A Walk Through Completely Normal Bihyperideals in Hypersemi groups

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Abstract — *In this paper, we introduce completely normal hyperideal and bi-hyperideal in normal hypersemigroups. We prove that a hypersemigroup H is completely regular if and only if every bi-hyperideal of H is semiprime. Then, we prove that the center of a normal regular hypersemigroup is regular. We also prove several equivalent conditions connecting normal hypersemigroups, idempotent hypersemigroups, completely regular and viable hypersemigroups.*

Keywords — *hypersemigroup, normal hyperideal, bi-hyperideal.*

I. INTRODUCTION

A generalization is the abstraction of general concepts from specific instances by the calculation from particular Mathematical objects to general Mathematical objects by looking at the common properties and expanding their domains of proofs. Indeed, the importance of generalizations is not only limited to individual thinking or scientific thinking and that also refers to social communication. The practice of generalization is a powerful process that is indispensable part of passing from one algebraic structure to other algebraic structure for further advancement of the theory.

A semigroup is simply a set S which is closed under an associative binary operation. The notion of a semigroup is a real classical and natural one and it can be realized that semigroups, in the guise of N, have been present in Mathematics from its ancient discovery. There are two overlapping theories of semigroups: the topological theory and the algebraic theory. The algebraic theory is newer and mostly developed after the Second World War.

The notion of bi-ideal was introduced by Good, Hughes and the concept of (m, n)-ideal was given by Lajos [42], [44], [45], [46], [47], [48]. The present article is an attempt to give an overview of the study of certain aspects of the early theory in [38] with a possible profound effect on its subsequent development. In the recent years, various groups all over the world study the hypersemigroups in research programs applying the definition given by Marty [28] generalizing and extending the traditional erstwhile binary operation in a way that in classical algebraic structures, the composition of two elements is an element, and in the algebraic hyperstructures, the composition of two elements is a set. During forties, Marty, Krasner, Kuntzmann and Crolsot in France; Dresher, Ore, Eaton, Wall, Campalgne, Griffith and Prenowitz in the United States; Utuml in Japan, Dietzman and Vikrov in USSR and Zappa in Italy studied and enriched the general theory of hyperstructures and their applications in other branches of Mathematics, Sciences as well as in Social Sciences. For comprehensive bibliography on the subject matter, we refer some recent books and publications such as in [1], [2], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [29], [30], [31], [32], [33], [34], [35], [36], [37], [39], [40], [41], [43], [52]. Most recently, Basar et al. studied different aspects of hypersemigroups [3], [4], [5], [6], [7], [8], [9], [11], [12], [13].

II. PRELIMINARIES

Let H be a nonempty set, then the mapping $\circ: H \times H \to H$ is called hyperoperation or join operation on H, where $P^*(H) = P(H)\backslash\{0\}$ is the set of all nonempty subsets of H. Let A and B be two nonempty sets. Then, a hypergroupoid $(H,\circ)$ is called a semihypergroups if for every $x, y, z \in H$

$$x \circ (y \circ z) = (x \circ y) \circ z.$$

For subsets A, B of semihypergroup H, the product set $A \circ B$ of the pair (A, B) relative to H is defined as $A \circ B = \{a \circ b : a \in A, b \in B\}$ and for $A \subseteq H$, the product set $A \circ A$ relative to H is defined as $A^2 = A \circ A = A \circ A$. Note that A^0 is an identity operator. That is, $A^0 \circ H = H = H \circ A^0$. Also, $(A] = \{h \in H: h \leq aforsomea \in A\}$.

If there is no ambiguity, we identify hypersemigroup $(H,\circ)$ by H. A sub-hypersemigroup T of a hypersemigroup H is called normal if $h \circ T = T \circ h$ for all $h \in H$. A hypersemigroup H is called left (right) regular if for every element $h \in H$, there exists an element $a \in H$ such that $\{h\} = a \circ h \circ h \circ$ ($\{h\} = h \circ h \circ a$). A hypersemigroup H is called intra-regular if for any $h \in H$, there exist elements $a, b \in H$ such that $\{h\} = a \circ h \circ h \circ b$. A hypersemigroup H is called completely regular if for any element $h \in H$, there exists an element $a \in H$ such that $\{h\} = h \circ a$ and $h \circ a = a \circ h$. We denote and define the principal left hyperideal, right hyperideal and bi-hyperideal of H generated by $h \in H$ as follows: $(h)_L = h \cup H \circ h$, $(h)_R = h \cup h \circ H$, $(h)_B = h \cup h^2 \cup h \circ H \circ h$. An element a of H is called a zero element of H if $h \circ a = a \circ h = \{a\}$ for $h \in H$.

Notation 1. We denote by $\mathcal{B}(H)$ the set of all bi-hyperideals of H and $\mathfrak{B}(H)$ the set of nonempty subsets of the hypersemigroup H.

Definition 1.1 *A hypersemigroup H is called normal if $h \circ H = H \circ h$ for all $h \in H$.*

Definition 1.2 *A hyperideal I of a hypersemigroup H is called normal if $h \circ I = I \circ h$ for all $h \in H$.*

The concept of a viable semigroup was introduced by Putcha and Weissglass[33].

Definition 1.3 *A hypersemigroup H is called viable if $a \circ b = b \circ a$ whenever $a \circ b$ and $b \circ a$ are idempotents.*

Definition 1.4 *A nonempty subset S of a hypersemigroup H is semiprime if $h^2 \subseteq S$, $h \in H$ implies $h \in S$.*

A semihypergroup (H, o) is called hypergroup if $x \circ H = H \circ x =$ for all $(x, y) \in I$.

III. MAIN RESULTS

Lemma 3.1 *[1] . For a hypersemigroup H, the following conditions are equivalent:*

1. H is completely regular;
2. $\{h\} \subseteq h^2 \circ H \circ h^2$ for some $h \in H$;
3. H is left and right regular.

Theorem 3.1 *A hypersemigroup H is completely regular if and only if every bi-hyperideal of H is semiprime.*

Proof. Let H be completely regular and A be any bi-hyperideal of H and $h^2 \subseteq A$ and $h \in H$. Then, it implies by analogous of Lemma 3.1 that

$$\{h\} \subseteq h^2 \circ H \circ h^2 \subseteq A \circ H \circ A \subseteq A.$$

Hence, A is semiprime.

Conversely, every bi-hyperideal of H is semiprime. Then, since every one-sided ideal of a hypersemigroup is a bi-hyperideal, every left and right bi-hyperideal of H is semiprime. Also, from analogous of Lemma 3.1, H is left and right regular. Hence, by Lemma 3.1, H is completely regular.

Theorem 3.2 *Let H be a completely regular hypersemigroup. Then, $\mathcal{B}(H)$ is idempotent.*

Proof. Let A be any bi-hyperideal of completely regular hypersemigroup H and a any element of A. Then, by Lemma 3.1, we obtain

$$\{a\} \subseteq a^2 \circ H \circ a^2 \subseteq A^2 \circ H \circ A^2 \subseteq A.$$

Therefore, we have

$$A \subseteq A^2 \circ H \circ A^2 = A \circ (A \circ H \circ A) \circ A \subseteq A^3 \subseteq A^2 \subseteq A.$$

Hence, $A^2 = A$.

Lemma 3.2 *For a hypersemigroup H, the following conditions are equivalent:*

1. H is regular;
2. $\mathcal{B}(H)$ is regular.

Combining Theorem 3.2 and Lemma 3.2, we obtain the following:

Theorem 3.3 *For a completely regular hypersemigroup H, the following conditions are equivalent:*

(i). $\mathcal{B}(H)$ is idempotent;
(ii). $\mathcal{B}(H)$ is regular.

Theorem 3.4 *We now have the converse of the Theorem 3.2 as follows:*

(i) . H is regular;
(ii). H is left regular;
(iii). H is right regular;
(iv). H is intra- regular;
(v). H is completely regular;
(vi). $\{h\} \subseteq (h \circ H)^n$ for every element $h \in H$ and every integer $n \geq 2$;
(vii). $\mathcal{B}(H)$ is idempotent;
(viii). $\mathcal{B}(H)$ is completely regular;
(ix). $\mathcal{B}(H)$ is regular.

Proof. Since H is normal, it can be easily shown that $(i) - (vi)$ are equivalent.
By Lemma 3.2, (i) and (ix) are equivalent. By Theorem 3.2, $(v) \Rightarrow (vii)$.
It is clear that $(vii) \Rightarrow (viii)$ and $(viii) \Rightarrow (ix)$.

Theorem 3.5 *The center of a normal regular hypersemigroup is regular.*

Proof. Let a be any element of the center Z of a normal regular hypersemigroup H. Then, by Theorem 3.4, we have

$$\{a\} \subseteq (a \circ H)^3 \subseteq a \circ (a \circ H) \circ a.$$

Therefore, there exists an element $x \in H$ such that

$$\{a\} = a \circ (a \circ x) \circ a,$$

Now, we need to prove that $a \circ x \subseteq Z$. Let $b \in S$. Then, we have

$$\begin{aligned} b \circ (a \circ x) = (b \circ a) \circ x = (a \circ b) \circ x = a \circ (b \circ x) &= (a \circ a \circ x \circ a) \circ (b \circ x) \\ &= a \circ a \circ x(a \circ b) \circ x = a \circ a \circ x \circ (b \circ a) \circ x \\ &= a \circ a \circ (x \circ b) \circ (a \circ x) \\ &= a \circ (a \circ (x \circ b)) \circ (a \circ x) \\ &= a \circ (a \circ (x \circ b)) \circ (x \circ a) \\ &= a \circ ((x \circ b) \circ a \circ (x \circ a) \\ &= (a \circ (x \circ b))(a \circ x \circ a) \\ &= ((x \circ b) \circ a) \circ a \circ x \circ a \\ &= (x \circ b) \circ (a \circ a \circ x \circ a) \\ &= (x \circ b) \circ a = a \circ (x \circ b) \\ &= (a \circ x) \circ b \end{aligned}$$

Therefore, we have $a \circ x \subseteq Z$ and therefore, $\{a\} \subseteq a \circ Z \circ a$.

Hence, the center of Z is regular.

By a hypersemigroup analogue of [[50], Theorem 8], we have the following:

Lemma 3.3 *For a hypersemigroup H, the following assertions are equivalent:*

(i). H is a semilattice of hypergroups;

(ii). H is regular and $e \circ H = H \circ a$ for all idempotent elements e of H.

By a hypersemigroup analogue of [[49], Theorem 12], we have the following:

Lemma 3.4 *If a hypersemigroup H is a semilattice of hypergroups, then every bi-hyperideal of H is two-sided.*

By a hypersemigroup analogue of [[33], Theorem 6], we have the following:

Lemma 3.5 *If a hypersemigroups H is a semilattice of hypergroups, then it is viable.*

Theorem 3.6 *[10] The following assertions are equivalent for a hyperideal I of H:*

(i).I is normal;

(ii). $Y \circ I = I \circ Y$ for all $Y \in \mathcal{B}(H)$;

(iii). $(h)_B \circ I = I \circ (h)_B$ for all $h \in H$;

(iv). $(h)_B \circ I = I \circ (h)_L$ for all $h \in H$;

(v). $(h)_B \circ I = I \circ h$ for all $h \in H$;

(vi). $(h)_R \circ I = I \circ (h)_B$ for all $h \in H$;

(vii). $(h)_R \circ I = I \circ (h)_L$ for all $h \in H$;

(viii). $(h)_R \circ I = I \circ h$ for all $h \in H$;

(ix). $h \circ I = I \circ (h)_B$ for all $h \in H$;

(x). $h \circ I = I \circ (h)_L$ for all $h \in H$.

Theorem 3.7 *For a regular hypersemigroup H, the following conditions are equivalent:*

(i). H is normal;

(ii). $e \circ H = H \circ e$ for all idempotents e of H;

(iii). $[e]_B \circ H = H \circ [e]_B$;

(iv). $[H]_B \circ H = H \circ [e]_L$;

(v). $[e]_B \circ H = H \circ e$;

(vi). $[e]_R \circ H = H \circ [e]_B$;

(vii). $[e]_R \circ H = H \circ [e]_L$;

(viii). $[e]_R \circ H = H \circ e$;

(ix). $e \circ H = H[e]_B$;

(x). $e \circ H = H \circ [e]_L$;

(xi). $[e]_B \circ \mathcal{B}(H) = \mathcal{B}(H)[e]_B$;

(xii). $[e]_B \circ \mathcal{B}(H) = \mathcal{B}(H) \circ [e]_L$;

(xiii). $[e]_B \circ \mathcal{B}(H) = \mathcal{B}(H) \circ e$;

(xiv). $[e]_R \circ \mathcal{B}(H) = \mathcal{B}(H)[e]_B$;

(xv). $[e]_R \circ \mathcal{B}(H) = \mathcal{B}(H) \circ [e]_L$;

(xvi). $[e]_R \circ \mathcal{B}(H) = \mathcal{B}(H) \circ e$;

(xvii). $e \circ \mathcal{B}(H) = \mathcal{B}(H) \circ [e]_B$;

(xviii). $e \circ \mathcal{B}(H) = \mathcal{B}(H) \circ [e]_L$;

(xix). $e \circ \mathcal{B}(H) = \mathcal{B}(H) \circ e$;

(xx). $\mathcal{B}(H)$ is viable.

Proof. Since, the hypersemigroup H itself is a hyperideal of H, it follows from Theorem 3.6, (i)-(x) are equivalent. From this and Theorem 2.7 of [10], we can easily see that (i) and (xi)-(xix) are equivalent. Lemma 3.3 implies that a hypersemigroup is normal and regular if and only if it is semilattice of hypergroups.

Let H be normal. Then, by Theorem 2.7 of [10] and Lemma 3.3, $\mathcal{B}(H)$ is normal and regular. Thus, $\mathcal{B}(H)$ is semilattice of hypergroups. So, by Lemma 3.5, $\mathcal{B}(H)$ is viable. Hence, $(i) \Rightarrow (xx)$.

Conversely, we assume that $\mathcal{B}(H)$ is viable. Since H is regular by Lemma 3.2, $\mathcal{B}(H)$ is regular. Let A be any bi-hyperideal of H. Then, for some some $X \in \mathcal{B}(H)$, we obtain the following:

$$A = A \circ X \circ A \subseteq A \circ H \circ A \subseteq A.$$

We have, $A = A \circ H \circ A$ and both $A \circ H$ and $H \circ A$ are idempotent of $\mathcal{B}(H)$.

Since $\mathcal{B}(H)$ is viable, we have $A \circ H = H \circ A$. Since A is any bi-hyperideal of H, by Theorem 2.7 of [10], H is normal. Hence, $(xx) \Rightarrow (i)$.

Conversely, let H be normal and regular. Then, by Theorem 3.4, $\mathcal{B}(H)$ is completely regular. Since H is a semilattice of hypergroups, it follows by Lemma 3.4, that every bi-hyperideal of H is two-sided. Therefore, we obtain that $(ii) \Rightarrow (vi)$.

Theorem 3.8 *The following conditions are equivalent for hypersemigroups H:*

(i). H is semilattice of hypergroups;

(ii). H is regular;

(iii). $\mathcal{B}(H)$ is a semilattice of hypergroups;

(iv). $\mathcal{B}(H)$ is regular and normal;

(v). $\mathcal{B}(H)$ is regular and viable;

(vi). $\mathcal{B}(H)$ is a completely regular hypersemigroup if every hyperideal of H is two-sided.

Proof. It is sufficient to show that (ii) and (xi) are equivalent. Let (xi) holds. Since $\mathcal{B}(H)$ is regular, H is regular by Lemma 3.2. To show that H is normal, let A be any bi-hyperideal of H. Since, $\mathcal{B}(H)$ is completely regular, there exists a bi-hyperideal X of H such that

$$A = A \circ X \circ A, A \circ X = X \circ A.$$

Then, since every bi-hyperideal of H is two-sided, we have

$$\begin{aligned} A \circ H &= (A \circ X) \circ H = (A \circ X) \circ (A \circ H) \\ &= X \circ (A \circ A \circ H) \\ &\subseteq X \circ A \\ &\subseteq H \circ A \end{aligned}$$

Similarly, we can show that the converse inclusion holds. Hence

$$A \circ H = H \circ A$$

for all $A \subseteq \mathcal{B}(H)$.

Then, it follows from Theorem 2.7 of [10], that H is normal. Hence, $(vi) \Rightarrow (ii)$.

As an immediate consequence of (i) and (ii) of Theorem 3.8, we have

Corollary 3.1 *For an idempotent hypersemigroup H, the following conditions are equivalent:*

(i). H is normal;

(ii). H is commutative.

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