On The Classes of $(1,2)^* - \beta c$-Open Sets In Bitopological Spaces

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Abstract: A new class of generalized open sets in a bitopological space, called $(1,2)^* - \beta c$ open sets, is introduced and studied. This class is contained in the class of $(1,2)^* - \beta$-open sets and contains all $(1,2)^* - bc$ -open sets and$(1,2)^*$- $\theta -$ semi-open sets. We show that the class of$(1,2)^* - \beta c$ -open sets generates the same bitopology as the class of $\theta - (1,2)^*$-open sets in alexandroff spaces.

Keywords — *(1,2)* -βc open sets, generated by (1,2)*--βc open sets ,* $\tau_{1,2-\delta}$,$\tau_{1,2-\beta c}$

1. Introduction :

Levine introduced generalized closed set in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved the notions of $(1,2)^*$ -semi-open sets, $(1,2)^*$-α-open sets, $(1,2)^*$-pre open sets , $(1,2)^*$-semi-preopen sets or $(1,2)^* - \beta$-open sets have been introduced and studied respectively

Lellis Thivagar.M and O.Ravi gave a new class of $(1,2)^*$ generalized open sets which contained $(1,2)^*$-semi open sets and $(1,2)^*$ -semi preopen sets call it $(1,2)^*$ - β -open sets. Chandrashekhara Rao and Vaithilingham introduced a special class of $(1,2)^*$-b-open sets name it $(1,2)^*$-bc open sets and studied its fundamental properties and compare it with some other types of sets.khalaf.A and Ameen.z.[10][11] introduced sc-open sets and sc-continuity in topological space and pc-open sets and pc-continuity in general topology. In this paper we introduce a new class of sets namely $(1,2)^*$-βc-open sets in bitopological spaces and investigate the bitopology generated by its family, then compare this bitopology with other well known bitopological spaces. Throughout this paper (X, τ_1, τ_2) (briefly X) will denote bitopological space.

Definition 1.1; Let S be a subset of a bitopological space X then

(i) The $\tau_{1,2}$-closer of S,denoted by $\tau_{1,2}$-cl(S), is defined as $\cap \{F; S \subseteq F \text{ and } F \text{ is } \tau_{1,2}$-closed$\}$

(ii) The $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S),is defined as $\cup\{F; S \subseteq F \text{ and } F \text{ is } \tau_{1,2}$-open$\}$

Definition 1.2 [2], [3], [4], [5], [6], [7] : Let X be a bitopological space, a nonempty subset A of X is called:

1. $(1,2)^*$-semi-open if A $\subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A)).
2. $(1,2)^*$-pre-open if A $\subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)).
3. $(1,2)^*$-α-open if A $\subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))).
4. $(1,2)^*$-semi-preopen or $(1,2)^*$- β-open if A $\subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))).
5. $(1,2)^*$-b-open if A $\subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\cup \tau_{1,2}$-int($\tau_{1,2}$-cl(A)).
6. $(1,2)^*$-regular-open if A = $\tau_{1,2}$-int(cl(A)).

The complement of $(1,2)^*$-semi-open sets, $(1,2)^*$-α-open sets, $(1,2)^*$-pre-open sets, $(1,2)^*$-semi-preopen sets, $(1,2)^*$-b-open sets and $(1,2)^*$-regular open sets are $(1,2)^*$-semi-closed sets, $(1,2)^*$-α-closed sets, $(1,2)^*$-pre-closed sets, $(1,2)^*$-semi-preclosed sets, $(1,2)^*$-b-closed and $(1,2)^*$-regular closed sets respectively. The family of $(1,2)^*$ -semi-open sets, $(1,2)^*$ -α-open sets, $(1,2)^*$ -pre-open sets, $(1,2)^*$ -semi-preopen sets, $(1,2)^*$ -b-open, $(1,2)^*$- regular open and $(1,2)^*$-regular closed sets are denoted by $(1,2)^*$-SO(X), $(1,2)^*$-αO(X), $(1,2)^*$-PO(X), $(1,2)^*$-SPO(X), $(1,2)^*$- BO(X), $(1,2)^*$-RO(X) and $(1,2)^*$-RC(X) respectively.

Definition 1.3 [10] : A subset A of a space X is called $(1,2)^*$-sc-open if for each x ∈ A ∈ $(1,2)^*$-SO(X), there exists a $(1,2)^*$-closed set F such that x ∈ F ⊆ A. The family of all $(1,2)^*$-sc-open subsets of a bitopological space (X, τ_1, τ_2) is denoted by $(1,2)^*$-SCO(X).

Definition 1.4 [11] : A $(1,2)^*$- pre-open subset of a space X is called $(1,2)^*$-pc-open if for each x ∈ A, there exists a $(1,2)^*$-closed set F such that x ∈ F ⊆ A. The family of all $(1,2)^*$-pc-open subsets of a bitopological space (X, τ_1, τ_2) is denoted by $(1,2)^*$-PCO(X).

Definition 1.5 [7] : Let X be a bitopological space, a nonempty subset A of X is called $(1,2)^*$-bc-open if :

i. A is $(1,2)^*$-b-open and

ii. for each x ∈ A there is a $(1,2)^*$-closed set F such that x ∈ F ⊆ A.

The family of all $(1,2)^*$-bc-open subsets of a bitopological space (X, τ_1, τ_2) is denoted by $(1,2)^*$-BCO(X).

Definition 1.6 [9] : A set A of a bitopological space is called $(1,2)^*$- θ-open if for each x ∈ A there exists an $\tau_{1,2}$-open set G such that, x ∈ G ⊆ $\tau_{1,2}$-cl(G) ⊆ A.

Definition 1.7 [8] : A set A of a bitopological space is called $(1,2)^*$-δ-open if for each x ∈ A there exists an $\tau_{1,2}$-open set G such that, x ∈ G ⊆ $\tau_{1,2}$-int($\tau_{1,2}$-cl(G)) ⊆ A.

Remark : The collection of $(1,2)^*$- θ-open sets in a bitopological space (X, τ_1, τ_2) forms the bitopology on X which is denoted by $\tau_{1,2-\theta}$. Also the family of $(1,2)^*$-δ-open sets in a bitopological space (X, τ_1, τ_2) forms a bitopology $\tau_{1,2-\delta}$- such that $\tau_{1,2-\delta} \subseteq \tau_{1,2}$

Definition 1.8 [9] : A subset A of a space X is θ-$(1,2)^*$-semi-open if for each x ∈A, there exists a $(1,2)^*$-semi-open set G such that x ∈ G ⊆ $\tau_{1,2}$-cl(G) ⊆A.

Definition 1.9 [6], [12] : A bitopological space X is called:

i. A locally indiscrete if and only if every $\tau_{1,2}$-open set is $\tau_{1,2}$-closed.

ii. A regular space if for each x ∈ X and for each $\tau_{1,2}$-open set G containing x, there exist an $\tau_{1,2}$-open set H such that x ∈ H ⊆ $\tau_{1,2}$-cl(H) ⊆G.

iii. An extremely disconnected if the closure of any $\tau_{1,2}$-open set is $\tau_{1,2}$-open.

Definition 1.10 [12] : A bitopological space X is said to be an alexandroff space, if any arbitrary intersection of $\tau_{1,2}$-open sets is $\tau_{1,2}$-open.

Remark : Note that in an alexandroff space, arbitrary union of $\tau_{1,2}$-closed sets is $\tau_{1,2}$-closed.

Lemma 1.11 [5] : Arbitrary union of $(1,2)^*$-β-open sets is $(1,2)^*$-β-open set.

Lemma 1.12 [5] : For every $\tau_{1,2}$-open set G in a bitopological space X and every A ⊆ X we have, $\tau_{1,2}$- cl(A) ∩ G ⊆ $\tau_{1,2}$-cl(A ∩G).

Theorem 1.13 [5] : If V is $\tau_{1,2}$-open and A is $(1,2)^*$-β-open, then V ∩ A is $(1,2)^*$-β -open.

2. $(1,2)^*$-βc-Open Sets

Now we consider a new class of generalized $\tau_{1,2}$-open sets.

Definition 2.1 : Let X be a bitopological space, a nonempty subset A of X is called $(1,2)^*$-βc-open set if :

i. A is $(1,2)^*$-β-open and

ii. for each x ∈A there is a $\tau_{1,2}$-closed set F such that x ∈F ⊆A. The family of all $(1,2)^*$-βc-open sets in X is denoted by $(1,2)^*$-βco(X).

Proposition 2.2 : A subset A in a bitopological space X is $(1,2)^*$-βc-open if and only if A is $(1,2)^*$-β-open and it is a union of $\tau_{1,2}$-closed sets.

Proof : Clear from the definition 2.1.

Remark : Every $(1,2)^*$-βc -open is $(1,2)^*$-β-open but the converse need not be true as in the following example.

Example 2.3 : Let X = {a, b, c} with a topology τ_1 = {∅, X, },$\tau_2 = \{\{a\}, \{b\}, \{a, b\}\}$, then:

i. The family of $\tau_{1,2}$-closed sets is: $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$.

ii. The family of $(1,2)^{*}\beta$-open sets is: $\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$.

iii. The family of $(1,2)^{*}$-βc-open sets is: $\{\emptyset, X, \{a,c\}, \{b,c\}\}$.

iv. Here {a} is $(1,2)^{*}$-β open set but not $(1,2)^{*}$-βc-open set.

Remark : The $(1,2)^{*}$-βc-open set need not be a $\tau_{1,2}$-closed set as in the following example.

Example 2.4 :

Let x={a,b,c,d}with bi-topology $\tau_1 = \{\emptyset, X, \{a,d\}, \{c,d\}, \{a,c,d\}\}$,

$\tau_2 = \{\emptyset, X, \{d\}, \{a,b\}, \{a,b,d\}$ then

$\tau_{1,2} - o(x) = \{\emptyset, X, \{d\}, \{a,b\}, \{a,d\}, \{c,d\}, \{a,c,d\}, \{a,b,d\}\}$

$\tau_{1,2} - c(x) = \{\emptyset, X, \{d\}, \{a,b,c\}, \{c,d\}, \{b,c\}, \{a,b\},, \{c\}\}$ and

$(1,2)^{*}$-$\beta o(x) = \{\emptyset, X, \{a\}, \{d\}, \{a,b\}, \{a,d\}, \{b,d\},, \{c,d\}, [a,b,d\}, \{b.c.d\}, \{a,c,d\}\}$

$(1,2)^{*}$-$\beta co(x) = \{\emptyset, X, \{c,d\}, \{b.c.d\}, \{a,b\}, \{b,d\}\}$

Clearly , the set {b,c,d} is a $(1,2)^{*}$-$\beta co(x)$ but it is not a $\tau_{1,2}$-closed set in X.

Proposition 2.5 : Arbitrary union of $(1,2)^{*}$-βc-open sets is $(1,2)^{*}$-βc-open set.

Proof : First, Let $\{A_\alpha : \alpha \in \Delta\}$ be a family of $(1,2)^{*}$-βc-open sets in a bi-topological space X, then A_α is $(1,2)^{*}$-β-open set for each $\alpha \in \Delta$ so by Lemma 1.12, $\bigcup_{\alpha\in\Delta} A_\alpha$ is $(1,2)^{*}$-β-open set. Second, let $x \in \bigcup_{\alpha\in\Delta} A_\alpha$, then there exists $\gamma \in \Delta$ such that $\in A_\gamma$. Since A_γ is $(1,2)^{*}$-βc-open set, there exists a $\tau_{1,2}$-closed set F such that x ∈ F $\subseteq A_\gamma \subseteq \bigcup_{\alpha\in\Delta} A_\alpha$ so, x ∈ F $\subseteq \bigcup_{\alpha\in\Delta} A_\alpha$ and $\bigcup_{\alpha\in\Delta} A_\alpha$ is $(1,2)^{*}$-βc-open set.

Proposition 2.6 : A subset A in a bi-topological space X is $(1,2)^{*}$-βcopen if and only if for each x ∈ A, there exists $(1,2)^{*}$-βc-open set B such that x ∈ B ⊆ A.

Proof : Clearly by using definition 2.1 and proposition 2.5.

Proposition 2.7: If the bi-topological space X is locally indiscrete, then every $(1,2)^{*}$-semi-open set is $(1,2)^{*}$-βc-open.

Proof: Let A be a $(1,2)^{*}$ -semi-open, then $A \subseteq \tau_{1,2} - cl(\tau_{1,2} - int(A)) \subseteq \tau_{1,2} - cl(\tau_{1,2} - int(\tau_{1,2} - cl(A)))$ so A is $(1,2)^{*}$-β-open. Since X is locally indiscrete, then $\tau_{1,2}$ -int(A) is closed and $A \subseteq \tau_{1,2} - cl(\tau_{1,2} - int(A)) = \tau_{1,2} - int(A)$ which implies, A is $\tau_{1,2}$-open set and for any x ∈ A, x ∈$\tau_{1,2}$- int (A) ⊆ A. Hence, A is $(1,2)^{*}$-βc −open.

Remark : Since every $\tau_{1,2}$-open set is $(1,2)^{*}$-semi-open, then by proposition 2.7, in a locally indiscrete space, every $\tau_{1,2}$-open set is $(1,2)^{*}$-βc-open set.

Proposition 2.8 : Let X be a bi-topological space, if X is regular space, then every $\tau_{1,2}$-open set is a $(1,2)^{*}$-βc-open set in X.

Proof : Let A be $\tau_{1,2}$-open, then A is $(1,2)^*$-β-open. Since X is regular, then for each $x \in A$, there exists an $\tau_{1,2}$ -open set G such that $x \in G \subseteq \tau_{1,2} - cl(G) \subseteq A$. So that, $x \in \tau_{1,2} - cl(G) \subseteq A$ and therefore, A is $(1,2)^*$-βc-open set.

Proposition 2.9 : Let X be an extremely disconnected bi-topological space and let A and B be subsets of X. If A is a $(1,2)^*$-βc-open set and B is $(1,2)^*$-regular open, then $A \cap B$ is a $(1,2)^*$-βc-open set.

Proof : Let X be an extremely disconnected bi-topological space and let A and B be subsets of X. If A is a $(1,2)^*$-βc-open set and B is $(1,2)^*$-regular open, then $A \cap B$ is a $(1,2)^*$-β-open. If $x \in A \cap B$ then, there exists a $\tau_{1,2}$-closed set F such that $x \in F \subseteq A$ and so, $x \in F \cap B \subseteq A \cap B$, but B is $(1,2)^*$regular open in an extremely disconnected space so, B is $\tau_{1,2}$-closed and hence $F \cap B$ is $\tau_{1,2}$-closed set. Therefore, $A \cap B$ is a $(1,2)^*$-βc-open set.

Corollary 2.10 : Let X be an extremely disconnected bi-topological space and let B be a $(1,2)^*$ −regular open subset of X, then B is a $(1,2)^*$-βc-open set.

Proof : Using proposition 2.9, if we let $A = X$ and B is a $(1,2)^*$-regular open set, then $A \cap B = B$ is a $(1,2)^*$-βc-open set.

Proposition 2.11 : Let X be an extremely disconnected topological space and let A be a $(1,2)^*$ −δ-open subset of X, then A is a $(1,2)^*$ −βc-open set.

Proof : Let X be an extremely disconnected bi-topological space and let A be a nonempty $(1,2)^*$-δ-open subset of X, then for any $x \in A$ there exists an $\tau_{1,2}$ -open set G_x such that $x \in G_x \subseteq \tau_{1,2} - int(\tau_{1,2} - cl(G_x) \subseteq A$ which implies $\bigcup_{\propto \in \triangle} G_x$= A is an $\tau_{1,2}$-open set and so a $(1,2)^*$ - β -open, but X is extremely disconnected so that, $\tau_{1,2} - int(\tau_{1,2} - cl(G_x)) = \tau_{1,2} - cl(G_x)$ and $x \in \tau_{1,2} - cl(G_x) \subseteq A$. Therefore, A is a $(1,2)^*$-βc-open set.

Proposition 2.12 : Let X be a bi-topological space with a subset A. If A is a $\theta - (1,2)^*$-semi open set then, A is a $(1,2)^*$-βc-open set.

Proof : Let A be a $\theta - (1,2)^*$-semi open set then, for each $x \in A$ there exists a $(1,2)^*$-semi open set $G_x$such that $x \in G_x \subseteq \tau_{1,2} - cl(G_x) \subseteq A$ which implies, $A = \bigcup_{\propto \in \triangle} G_x$ which means A is a union of $(1,2)^*$-semi open sets and therefore, A is $(1,2)^*$-semi open and so $(1,2)^*$-β-open set.

Also,$A = \bigcup_{\propto \in \triangle} \tau_{1,2} - cl(G_x)$ which is a union of $\tau_{1,2}$-closed sets. Therefore, A is a $(1,2)^*$-βc-open set.

Corollary 2.13 : Let X be a bi-topological space with a subset A. If A is a $\theta - (1,2)^*$open set, then A is a $(1,2)^*$-βc-open set.

Proof : Since every $\theta - (1,2)^*$-open set is $\theta \dot{-} (1,2)^*$-semi open set, then by proposition 2.12 we get the result.

Remark :If $(1,2)^*$-semi-open sets ⇒ $(1,2)^*$-b-open sets ⇒ $(1,2)^*$-β-open sets, then $(1,2)^*$-sc-open sets ⇒ $(1,2)^*$-bc-open sets ⇒ $(1,2)^*$-βc-open sets.

Remark :If $(1,2)^*$-pre-open sets ⇒ $(1,2)^*$-b-open sets ⇒ $(1,2)^*$-β-open sets, then $(1,2)^*$-pc-open sets ⇒ $(1,2)^*$-bc-open sets ⇒ $(1,2)^*$-βc-open sets.

Proposition 2.14 : Let (X, τ_1, τ_2) be a topological space, then $(1,2)^*$-SCO(X) ∪$(1,2)^*$-PCO(X)⊆ $(1,2)^*$-BCO(X) ⊆$(1,2)^*$-βCO(X)

Proposition 2.15 : Let (X, τ_1, τ_2) be alexandroff space, then $(1,2)^*$-SCO(X) = $(1,2)^*$-BCO(X) = $(1,2)^*$-βCO(X).

Proof : By proposition 2.14, $(1,2)^*$-SCO(X) ⊆ $(1,2)^*$-BCO(X) ⊆ $(1,2)^*$-βCO(X). If A ∈ $(1,2)^*$-βCO(X), then A is a union of $\tau_{1,2}$-closed sets in alexandroff space which implies, A is $\tau_{1,2}$-closed. But A ⊆ $\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))) = $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) and so that, A is $(1,2)^*$-semi-open and hence, A ∈ $(1,2)^*$-SCO(X). Therefore, $(1,2)^*$-SCO(X) ⊆ $(1,2)^*$-BCO(X) ⊆ $(1,2)^*$-βCO(X) ⊆ $(1,2)^*$-SCO(X) and so, $(1,2)^*$-SCO(X) = $(1,2)^*$-BCO(X)= $(1,2)^*$-βCO(X).

3. Generated by $(1,2)^*$-βc-Open Sets

Although none of $(1,2)^*$-SO(X), $(1,2)^*$-PO(X), $(1,2)^*$-BO(X) and $(1,2)^*$-βO(X) is a bi-topology on X, each of these classes generates a bi-topology in a natural way. Njastad showed that $\tau_{12-\alpha}$ was the bi-topology generated by $(1,2)^*$-SO(X) and Andrijevic showed that $\tau_{12-\gamma}$ and $\tau_{12-\beta}$ were the bi-topologies generated by $(1,2)^*$-PO(X) and $(1,2)^*$-BO(X) respectively and moreover he proved that $\tau_{12-\gamma}$ = $\tau_{12-\beta}$ = $\tau_{1,2-spo}$ which generated by $(1,2)^*$-SPO(X) or $(1,2)^*$-βO(X). In this section we shall study the bi-topology generated by the class $(1,2)^*$-βCO(X) and we denoted it by $\tau_{1,2-\beta c}$.

Definition 3.1 : A subset A of a bi-topological space (X, τ_1, τ_2) is called a $(1,2)^*$-βc -set if A ∩B ∈$(1,2)^*$-β CO(X) for all B ∈$(1,2)^*$-β CO(X).

The class of all $(1,2)^*$-βc sets in (X, τ_1, τ_2) will be denoted by $\tau_{1,2-\beta c}$.

Theorem 3.2 : $\tau_{1,2-\beta c}$. is a bi-topology on X.

Proof :

(1) Clearly ∅ and X ∈$\tau_{1,2-\beta c}$

(2) Let $\{\bigcup_{\alpha\in\triangle} A_\alpha \in \Delta\} \subseteq \tau_{\beta c}$. Then A_α∩B ∈$(1,2)^*$-βCO(X) for all B ∈$(1,2)^*$-βCO(X). Therefore, $(\bigcup_{\alpha\in\triangle} A_\alpha \in)$∩B = $_{\alpha}$ $_{\triangle}\{A_\alpha \cap$ B} where A_α∩B is $(1,2)^*$-βc-open set and since arbitrary union of $(1,2)^*$-βc-open sets is $(1,2)^*$-βc-open, it follows that, $\bigcup_{\alpha\in\triangle} A_\alpha$ $_{\triangle}\{A_\alpha$∩B} is a $(1,2)^*$-βc-open set for each B∈$(1,2)^*$-βCO(X) and hence, $(\bigcup_{\alpha\in\triangle} A_\alpha)$∩B is $(1,2)^*$-βc-open set. Therefore, $\bigcup_{\alpha\in\triangle} A_\alpha \in \tau_{1,2-\beta c}$.

(3) Let C and D ∈$\tau_{1,2-\beta c}$. Then (C ∩ D) ∩ B = C ∩ (D ∩ B) ∈$(1,2)^*$-βCO(X) for all B ∈$(1,2)^*$-βCO(X)

and hence $C\cap D \in \tau_{1,2-\beta c}$. By (1), (2) and (3) we have $\tau_{1,2-\beta c^{\cdot}}$ is a bi-topology on X.

Theorem 3.3 : $\tau_{1,2-\beta c^{\cdot}} \subseteq (1,2)^{*}$-$\beta CO(X,\tau_1,\tau_2)$ for any $\tau_{1,2}$.

Proof : Let $A \in \tau_{1,2-\beta c^{\cdot}}$ then for any $(1,2)^{*}$-βc-open set B in X we have, $A\cap B$ is a $(1,2)^{*}$-βc-open set. In particular, if we let B = X, then $A\cap X = A$ is $(1,2)^{*}$-βc-open set and $A \in (1,2)^{*}$-$\beta CO(X,\tau_1,\tau_2)$. Hence, $\tau_{1,2-\beta c^{\cdot}} \subseteq (1,2)^{*}$-$\beta CO(X,\tau_1,\tau_2)$

Remark : Note that $(1,2)^{*}$-$\beta CO(X,\tau_1,\tau_2) \subseteq \tau_{1,2-\beta c^{\cdot}}$ is not true in general.

Consider the following example.

Example 3.4 : Let X = {a, b, c} with the bi-topology τ_{12}= {∅, X, {a}, {b}, {a, b}}, then. $(1,2)^{*}$-β CO (X,τ_1,τ_2) = {∅, X, {c,d}, {b, d},{b,c,d},{a,b}}here A={a,b} and B={b,d} $\tau_{12-\beta c} = \{\emptyset, X, \{b\}, [d\}, \{b. d\}\}$ here $\{c,d\} \in (1,2)^{*}$ - $\beta co(x)$but $\{c,d\} \notin \tau_{12-\beta c}$ because $\{c,d\} \cap \{b,d\}\} = \{d\} \notin (1,2)^{*}$-$\beta co(X,\tau_1,\tau_2)$

Theorem 3.5 : A subset A of a bi-topological space (X,τ_1,τ_2) is closed in (X, $\tau_{1,2-\beta c}$) if and only if A $\cup$ B is $(1,2)^{*}$-βc -closed for any $(1,2)^{*}$-βc -closed set B in X.

Proof : A subset A is closed in (X, $\tau_{1,2-\beta c}$) if and only if A^c is $\tau_{1,2}$ −open in (X, $\tau_{1,2-\beta c}$) if and only if $A^c \cap C$ is $(1,2)^{*}$-βc-open for any $(1,2)^{*}$-βc-open set C if and only if ($A^c \cap C)^C = A \cup C^c$ is $(1,2)^{*}$-βc-closed for any $(1,2)^{*}$-βc-closed set C^c that is, $A \cup B$ is $(1,2)^{*}$-βc-closed for any $(1,2)^{*}$-βc-closed set B in X.

Theorem 3.6 : Let (X,τ_1,τ_2) be a locally indiscrete space, then $\tau_{1,2} \subseteq \tau_{1,2-\beta c}$

Proof : Let (X,τ_1,τ_2) be a locally indiscrete bi-topological space and consider $A\in\tau_{1,2}$, then by Theorem 1.12, for any $(1,2)^{*}$-βc-open set B in X we have, $A\cap B$ is $(1,2)^{*}$-β -open set. But B is $(1,2)^{*}$-βc-open so that, B = $\bigcup_{\alpha\in\triangle} A_\alpha F_\alpha$ where F_α is a $\tau_{1,2}$ −closed set for each $\alpha\in\Delta$ which implies, $A\cap B$ = $\bigcup_{\alpha\in\triangle} A_\alpha (A \cap F_\alpha)$ which union of $\tau_{1,2}$ -closed sets because A is $\tau_{1,2}$-closed in a locally indiscrete space and hence, $A\cap B$ is $(1,2)^{*}$-βc -open set for any $(1,2)^{*}$-βc -open set B in X. So that, $A\in\tau_{1,2-\beta c}$ and $\tau_{1,2}\subseteq\tau_{1,2-\beta c}$

Theorem 3.7 : Let (X,τ_1,τ_2) be T_1 space, then $\tau_{1,2} \subseteq \tau_{1,2-\beta c}$

Proof : Let X be a T_1 topological space and consider $A \in\tau_{1,2}$, then for any $(1,2)^{*}$-βc-open set B in X we have, $A\cap B$ is β-open set. But $A\cap B = \bigcup_{\alpha\in\triangle} A_\alpha$ $_{x\in A\cap B}$ {x} where {x} is a $\tau_{1,2}$ −closed set in T_1 space. So that, $A\cap B$ is $(1,2)^{*}$-βc -open set for any $(1,2)^{*}$-βc -open set B in X and so, $A \in\tau_{1,2-\beta c}$. Hence, $\tau_{1,2} \subseteq \tau_{1,2-\beta c}$ for any T_1 space.

Theorem 3.8 Let (X,τ_1,τ_2) be a regular bitopological space. Then, $\tau_{1,2} \subseteq \tau_{1,2-\beta c}$.

Proof : Let X be a regular space and let $A\in\tau_{1,2}$, then for any $(1,2)^*$-βc -open set B in X we have, $A\cap B$ is $(1,2)^*$-β open set. Since X is regular, there is an $\tau_{1,2}$-open set G_x such that $x\in G_x\subseteq\tau_{1,2}$-cl($G_x$) ⊆A. Since B is $(1,2)^*$-βc-open set, there is a $\tau_{1,2}$-closed set F_x such that $x\in F_x\subseteq A$. So that, for any $x\in A\cap B$ we have, $x\in\tau_{1,2}$-cl($G_x\cap F_x\subseteq A\cap B$ where $\tau_{1,2}$-cl(G_x)$\cap F_x$ is a $\tau_{1,2}$-closed set. Hence, $A\cap B$ is $(1,2)^*$-βc-open set for any $(1,2)^*$-βc-open set B in X. So, $A\in\tau_{1,2-\beta c}$ and therefore, $\tau_{1,2}\subseteq\tau_{1,2-\beta c}$

Theorem 3.9 : For a space (X,τ_1,τ_2) and $x\in X$ the following are equivalent:

(1) $\{x\}\in(1,2)^*$-βCO(X).

(2) $\{x\}\in\tau_{1,2-\beta c}$.

Proof : ($1\Rightarrow 2$) Let $\{x\}\in(1,2)^*$-βCO(X), then for any $(1,2)^*$-βc-open set B in X we have, either $\{x\}\cap B=\emptyset$which is $(1,2)^*$-βc-open set or $\{x\}\cap B=\{x\}$ which is a $(1,2)^*$-βc-open set. Hence, in both cases, $\{x\}\cap B\in(1,2)^*$-βCO(X) and therefore, $\{x\}\in\tau_{1,2-\beta c}$.

($2\Rightarrow 1$) Let $\{x\}\in\tau_{1,2-\beta c}$ then by theorem 3.3, $\tau_{1,2-\beta c}\subseteq(1,2)^*$-$\beta$CO(X) and hence, $\{x\}\in(1,2)^*$-βCO(X).

Theorem 3.10 : Let (X,τ_1,τ_2) be an alexandroff space. Then, the following are equivalent:

(1) A is a $\tau_{1,2}$ −clopen set in (X,τ_1,τ_2)

(2) $A\in\tau_{1,2-\beta c}$.

Proof : ($1\Rightarrow 2$) Let A be a $\tau_{1,2}$ −clopen set in (X,τ_1,τ_2). Then, for any $(1,2)^*$-βc-open set B in X we have, $A\cap B$ is $(1,2)^*$-β-open and for any $x\in A\cap B$ there is a $\tau_{1,2}$-closed set $F\subseteq B$ such that $x\in A\cap F\subseteq A\cap B$ where $A\cap F$ is a $\tau_{1,2}$-closed set. Hence, $A\cap B$ is $(1,2)^*$-βc-open and so, $A\in\tau_{1,2-\beta c}$.

($2\Rightarrow 1$) Let $A\in\tau_{1,2-\beta c}$, then by theorem 3.3, $A\in(1,2)^*$-βCO(X) and so, A= $\bigcup_{\alpha\in\triangle}A_\alpha F_\alpha$ where F_α is $\tau_{1,2}$closed for each $\alpha\in\Delta$. Since X is alexandroff space, A is $\tau_{1,2}$ −closed but A is $(1,2)^*$-β-open and if A is not $\tau_{1,2}$ −open then, $A\subseteq\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))) = $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) ⊆ $\tau_{1,2}$-cl(A) =A which implies, A f= A which a contradiction. So, A must be an open set and therefore, A is a $\tau_{1,2}$-clopen set.

Corollary 3.11 : In an alexandroff space (X,τ_1,τ_2) $\tau_{1,2-\beta c}\subseteq\tau_{1,2}$

Proof : Let $A\in\tau_{1,2-\beta c}$ then by theorem 3.10, A is a $\tau_{1,2}$-clopen set in X and hence, $A\in\tau_{1,2}$. Hence, $\tau_{1,2-\beta c}\subseteq\tau_{1,2}$.

Theorem 3.12 : Let (X,τ_1,τ_2) be a bi-topological space, then $(1,2)^*$RC(X) $\subseteq\tau_{1,2-\beta c}$

Proof : Let A be a regular $\tau_{1,2}$-closed set in (X,τ_1,τ_2), then for any $(1,2)^*$-βc-open set B we have, $A\cap B\subseteq A\cap\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(B)))⊆ $\tau_{1,2}$-cl($A\cap\tau_{1,2}$-int($\tau_{1,2}$-cl(B)))⊆ $\tau_{1,2}$-cl($\tau_{1,2}$- cl($\tau_{1,2}$-int(A))$\cap\tau_{1,2}$-int($\tau_{1,2}$-cl(B)))⊆$\tau_{1,2}$-cl($\tau_{1,2}$-int(A)$\cap\tau_{1,2}$-int($\tau_{1,2}$-cl(B)))⊆ $\tau_{1,2}$-cl($\tau_{1,2}$- int($A\cap\tau_{1,2}$-cl(B))) ⊆$\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl($A\cap B$))). So that, $A\cap B$ is $(1,2)^*$-β-open and for any $x\in A\cap B$ there is a

$\tau_{1,2}$-closed set F ⊆B such that x ∈ A∩F⊆A∩B where A ∩F is $\tau_{1,2}$- closed because A is $\tau_{1,2}$-closed. Hence, A ∩B is $(1,2)^{*}$-βc-open set for any $(1,2)^{*}$-βc-open set B in X and therefore, A ∈ $\tau_{1,2-\beta c}$.

Theorem 3.13 : Let (X, τ_1, τ_2) be an extremely disconnected bi-topological space. Then,$\tau_{1,2-\delta} \subseteq \tau_{1,2-\beta c}$.

Proof : Let X be an extremely disconnected space and A∈$\tau_{1,2-\delta}$, then A is $\tau_{1,2}$-open and for any $(1,2)^{*}$-βc-open set B in X we have, A∩B is $(1,2)^{*}$-β-open. If x ∈ A∩B then, since X is extremely disconnected, there is an $\tau_{1,2}$-open set G such that x ∈ G⊆$\tau_{1,2}$int($\tau_{1,2}$-cl(G))⊆A and since B is $(1,2)^{*}$-βc-open, there is a $\tau_{1,2}$-closed set F such that x ∈F⊆B which implies, x∈$\tau_{1,2}$-int($\tau_{1,2}$-cl(G))∩F⊆A∩B where $\tau_{1,2}$-cl(G) is $\tau_{1,2}$-open in extremely disconnected space and so $\tau_{1,2}$-int($\tau_{1,2}$-cl(G))∩F = $\tau_{1,2}$-cl(G) ∩F which is a $\tau_{1,2}$-closed set. So that, A∩B is $(1,2)^{*}$-βc-open set for any $(1,2)^{*}$-βc-open set B in X and so, A∈ $\tau_{1,2-\beta c}$. Therefore, $\tau_{1,2-\delta} \subseteq \tau_{1,2-\beta c}$.

Theorem 3.14: Let (X, τ_1, τ_2) be an extremely disconnected alexandroff space,

then $\tau_{1,2-\delta} = \tau_{1,2-\beta c}$

Proof : Let X be an alexandroff space such that A∈$\tau_{1,2-\beta c}$, then A is a $\tau_{1,2}$ −clopen set and so for any x ∈ A we have, x ∈ A⊆$\tau_{1,2}$-int($\tau_{1,2}$-cl(A))⊆A which implies that A is $(1,2)^{*}$- δ -open and hence, A ∈$\tau_{1,2-\delta}$. So that, $\tau_{1,2-\beta c} \subseteq \tau_{1,2-\delta}$. But $\tau_{1,2-\delta} \subseteq \tau_{1,2-\beta c}$

Therefore, $\tau_{1,2-\delta} = \tau_{1,2-\beta c}$.

Conclusion : Its importance to significant in various area of Mathematics and related Sciences. In this paper we studied the concept of (1,2)*Bc- Open Sets in bitopological spaces and some of the properties of (1,2)*Bc-Open sets are discussed. This shall be extended in the future research with some applications.

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