2 - (\lambda+\beta+\gamma)\psi + \lambda\beta = 0$$

The roots are $\frac{\alpha \pm \sqrt{\alpha^2 - 4\lambda\beta}}{2}$ *where* $\alpha = \lambda+\gamma+\beta,$ Let the two roots be ψ_1, ψ_2 where the second root ψ_2 is a unique real root lies within [0,1).

Substituting ψ_2 in equation (9),

$$Q_0 = \frac{\gamma}{(\lambda+\gamma) - \left(\frac{\alpha - \sqrt{\alpha^2 - 4\lambda\beta}}{2}\right)} \qquad (11)$$

$$= \frac{\gamma}{(\alpha-\beta)-\left(\frac{\alpha-\sqrt{\alpha^2-4\lambda\beta}}{2}\right)}$$

$$= \frac{\gamma\left(\frac{\alpha-\sqrt{\alpha^2-4\lambda\beta}}{2\lambda\beta}\right)}{1-\beta\left(\frac{\alpha-\sqrt{\alpha^2-4\lambda\beta}}{2\lambda\beta}\right)} \tag{12}$$

Expanding binomially,

$$Q_0 = \sum_{n=0}^{\infty}\beta^n\left(\frac{\alpha-\sqrt{\alpha^2-4\lambda\beta}}{2\lambda\beta}\right)^{n+1} + \gamma\sum_{n=0}^{\infty}\beta^n\left(\frac{\alpha-\sqrt{\alpha^2-4\lambda\beta}}{2\lambda\beta}\right)^{n+1} \tag{13}$$

Now we obtain the remaining steady state probabilities in terms of Q_0 from the equation (7) we get,

$$\frac{Q_n}{Q_{n-1}} = \frac{\lambda}{(\lambda+\gamma+\beta)-\frac{\lambda\beta}{(\lambda+\gamma+\beta)-\frac{\lambda\beta}{(\lambda+\gamma+\beta)-.....}}}$$

By similar arguments, as before, the above equation is written as,

$$\frac{Q_n}{Q_{n-1}} = \frac{\lambda}{\frac{\alpha+\sqrt{\alpha^2-4\lambda\beta}}{2}} \quad for \;\; n=1,2....... \tag{14}$$

$$Q_n = \left(\frac{\alpha-\sqrt{\alpha^2-4\lambda\beta}}{2\beta}\right)^n Q_0 \;\; for \;\; n=1,2....... \tag{15}$$

Where $\alpha=\lambda+\beta+\gamma$

Hence from the equations (13), (15) gives steady state probabilities Q_0 & Q_n for n=1, 2….

Theorem 1:

For $\gamma>0$ the steady state distribution of the n number of customers in the system is $\{\pi_n; n\geq 0\}$ of the M/M/1 queue with catastrophe corresponds to

$$\pi_0 = (1-\rho) \tag{16}$$

$$\pi_n = (1-\rho)\rho^n \tag{17}$$

where, $\rho = \frac{(\lambda+\beta+\gamma)-\sqrt{\lambda^2+\beta^2+\gamma^2+2\lambda\gamma+2\beta\gamma-2\lambda\beta}}{2\beta}$ (18)

Proof:

From (11),

$$Q_0 = \frac{\gamma}{(\lambda+\gamma)-\left(\frac{\alpha-\sqrt{\alpha^2-4\lambda\beta}}{2}\right)} \quad where \;\; \alpha=\lambda+\beta+\gamma$$

$$\pi_0 = Q_0 = \frac{2\gamma}{2(\lambda+\gamma)-(\lambda+\beta+\gamma)+\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}}$$

$$= \frac{2\gamma}{(\lambda-\beta+\gamma)+\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}}$$

$$= 1-\left[\frac{(\lambda+\beta+\gamma)-\sqrt{\lambda^2+\beta^2+\gamma^2+2\lambda\gamma+2\beta\gamma-2\lambda\beta}}{2\beta}\right]$$

$$= 1-\rho$$

From equation (15),

$$\pi_n = Q_n = \left(\frac{\alpha-\sqrt{\alpha^2-4\lambda\beta}}{2\beta}\right)^n \pi_0$$

$$= \pi_0\left(\frac{(\lambda+\beta+\gamma)-\sqrt{\lambda^2+\beta^2+\gamma^2+2\lambda\gamma+2\beta\gamma-2\lambda\beta}}{2\beta}\right)^n$$

$$= (1-\rho)\rho^n, n = 1,2.....$$

Theorem 2:

If $\gamma > 0$, the asymptotic behavior of the average queue length L_q at steady state is

$$L_q = \frac{\lambda-\beta}{\gamma}+\frac{2\beta}{2(\lambda+\gamma)-\left[(\lambda+\beta+\gamma)-\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}\right]}$$

III. BALANCE EQUATIONS

The average queue length of the system is,

$$L_q = \frac{\lambda-\beta}{\gamma}+\frac{2\beta}{2(\lambda+\gamma)-\left[(\lambda+\beta+\gamma)-\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}\right]} \quad (19)$$

$$\textit{where } \beta = \mu_1(5+2p)+4\mu_2+2\mu_3$$

The waiting time of a customer in all the three queues is,

$$W_q = \frac{L_q}{\lambda} \quad (\textit{By Little's formula})$$

$$= \frac{1}{\lambda}\left[\frac{\lambda-\beta}{\gamma}+\frac{2\beta}{2(\lambda+\gamma)-\left[(\lambda+\beta+\gamma)-\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}\right]}\right] \quad (20)$$

The average number of customers in the system is,

$$L_s = L_q+\frac{\lambda}{\mu} \quad (\textit{By Little's formula})$$

$$= \frac{\lambda-\beta}{\gamma}+\frac{2\beta}{2(\lambda+\gamma)-\left[(\lambda+\beta+\gamma)-\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}\right]}+\frac{\lambda}{\mu} \quad (21)$$

The waiting time of a customer in the system is,

$$W_s = \frac{L_s}{\lambda} \quad (By\ Little\text{'s}\ formula)$$

$$= \frac{1}{\lambda}\left[\frac{\lambda-\beta}{\gamma} + \frac{2\beta}{2(\lambda+\gamma)-\left[(\lambda+\beta+\gamma)-\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}\right]} + \frac{\lambda}{\mu}\right] \tag{22}$$

The average queue length for node 1 is,

$$L_{q_1} = \frac{\lambda-\beta}{\gamma} + \frac{2\beta}{2(\lambda+\gamma)-\left[(\lambda+\beta+\gamma)-\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}\right]} \tag{23}$$

$$where\ \beta = \mu_1(2p+1)$$

The average queue length for node 2 is,

$$L_{q_2} = \frac{\lambda-\beta}{\gamma} + \frac{2\beta}{2(\lambda+\gamma)-\left[(\lambda+\beta+\gamma)-\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}\right]} \tag{24}$$

$$where\ \beta = \mu_1(1-p)+\mu_2(1-q)$$

The average queue length for node 3 is,

$$L_{q_3} = \frac{\lambda-\beta}{\gamma} + \frac{2\beta}{2(\lambda+\gamma)-\left[(\lambda+\beta+\gamma)-\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}\right]} \tag{25}$$

$$where\ \beta = \mu_1(3-2p)+\mu_2(3-2q)+\mu_3$$

The remaining parameters $W_{q_1}, L_{s_1}, W_{s_1}, W_{q_2}, L_{s_2}, W_{s_2}, W_{q_3}, L_{s_3}, W_{s_3}$ are calculated using Little's formula.

IV. PARTICULAR CASE

When $\mu_1 = \mu \;\&\; \mu_2 = \mu_3 = 0$ *then* $p = 0$ i.e. there is only one node and also when $s = 0$ i.e. when there is no retrial then we get $\beta = \mu$.

The asymptotic behavior of the average queue length L_q when $\gamma > 0$ is

$$L_q = \frac{\lambda-\mu}{\gamma} + \frac{2\mu}{2(\lambda+\gamma)-\left[(\lambda+\mu+\gamma)-\sqrt{(\lambda+\mu+\gamma)^2-4\lambda\mu}\right]} \tag{26}$$

A. Case 1

The equation (26) coincides exactly with *Krishnakumar and Arivudainambi* [3].

i.e.,If $\gamma > 0$,the asymptotic behavior of the mean system size $m(t)$ is given by,

$$m(t) = \frac{\lambda-\mu}{\gamma} + \frac{2\mu}{2(\lambda+\gamma)-\left[(\lambda+\mu+\gamma)-\sqrt{(\lambda+\mu+\gamma)^2-\alpha^2}\right]} \qquad as\ t\to\infty$$

Where, $\alpha = 2\sqrt{\lambda\mu}$

B. Case 2

The equation (26) coincides exactly with *Thangaraj and Vanitha* [17] by taking q=1 in that paper. i.e., The asymptotic behavior of the mean system size $m(t)$ when $\gamma > 0$ is

$$m(t) = \frac{\lambda-\mu q}{\gamma} + \frac{2\mu q}{2(\lambda+\gamma)-\left[(\lambda+\mu q+\gamma)-\sqrt{(\lambda+\mu q+\gamma)^2-4\lambda\mu q}\right]} \qquad as\ t\to\infty$$

C. Case 3

The equation (26) coincides exactly with *Shanmugasundaram and Chitra* [12] by taking β=0 & q=1 in that paper. i.e., The asymptotic behavior of the average queue length $h(t)$ when $\nu > 0$ is

$$h(t)=\frac{\lambda-\omega}{\nu}+\frac{2\omega}{2(\lambda+\nu)-\left[(\lambda+\omega+\nu)-\sqrt{(\lambda+\omega+\nu)^{2}-4\lambda\omega}\right]} \qquad as\ t\to\infty$$

Where $\omega=(\beta+q)\mu_1+(p+\alpha)\mu_2$

V. NUMERICAL EXAMPLE

A. Calculation for Node 1

For $p=0.5, \mu_1=5, \mu_2=8, \mu_3=13, \lambda=1,2,3......10, \gamma=1,3,5,7,10$

Calculation of the average queue length for the node 1: For the arrival rate λ from 1 to 10 and for γ (the catastrophe effect) from 1 to 10, the average queue length of node 1 is calculated in Table I. From Fig.2 it is clear that as the arrival rate increases, the average queue length of node 1 increases and as the γ value increases, the average queue length of node 1 decreases.

TABLE I

λ/γ	1	3	5	7	10
1	0.0990	0.0817	0.0697	0.0609	0.0512
2	0.2170	0.1736	0.1457	0.1259	0.1050
3	0.3589	0.2770	0.2283	0.1952	0.1612
4	0.5311	0.3930	0.3177	0.2688	0.2198
5	0.7417	0.5226	0.4142	0.3466	0.2808
6	1.0000	0.6667	0.5177	0.4286	0.3440
7	1.3166	0.8257	0.6283	0.5146	0.4095
8	1.7016	1.0000	0.7457	0.6046	0.4770
9	2.1623	1.1893	0.8697	0.6983	0.5466
10	2.7016	1.3930	1.0000	0.7956	0.6180

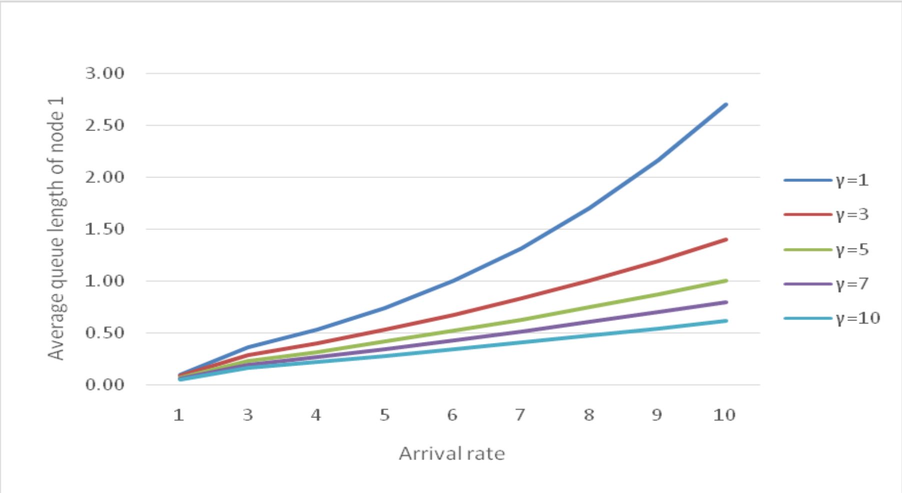

Fig.2 Average Queue Length of Node 1

Calculation of the waiting time of a customer in the queue for the node 1: For the arrival rate λ from 1 to 10 and for γ (the catastrophe effect) from 1 to 10, the waiting time of a customer in the queue of node 1 is calculated in Table II. From Fig.3 it is clear that as the arrival rate increases, the waiting time of a customer in the queue of node 1 increases and as the γ value increases, the waiting time of a customer in the queue of node 1 decreases.

TABLE II

λ/γ	1	3	5	7	10
1	0.0990	0.0817	0.0697	0.0609	0.0512
2	0.1085	0.0868	0.0728	0.0630	0.0525
3	0.1196	0.0923	0.0761	0.0651	0.0537
4	0.1328	0.0982	0.0794	0.0672	0.0550
5	0.1483	0.1045	0.0828	0.0693	0.0562

6	0.1667	0.1111	0.0863	0.0714	0.0573
7	0.1881	0.1180	0.0898	0.0735	0.0585
8	0.2127	0.1250	0.0932	0.0756	0.0596
9	0.2403	0.1321	0.0966	0.0776	0.0607
10	0.2702	0.1393	0.1000	0.0796	0.0618

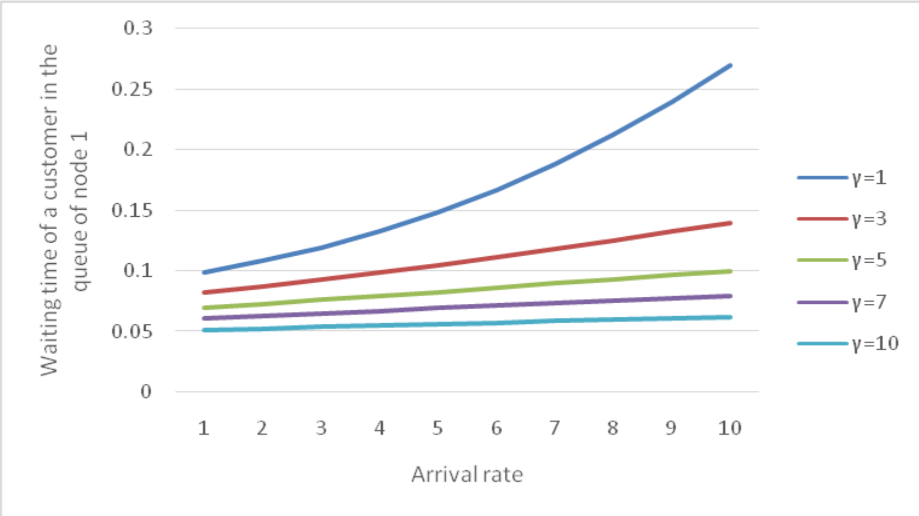

Fig. 3 Waiting Time of a Customer in the Queue of Node 1

For $p = 0.5, \mu_1 = 5, 5.2, 5.4, ...6.8, \mu_2 = 8, \mu_3 = 13, \lambda = 5, \gamma = 1, 3, 5, 7, 10$

Calculation of the number of customers in the node 1: For the service rate from 5 to 6.8 and for γ (the catastrophe effect) from 1 to 10, the number of customers in the node 1 is calculated in Table III. From Fig. 4 it is clear that as the service rate increases and for various values of γ, the number of customers in the node 1decreases.

TABLE III

μ_1/γ	1	3	5	7	10
5	1.7417	1.5226	1.4142	1.3466	1.2808
5.2	1.6654	1.4659	1.3643	1.3001	1.2370
5.4	1.5953	1.4132	1.3178	1.2567	1.1962
5.6	1.5308	1.3640	1.2743	1.2162	1.1581
5.8	1.4712	1.3180	1.2336	1.1783	1.1224
6	1.4159	1.2749	1.1954	1.1426	1.0890
6.2	1.3646	1.2344	1.1594	1.1091	1.0576
6.4	1.3168	1.1963	1.1256	1.0776	1.0280
6.6	1.2723	1.1605	1.0936	1.0477	1.0000
6.8	1.2306	1.1267	1.0634	1.0195	0.9736

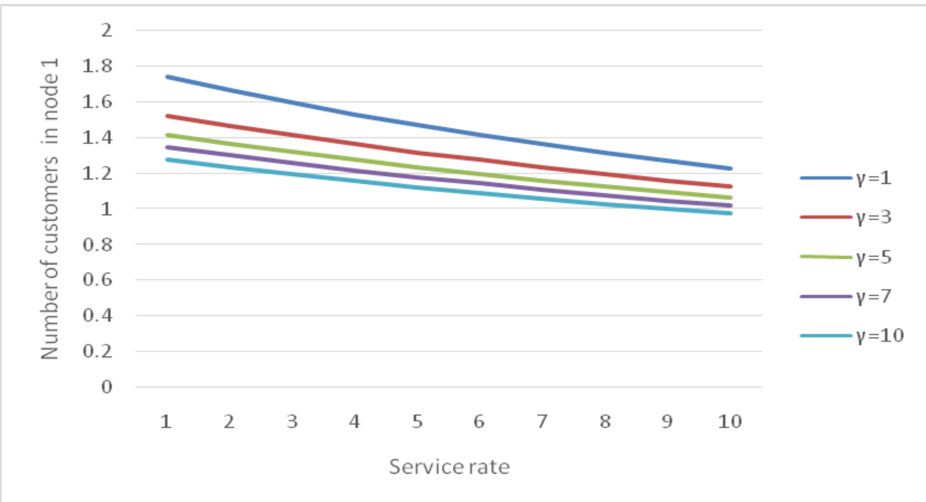

Fig. 4 Number of Customers in Node 1

Calculation of the waiting time of a customer in the node 1: For the service rate from 5 to 6.8 and for γ (the catastrophe effect) from 1 to 10, the waiting time of a customer in the node 1 is calculated in Table IV. From Fig. 5 it is clear that as the service rate increases and for various values of γ, the waiting time of a customer in the node 1 decreases.

TABLE IV

μ_1/γ	1	3	5	7	10
5	0.3483	0.3045	0.2828	0.2693	0.2562
5.2	0.3331	0.2932	0.2729	0.2600	0.2474
5.4	0.3191	0.2826	0.2636	0.2513	0.2392
5.6	0.3062	0.2728	0.2549	0.2432	0.2316
5.8	0.2942	0.2636	0.2467	0.2357	0.2245
6	0.2832	0.2550	0.2391	0.2285	0.2178
6.2	0.2729	0.2469	0.2319	0.2218	0.2115
6.4	0.2634	0.2393	0.2251	0.2155	0.2056
6.6	0.2545	0.2321	0.2187	0.2095	0.2000
6.8	0.2461	0.2253	0.2127	0.2039	0.1947

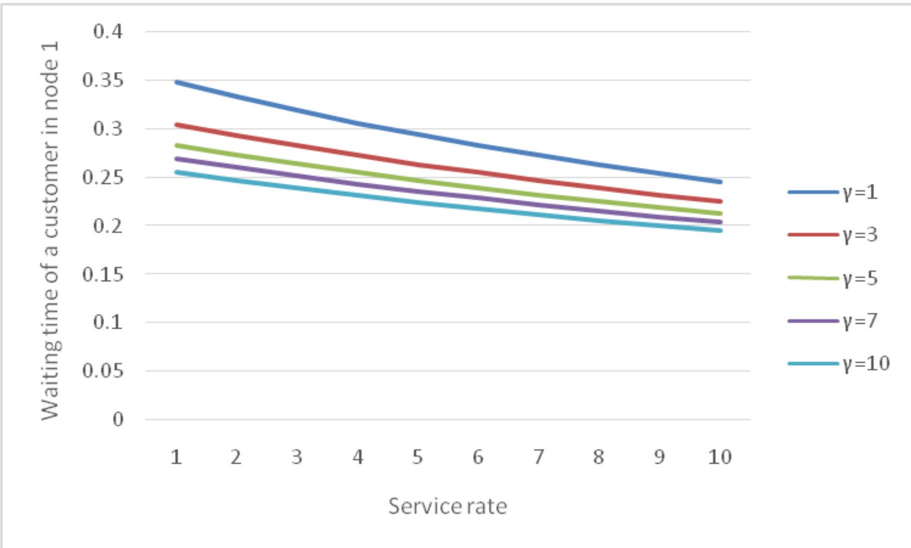

Fig. 5 Waiting Time of a Customer in Node 1

B. Calculation for the system

For $p = 0.5, \mu_1 = 5, \mu_2 = 8, \mu_3 = 13, \lambda = 1, 2, 3......10, \gamma = 1, 3, 5, 7, 10$

Calculation of the number of customers in all the three queues for the system: For the arrival rate λ from 1 to 10 and for γ (the catastrophe effect) from 1 to 10, the number of customers in all the three queues is calculated in Table V. From Fig.6 it is clear that as the arrival rate increases, the number of customers in all the three queues increases and as the γ value increases, the number of customers in all the three queues decreases.

TABLE V

λ/γ	1	3	5	7	10
1	0.0114	0.0111	0.0109	0.0106	0.0103
2	0.0230	0.0225	0.0220	0.0215	0.0208
3	0.0349	0.0341	0.0333	0.0325	0.0315
4	0.0470	0.0459	0.0448	0.0438	0.0424
5	0.0595	0.0580	0.0566	0.0553	0.0535
6	0.0722	0.0704	0.0687	0.0671	0.0648
7	0.0853	0.0831	0.0810	0.0790	0.0763
8	0.0986	0.0961	0.0936	0.0913	0.0880
9	0.1123	0.1093	0.1065	0.1038	0.1000
10	0.1264	0.1229	0.1196	0.1165	0.1122

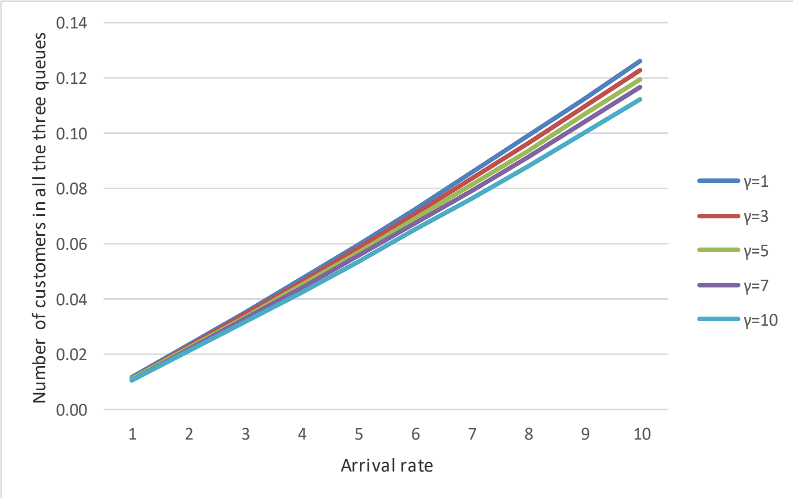

Fig. 6 Number of Customers in all the Three Queues

***Calculation of the waiting time of a customer in all the three queues for the system*:** For the arrival rate λ from 1 to 10 and for γ (the catastrophe effect) from 1 to 10, the waiting time of a customer in all the three queues is calculated in Table VI. From Fig.7 it is clear that as the arrival rate increases, the waiting time of a customer in all the three queues increases and as the γ value increases, the waiting time of a customer in all the three queues decreases.

TABLE VI

λ/γ	1	3	5	7	10
1	0.0114	0.0111	0.0109	0.0106	0.0103
2	0.0115	0.0112	0.0110	0.0107	0.0104
3	0.0116	0.0114	0.0111	0.0108	0.0105
4	0.0118	0.0115	0.0112	0.0110	0.0106
5	0.0119	0.0116	0.0113	0.0111	0.0107
6	0.0120	0.0117	0.0114	0.0112	0.0108
7	0.0122	0.0119	0.0116	0.0113	0.0109
8	0.0123	0.0120	0.0117	0.0114	0.0110
9	0.0125	0.0121	0.0118	0.0115	0.0111
10	0.0126	0.0123	0.0120	0.0117	0.0112

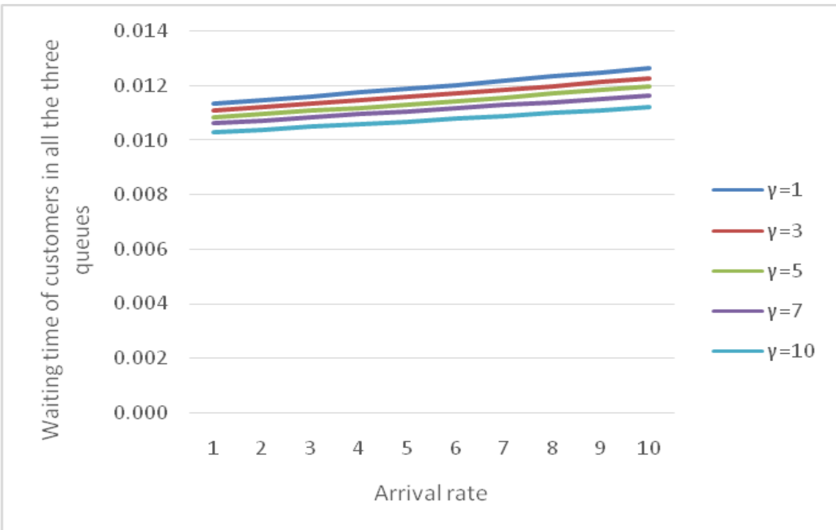

Fig .7 Waiting Time of a Customer in all the Three Queues

For $p = 0.5, \mu_1 = 5, 5.2, 5.4, ...6.8, \mu_2 = 8, \mu_3 = 13, \lambda = 5, \gamma = 1, 3, 5, 7, 10$

Calculation of the number of customers in the system: For the service rate from 5 to 6.8 and for γ (the catastrophe effect) from 1 to 10, the number of customers in the system is calculated in Table VII. From Fig.8 it is clear that as the service rate increases and for various values of γ, the number of customers in the system decreases.

TABLE VII

μ_1/γ	1	3	5	7	10
5	0.6364	0.6349	0.6336	0.6322	0.6304
5.2	0.6312	0.6297	0.6284	0.6271	0.6253
5.4	0.6260	0.6246	0.6233	0.6221	0.6203
5.6	0.6210	0.6196	0.6183	0.6171	0.6154
5.8	0.6160	0.6147	0.6134	0.6122	0.6106
6	0.6111	0.6098	0.6086	0.6074	0.6058
6.2	0.6063	0.6050	0.6038	0.6027	0.6011
6.4	0.6015	0.6003	0.5992	0.5981	0.5965
6.6	0.5969	0.5957	0.5946	0.5935	0.5920
6.8	0.5923	0.5911	0.5900	0.5890	0.5875

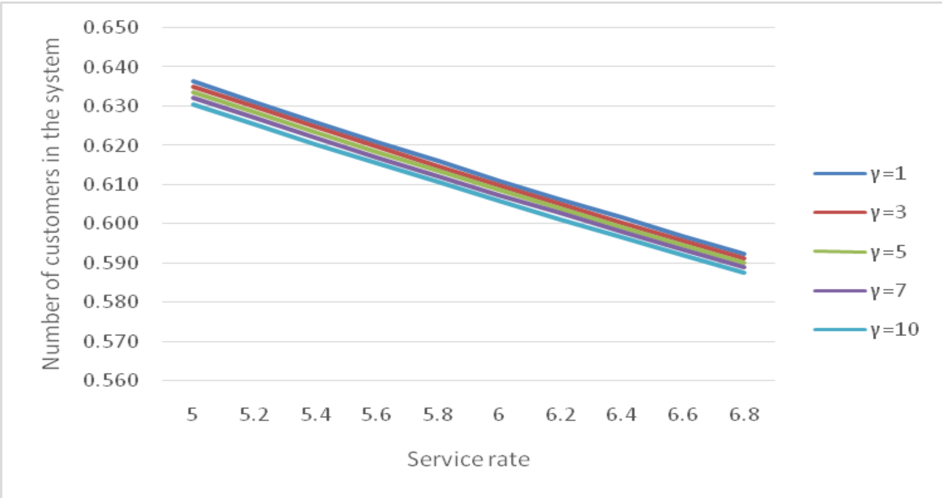

Fig. 8 Number of Customers in the System

Calculation of the waiting time of a customer in the system: For the service rate from 5 to 6.8 and for γ (the catastrophe effect) from 1 to 10, the waiting time of a customer in the system is calculated in Table VIII. From Fig. 9 it is clear that as the service rate increases and for various values of γ, the waiting time of a customer in the system decreases.

TABLE VIII

μ_1/γ	1	3	5	7	10
5	0.1273	0.1270	0.1267	0.1264	0.1261
5.2	0.1262	0.1259	0.1257	0.1254	0.1251
5.4	0.1252	0.1249	0.1247	0.1244	0.1241
5.6	0.1242	0.1239	0.1237	0.1234	0.1231
5.8	0.1232	0.1229	0.1227	0.1224	0.1221
6	0.1222	0.1220	0.1217	0.1215	0.1212
6.2	0.1213	0.1210	0.1208	0.1205	0.1202
6.4	0.1203	0.1201	0.1198	0.1196	0.1193
6.6	0.1194	0.1191	0.1189	0.1187	0.1184
6.8	0.1185	0.1182	0.1180	0.1178	0.1175

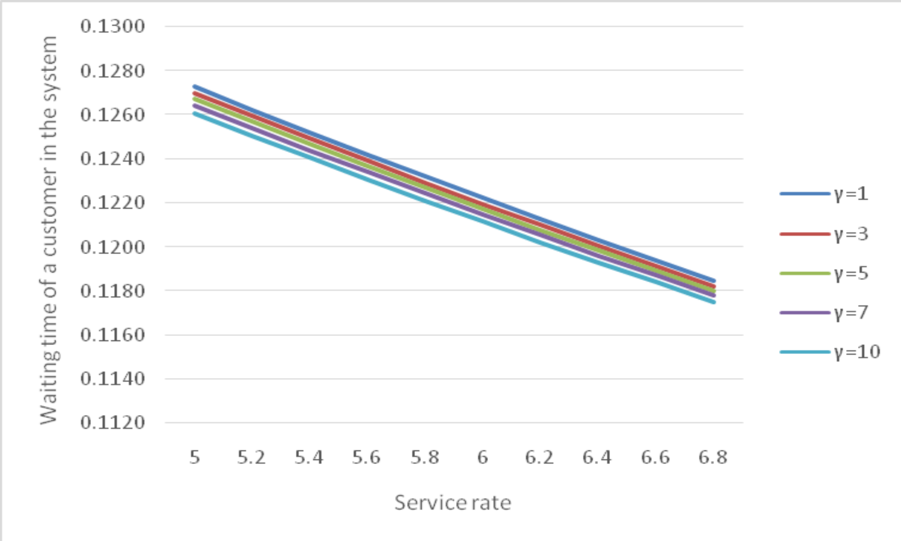

Fig. 9 Waiting Time of a Customer in the System

VI. CONCLUSION

Here we derived the steady state probabilities, the stationary probabilities of the giving network. The numerical examples shows that as the arrival rate (λ) increases, the number of customers in all the three queues and the waiting time of a customer in all the three queues increases and as the catastrophic effect (γ) increases, the number of customers in all the three queues and the waiting time of all the three queues decreases. Also as the service rate increases for various γ values, the number of customers in the system and waiting time of a customer in the system decreases. Particular cases are also derived. It shows the feasibility of the model.

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