Vague Join Semilattices

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Abstract

In this paper we introduce vague join semi lattice, vague join cut-set on semi-lattice and studied their properties. Further we investigate the development of some important results and theorems about vague join semi lattice, vague meet $A_{(\alpha\ \beta)}$*cut - set on semi-lattices.*

Keywords*:* *Vague set , L-vague set ,L-vague cut-set, Vague group, L-vague group, L-vague semiring, L-vague ideal.*

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Introduction

The concept of Lattice was first defined by Dedekind in 1897 and then developed by Birkhoft.G,., imposed an operation an open problem "Is there a common abstraction which includes Boolean algebra, Boolean rings and lattice ordered group or L-group is an algebraic structure connecting lattic and group.To answer this problem many common abstractions, namely dually residuated lattice ordered semigroups, commutative lattice ordered groups. lattice ordered rings, latice ordered near rings and lattice ordered semirings are presented.Amoung them the algebraic structure lattice ordered semirings or L-semiring was introduced by RangaRao.P.,[13].Also the concept proposed by Zadeh.L.A.[16] defining a fuzzy subset A of a given universe X characterizing the membership of an element x of X belonging to A by means of a membership function $\mu_A(x)$ defined from X in to [0 1] has revolutionized the theory of Mathematical modeling. Decision making etc.,in handling the imprecise real life situations mathematically. Now several branches of fuzzy mathematics like fuzzy algebra,fuzzy topology ,fuzzy control theory ,fuzzy measure theory etc.,haveemerged.But in the decision making, the fuzzy theory takes care of membership of an element x only, that is the evidence against x belonging to A .It is felt by several decision makers and researchers that in proper decision making, the evidence belongs to A and evidence not belongs to A are both necessary .and how much X belongs to A or how much x does not belongs to A are necessary.
Several generalizations of Zadeh's fuzzy set theory have been proposed, such as L-fuzzy sets [4]. Interval valued fuzzy sets ,Intuitionistic fuzzy sets by Atanassov.K.T [1] ,Vague sets [3] are mathematically equivalent. Any such set A of a given Universe X can be charactrierized by means of a pair of function (t_A , f_A) where $t_A : X \rightarrow [0\ \ 1]$ and $f_A : X \rightarrow [0\ \ 1]$ such that $0 \leq t_A(x) + f_A(x) \leq 1$ for all x in X.The set $t_A(x)$ is called the truth function and the set $f_A(x)$ is called false function or non membership function and $t_A(x)$ gives the evidence of how much x belongs to A $f_A(x)$ gives the evidence of how much x does not A.These concepts are being applied in several areas like decision-making, fuzy control, knowledge discovery and fault diagonsisetc.It is believed the vague sets (or equivalently instuitionistic fuzzy sets) will more useful in decision making, and other areas of Mathematical modeling. Through Atanassov's instuitionistic fuzzy sets,Gau and Buehrer and some other areas of Mathematical modeling. Since then the theory fuzzy sets developed extensively and embraced almost all subjects like engineering science and technology. But the membership function $\mu_{A\ (x)}$ gives only a approximation belong to A .To avid this and obtain a better estimation and analysis of data decision making. Gau.W.L and Bueher D.J. [3] have initiated the study of vague sets with the hope that they form a better tool to understand, interpret and solve real life problems which are in general vague, than the theory of vague sets do. Ranjit Biswas[9]initiated the study of vague groups by Ramakrishna.N [6],[7],[8] and Eswarlal.T [2]are grate extended the study of vague algebra. The objective of this

paper is to contribute further to the study of vague algebra by introducing the concepts of L-vague cut-set, L-vague semiring of a L-semirings respectively.

I. Preliminaries

In this section we briefly present the necessary material on lattices, Boolean lattices Brouwerian lattices and illustrate with examples.

Definition 1.1 [4]
A poset (L, $\leq$) is called a lattice if sup{x, y}also denoted by (x v y) and inf {x, y} also denoted by (x $\wedge$ y) exists for every pair of elements x, y in L.

Definition 1.2
A lattice (L, $\leq$) in which every subset of L has g.l.b and l.u.b in it is called a complete lattice.

Definition 1.3[4] A lattice L is said to be distributive if it satisfies the
x $\wedge$ (y $\vee$ z) = (x $\wedge$ y)v(x $\wedge$ z) and x v(y $\wedge$ z) = (xv y) $\wedge$ (x v z) for all x, y in L.

Definition 1.4[4] A lattice L is said to be bounded if L has least element and greatest element .Usually least element of L is denoted by $\mathbf{0_L}$ and greatest element is denoted by $\mathbf{1_L}$

Definition 1.5[4] A join semi lattice or semilattice is a non-empty set S with binary operation n

" $\vee$ " defined on it satifies the following.

1.Idempotent: (a $\vee$ a)=a

2.commutative law :aVb=b $\vee$ a

3.Associative law: a $\vee$ (b $\vee$ c) = (a $\vee$ b) $\vee$ c for all a,b,c $\in S$

4.Any two elements in S have a least upper bound.

Definition 1.6[4] A semi lattice is a non-empty set S with binary relation n " $\leq$ " defined on it

defined on it satifies the following.

1. $\leq$ is reflexive : a$\leq$a for all a $\in S$
2. $\leq$ isanti symmetric : a$\leq$b and b$\leq$a $\Rightarrow$ a=b
3. $\leq$ istranstative law: a$\leq$b,b$\leq$c $\Rightarrow$ a$\leq$c ,for all a,b.c$\in S$.

Definition 1.7 Let A be a join semi lattice. A fuzzy set μ_A: X $\rightarrow$ [0 1] called fuzzy join semi lattice of A if it satisfies the following property

1.μ_A(x v y) $\geq$ min { μ_A(x), μ_A(y) } for all x,y in X.

Example 1.8. Let A={ 0, x_1,x_2, $x_{3\ ,1}$ } be a semi lattice. Then it's a fuzzy join semi lattice of μ_A of A given by $\mu_{A=}$ {(0, 0.8) (x_1 , 0.7)(x_2, 0.5), (x_3, 0.6) }.

Definition 1.9 [10] Let A=(t_A, f_A) and B=(t_B, f_B) be two vague sets of sets X then their intersection is defined as (A∩B)= ((t_A ∩$t_{B)}$, (f_A∩$f_{B)}$ **).**
Where(t_A ∩$t_{B)=}$min{ $t_A{}^{(x)}$, $t_B{}^{(y)}$ } and (f_A ∩$f_{B)=}$max{ $f_A{}^{(x)}$, $f_B{}^{(y)}$ } .

Definition 1.10 [10] Let A= (t_A, f_A) be a set of X its complement is defined as A^1= (t_A^1, f_A^1)
Where t_A^1=1-$f_A$and f_A^1 =1-t_A.

Definition 1.9 [10] The vague set A of a set X with$t_A(x)$=0 and $f_A(x)$=1,for x ∈,for all x∈ X is called the zero vague set of X.It is denoted by $\bar{0}$=(0,1).

Definition 1.10 [10] The vague set A of a set X with$t_A(x)$=1 and $f_A(x)$=0,for x ∈,for all x∈ X is called the unit vague set of X. It is denoted by $\bar{1}$=(1,0).

Definition 1.11A vague sets A= (t_A, f_A),B= A= (t_B, f_B)where t_A :X → [0, 1] , f_A :X → [0, 1]
0 ≤$t_A(x)$ + $f_A(x)$≤ 1.

Definition 1.12 Vague sets A and B are equal, written as A=B if and only if A ⊆B and
B ⊆A i.e. t_B =t_B and f_B =f_B.

Definition 1.13 [10] Let A=(t_A, f_A) and B=(t_B, f_B) be two vague sets of sets X then their intersection is defined as (A ∪ B)= ((t_A ∪ t_B), (f_A ∪ f_B) **).**
Where(t_A ∪ t_B)=max{ $t_A^{(x)}$, $t_B^{(y)}$ } and (f_A ∩f_B)=min{ $f_A^{(x)}$, $f_B^{(y)}$ } .
Definition 1.14 [10] The interval [$t_A^{(x)}$,1-$f_A^{(x)}$] is called the vague value of x ∈ A,and it is denoted by $V_A^{(x)}$= [$t_A^{(x)}$,1-$f_A^{(x)}$].
1. $t_A(x+y) \geq \wedge\{ t_A(x), t_A(y)\}$ 2. $t_A(xy) \geq \wedge\{ t_A(x), t_A(y)\}$
3. $t_A(x \vee y) \geq \wedge\{ t_A(x), t_A(y)\}$ 4.$t_A(x \wedge y) \geq \wedge\{ t_A(x), t_A(y)\}$.

II. VAGUE JOIN SEMILATTICE

We now introduce the concept of vague join semilattice,unit vague value and zero vague value on vague Jion semilattice with suitable examples.

Definition 2.1.: Let X be a join semilattice and A=(t_A , f_A) be a vague set of X is said to be vague join semi lattice wheret_A :X ⟶[0 ,1] , f_A: X ⟶ [0,1] such that 0 ≤$t_A(x)$+$f_A(x)$ ≤ 1 for all x in X. If it satisfies the following conditions.

1. $t_A(x \vee y) \geq \min \{ t_A(x), t_A(y)$ and 2. $f_A(x \vee y) \geq \max \{ f_A(x), f_A(y)$.

Example 2.2 : Let X={0,a,b.c,1} be a semi lattice and A={ (0,0.5,0.1),(a,0.6,0.3),(b,0.7,0.2),(c,0.8,0.2),(1,0.9,0.1)} is a vague set of a semi lattice of X.

Sol: - (i).$t_A(0 \vee a) \geq t_A(a)$=0.6 and min { $t_A(0)$, $t_A(a)$ }=min{0.5,0.6}=0.5

Implies $t_A(0 \vee a) \geq \min \{ t_A(0), t_A(a) \}$.

(ii).$t_A(a \vee b) \geq t_A(b)$=0.7 and min { $t_A(a)$, $t_A(b)$ }=min{0.6,0.7}=0.6

Implies $t_A(0 \vee a) \geq \min \{ t_A(0), t_A(a) \}$.

(iii).$t_A(b \vee c) \geq t_A(c)$=0.8 and min { $t_A(b)$, $t_A(c)$ }=min{0.7,0.8}=0.7

Implies $t_A(b \vee c) \geq \min \{ t_A(b), t_A(c) \}$.

(iv).$t_A(0 \vee 1) \geq t_A(1)$=0.8 and min { $t_A(0)$, $t_A(1)$ }=min{0.5,0.8}=0.5

Implies $t_A(0 \vee a) \geq \min \{ t_A(0), t_A(a) \}$.

(v).$f_A(0 \vee a) \geq f_A(a)$=0.6 and max { $f_A(0)$, $f_A(a)$ }= max{0.1,0.3}=0.5

Implies $f_A(0 \vee a) \geq \max \{ f_A(0), f_A(0) \}$.

(vi).$f_A(a \vee b) \geq f_A(b)$=0.3 and max { $f_A(a)$, $f_A(b)$ }= max{0.1,0.3}=0.3

Implies $f_A(a \vee b) \geq \max \{ f_A(0), f_A(0) \}$.

(vii).$f_A(b \vee c) \geq f_A(c)=0.8$ and $\max \{ f_A(b), f_A(c) \}= \max\{0.2,0.2\}=0.2$

Implies $f_A(b \vee c) \geq \max \{ f_A(b), f_A(c) \}$.

(viii).$f_A(0 \vee 1) \geq f_A(1)=0.2$ and $\max \{ f_A(0), f_A(1) \}= \max\{0.1,0.2\}=0.2$

Implies $f_A(0 \vee 1) \geq \max \{ f_A(0), f_A(1) \}$.

TherforeA={ (0,0.5,0.1),(a,0.6,0.3),(b,0.7,0.2),(c,0.8,0.2),(1,0.9,0.1)} is a vague join semi lattice of a semilattice X.

Definition .2.3 Let X be a semilattice and A=(t_A , f_A) be a vague set of a semi lattice X with $t_A^{(x)}=0$, $f_A^{(x)=1}$,for all $x \in X$ is called zero vague set of X.It is denoted by $\bar{0}$=(0,1).

Definition .2.4 Let X be a semilattice and A=(t_A , f_A) be a vague join semi lattice of a semi lattice X with $t_A^{(x)}=1$, $f_A^{(x)=0}$,for all $x \in X$ is called unit vague set of X.It is denoted by $\bar{1}$=(1,0).

Definition .2.5 Let X be a semilattice and A=(t_A , f_A) be a vague join semi lattice of a semi lattice X .And $K \in A$,K is called vague chain if Sup(K) is aa crisp chain of X set of X.

Theorem 2.6 : LetA=(t_A , f_A) be a vague join semi lattice of semi lattice X .Then sup of A=(t_A , f_A) is a crisp lattice of a semi lattice X.

Proof: LetA=(t_A , f_A) be a vague join semi lattice of semi lattice X

Since $t_A(x \vee y) \geq \wedge \min \{ t_A(x), t_A(y)\}$ and $f_A(x \vee y) \geq \max \{ f_A(x), f_A(y)$ for all $x,y \in X$.

It follows that $t_A(x \vee y) \geq 0$.

Also $t_A(x \vee y)$ and $f_A(x \vee y) \leq 1$ .Hence $(x \vee y) \in$ of A.

Thus sup of A is crisp lattice of semi lattice X.

Theorem 2.7 : LetA=(t_A , f_A) be a vague join semi lattice of semi lattice X .Then A^C=(t_A^C , f_A^C) is also vague join semi lattice of a semi lattice X.

Proof:LetA=(t_A , f_A) be a vague join semi lattice of semi lattice X .

$t_A(x \vee y) \geq \wedge \min \{ t_A(x), t_A(y)\}$ and $f_A(x \vee y) \geq \max \{ f_A(x), f_A(y)$ for all $x,y \in X$.

To show that A=(1- f_A , f_A) is a vague join semi lattice of semi lattice X .

LetB=A=(1-f_A , f_A)

$t_B(x \vee y) \geq \wedge (1- f_B^{(x)} ,f_B^{(y)} \} \leq \max \{ f_B(x), f_B(y)\} \leq \max \{ 1-f_B (x),1- f_B(y)\} \geq \min\{t_B(x), t_B(y)\}$.

Therefore $t_B(x \vee y) \geq \min\{t_B(x), t_B(y)\}$…….(2.7.1)

$f_B(x \vee y)= 1- f_B^{(xvy)} \leq \min\{f_B(x), f_B(y)\} \geq \max \{ 1-f_B (x),1- f_B(y)\} \geq \max\{f_B(x), f_B(y)\}$.

Therefore $f_B(x \vee y) \geq \max\{f_B(x), f_B(y)\}$…….(2.7.2).

Hence (2.7.1) and (2.7.2) the complement of vague join semi lattice of a semi lattice X is also vague join semi lattice X.

Theorem 2.8: LetA=(t_A , f_A) be a vague join semi lattice of semi lattice X .Then A=($t_A , 1-t_A$) is also vague join semi lattice of a semi lattice X.

Proof:LetA=(t_A , f_A) be a vague join semi lattice of semi lattice X .

$t_A(x \vee y) \geq \wedge \min \{ t_A(x), t_A(y)\}$ and $f_A(x \vee y) \geq \max \{ f_A(x), f_A(y)$ for all $x,y \in X$.

To show that A=(1- f_A , f_A) is a vague join semi lattice of semi lattice X .

LetB=A=(1-f_A , f_A)

$t_B(x \vee y) \geq \wedge$ ($1- f_B^{(x)}, f_B^{(y)}$ } $\leq$max { $f_B(x), f_B(y)$} $\leq$max { $1-f_B(x), 1- f_B(y)$}$\geq$min{$t_B(x), t_B(y)$}.

Therefore $t_B(x \vee y) \geq$min{$t_B(x), t_B(y)$}…….(2.8.1)

$f_B(x \vee y)= 1- f_B^{(xvy)} \leq$ min{$f_B(x), f_B(y)$} $\geq$max { $1-f_B(x), 1- f_B(y)$}$\geq$max{$f_B(x), f_B(y)$}.

Therefore $f_B(x \vee y) \geq$max{$f_B(x), f_B(y)$}…….(2.8.2).

Hence (2.8.1) and (2.8.2) the complement of vague join semi lattice of a semi lattice X is also vague join semi lattice X.

Theorem 2.9: If A,B are vague join semilattices of a semilattice X ,then their intersection (A ∩B) is also a vague join semi lattice of a lattice semi lattice X.

Proof:-Let A, B be two vague join semilattices of a semilattice X.

i.e. $t_A(x \vee y) \geq \wedge$min { $t_A(x), t_A(y)$} and $f_A(x \vee y) \geq$ max { $f_A(x), f_A(y)$ for all x,y∈ X.

To prove that (A ∩B) is also a vague join semi lattice of a lattice semi lattice X.

Let C=(A ∩B)

i.e. C ={ (x ,$t_A(x), f_A(x)$) : x ∈ X } is a vague join semi lattice of a lattice semi lattice X.

If $t_C(x)$=min{ $t_A(x, t_B(y)$ } and $f_C(x)$=max{ $f_A(x), f_B(y)$ }.

(i) $t_C(x \vee y) \geq \wedge$min { $t_A(x \vee y), t_B(x \vee y)$} =min{ min{ $t_A(x), t_A(y)$ }, min{ $t_B(x), t_B(y)$ }}

implies min {min { $t_A(x), t_B(x)$ }, min{ $t_A(y), t_B(y)$ }}=min{$t_C(x), t_C(y)$ }.

Therefore $t_C(x \vee y) \geq$ min {$t_C(x), t_C(y)$ } ………..(2.9.1)

(ii) If $f_C(x \vee y)=f_C(x)$=max{ $f_A(x), f_B(y)$ }

$f_C(x \vee y)$ =max { $f_A(x \vee y), f_B(x \vee y)$} =max{ max{ $f_A(x), f_A(y)$ }, max{ $f_B(x), f_B(y)$ }}

implies max {max{ $f_A(x), f_B(x)$ }, max{ $f_A(y), f_B(y)$ }}=max{$f_C(x), f_C(y)$ }.

Therefore $f_C(x \vee y) \geq$ max {$f_C(x), f_C(y)$ } for all x,y∈ X. ………..(2.9.2).

Therefore from (2.9.1) and (2.9.2) C=(A ∩B) is also a vague join semi lattice of a lattice semi lattice X.

Theorem 2.10 : If A,B are vague join semilattices of a semilattice X ,then their union (A ∪B) is also a vague join semi lattice of a lattice semi lattice X.

Proof:-Let A, B be two vague join semilattices of a semilattice X .

i.e. $t_A(x \vee y) \geq \wedge$min { $t_A(x), t_A(y)$} and $f_A(x \vee y) \geq$ max { $f_A(x), f_A(y)$ for all x,y∈ X.

To prove that (A ∩B) is also a vague join semi lattice of a lattice semi lattice X.

Let C=(A ∪ B)

i.e. C ={ (x ,$t_A(x), f_A(x)$) : x ∈ X } is a vague join semi lattice of a lattice semi lattice X.

If $t_C(x)$=min{ $t_A(x, t_B(y)$ } and $f_C(x)$=max{ $f_A(x), f_B(y)$ }.

(i) $t_C(x \vee y) \geq \wedge$min { $t_A(x \vee y), t_B(x \vee y)$} =min{ min{ $t_A(x), t_A(y)$ }, min{ $t_B(x), t_B(y)$ }}

implies min {min { $t_A(x), t_B(x)$ }, min{ $t_A(y), t_B(y)$ }}=min{$t_C(x), t_C(y)$ }.

Therefore $t_C(x \vee y) \geq$ min {$t_C(x), t_C(y)$ } ………..(2.10.1)

(ii) If $f_C(x \vee y)=f_C(x)=\max\{ f_A(x),f_B(y) \}$

$f_C(x \vee y) =\max \{ f_A(x \vee y), f_B(x \vee y)\} =\max\{ \max\{ f_A(x), f_A(y) \}, \max\{ f_B(x), f_B(y) \}\}$

implies $\max \{\max\{ f_A(x), f_B(x) \}, \max\{ f_A(y), f_B(y) \}\}=\max\{f_C(x), f_C(y) \}$.

Therefore $f_C(x \vee y) \geq \max \{f_C(x), f_C(y) \}$ for all $x,y\in X$. …………(2.10.2).

Therefore from (2.10.1) and (2.10.2) $C=(A \cup B)$ is also a vague join semi lattice of a lattice semi lattice X.

III. Vague cut-set on vague join semi lattices

We now introduce the concept of vague cut-set on vague join semilattice of a semilattice X with suitable examples.

Definition 3.1 Let X be a semi lattice and A be a vague join semilattice of X. Then the vague trueα cut set on vague semi lattice can be defined by $T_A\alpha$of A is$t_A\alpha=\{ x\in X : t_A(x) >\alpha \}$ where $\alpha \in [0\ \ 1]$.

Example 3.2

Let $X=\{0, x_1, x_2, x_3, 1 \}$ be a semi lattice and

$A=\{ (0,0.4,0.1),(x_1, 0.5,0.3),(x_2, 0.9,0.2),(x_3, 0.7,0.1),(1, 0.3,0.2) \}$ is a vague join semi lattice of a semi lattice X.

Then true $t_A\alpha=\{ x\in X : t_A(x) >\alpha \}$ where $\alpha \in [0\ \ 1]$.

Implies vague true α -cut set $t_A\alpha =t_{A0.4}=\{ (0,0.4,0.1),(x_1, 0.5,0.3),(x_2, 0.9,0.2),(x_3, 0.7,0.1) \}$,where $\alpha \in [0,1]$.

Definition 3.3. Let X be a semi lattice and A be a vague join semilattice of X. Then the vague false $\alpha -$ cut set on vague semi lattice can be defined by $F_A\alpha$of A is $f_A\alpha=\{ x\in X : f_A(x) <\beta \}$ where $\beta \in [0\ \ 1]$.

Example 3.4

Let $X=\{0, x_1, x_2, x_3, 1 \}$ be a semi lattice and

$A=\{ (0,0.4,0.1),(x_1, 0.5,0.3),(x_2, 0.9,0.2),(x_3, 0.7,0.1),(1, 0.3,0.2) \}$ is a vague join semi lattice of a semi lattice X.

Then true $f_A\alpha=\{ x\in X : f_A(x) <\beta\}$ where $\beta \in [0\ \ 1]$.

Implies vague true α -cut set $t_A\beta =t_{A0.3} =\{ (0,0.7,0.1),(x_1, 0.5,0.2),(x_2, 0.8,0.2),(x_3, 0.4,0.1) \}$,where $\beta = 0.3 \in [0,1]$.

Definition 3.5 : Let X be a semi lattice and A be a vague join semilattice of X. Then the vague (α ,β) cut set on vague semi lattice can be defined by $A_{(}(\alpha,\beta)$of A is

$A_{(}(\alpha,\beta)=\{ x \in X : t_A(x) \geq \alpha \text{ and } f_A(x) \leq \beta \}$ where $\alpha,\beta \in [0\ \ 1]$.

Example 3.6

Let $X=\{0, x_1, x_2, x_3, 1 \}$ be a semi lattice and

$A=\{ (0,0.4,0.1),(x_1, 0.5,0.3),(x_2, 0.9,0.2),(x_3, 0.7,0.1),(1, 0.3,0.2) \}$ is a vague join semi lattice of a semi lattice X.

Then true $t_A\alpha=\{ x\in X : t_A(x) >\alpha \}$ where $\alpha \in [0\ \ 1]$.

Implies vague cut set can be defined by$A(\alpha,\beta) ==\{ (0,0.4,0.3),(x_1, 0.5,0.3),(x_2, 0.8,0.2)\}$,where $\alpha,\beta \in [0,1]$.

Theorem.3.7: Let $A=(t_A, f_A)$ be a vague join of a semi lattice of semilattice X.Then the true vague true α cut set $t_A\alpha=\{ x \in X : t_A(x) >\alpha \}$ where $\alpha \in [0\ \ 1]$ is vague join semi lattice of X.

Proof: Let $A=(t_A, f_A)$ be a vague join of a semi lattice of semilattice X.

We shall show that $t_A\alpha$ is a vague join semi lattice of a semi lattice X.

Since by the definition of true α cut set $t_A\alpha$ for any $x ,y\in X$.

(i) $t_A(x)\geq \alpha$ and $t_C (y)\geq \alpha$ implies $t_A(x \vee y) \geq \{ \alpha,\alpha \}=\alpha$

min{ $t_A(x)$, $t_A(y)$ }, min{ $t_B(x)$, $t_B(y)$ }}

Therefore $t_C(x \vee y) \geq$ min {$t_C(x)$, $t_C(y)$ } ………..(3.7.1)

(i) $f_A(x) \leq \alpha$ and $f_A(y) \geq \alpha$ implies $f_A(x \vee y) \leq \{ \alpha, \alpha \} = \alpha$

min{ $f_A(x)$, $f_A(y)$ }, min{ $f_B(x)$, $f_B(y)$ }}

Therefore $f_A(x \vee y) \leq$ min {$f_A(x)$, $f_A(y)$ } ………..(3.7.2)

Theorem.3.8 : Let A=(t_A, f_A) be a vague join of a semi lattice of semilattice X. Then the unit vague set and zero vague sets are vague join semi lattices.

Proof: Let A=(t_A, f_A) be a vague join of a semi lattice of semilattice X.

Let B be the unit vague unit set.Let B=(t_A, 0) then $t_B = t_A$, $f_B = 0$.

Since by the definition of true α cut set $t_A\alpha$ for any x ,y∈ X.

Let x ,y∈ X.$t_B(x \vee y) \geq$ min { $t_A(x)$, $t_A(y)$} and $f_A(x \vee y) \geq$ max { $f_A(x)$, $f_A(y)$ for all x,y∈ X.

Let C be the zero vague set. Let C=(0,1-t_A) then $t_C = 0$, $f_C = 1 - t_A$

Let x ,y∈ X.

$t_C(x \vee y) \geq$ min { $t_C(x)$, $t_{AC}(y)$} and $f_A(x \vee y) \geq$ max { $f_C(x)$, $f_C(y)$ for all x,y∈ X.

Hence B and C are are vague join semi lattices of a semilattices X.

Theorem.3.9: Let A=(t_A, f_A) be a vague join of a semi lattice of semilattice X if and only if (α ,β) cut-set $A_{(}(\alpha,\beta)$={ x ∈ X : $t_A(x) \geq \alpha$ and $f_A(x) \leq \beta$ } where $\alpha, \beta \in [0\ \ 1]$.

is vague join semi lattice of X.

Proof: Let A=(t_A, f_A) be a vague join of a semi lattice of semilattice X.

Suppose that A is a vague join semi lattice X.

And x,y∈ (α,β)cut-set where α,β∈ [0 1] with $\alpha + \beta \leq 1$.

$t_A(x) \geq \alpha$ and $t_C(y) \geq \alpha$ implies $t_A(x \vee y) \geq \{ \alpha, \alpha \} = \alpha$

min{ $t_A(x)$, $t_A(y)$ }, min{ $t_B(x)$, $t_B(y)$ }}

Therefore $t_C(x \vee y) \geq$ min {$t_C(x)$, $t_C(y)$ } ………..(3.9.1)

$f_A(x) \leq \alpha$ and $f_A(y) \geq \alpha$ implies $f_A(x \vee y) \leq \{ \alpha, \alpha \} = \alpha$

min{ $f_A(x)$, $f_A(y)$ }, min{ $f_B(x)$, $f_B(y)$ }}

Therefore $f_A(x \vee y) \leq$ min {$f_A(x)$, $f_A(y)$ } ………..(3.9.2).

Hence A=(t_A, f_A) be a vague join of a semi lattice of semilattice X if and only if (α ,β) cut-set $A_{(}(\alpha,\beta)$={ x ∈ X : $t_A(x) \geq \alpha$ and $f_A(x) \leq \beta$ } where $\alpha, \beta \in [0\ \ 1]$.

is vague join semi lattice of X.

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