Strong Domination Polynomials of Queen Crown Graph

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Abstract ----Let G=(V,E) be a simple graph. A set S ⊆ V is called a dominating set if every vertex v ∈V is either a member of S or adjacent to a member of S. A set S ⊆ V is a Strong dominating set of G if for every vertex v ∈ V-S there exists a u ∈ S such that uv ∈ E and deg(u) ≥ deg(v). Let $\mathbf{Q_{m,n}}$ be a Queen crown graph which is obtained from two null graphs of order zero and taking one copy of null graph G_1 with m vetices , m ≥3 and another copy of null graph G_2 with n=2 vertices (that should be fixed) then joining the vertex of G_1 with an edge to every vertex of G_2 . Let Sd$(\mathbf{Q^j_{m,n}})$ be the family of strong dominating set of Queen crown graph with number of elements in the set j and let Sd$(\mathbf{Q_{m,n}}, j)$ =|**Sd**$(\mathbf{Q^j_{m,n}})$|.In this paper we establish $\mathbf{Q_{m,n}}$ and obtain a iterative formula for Sd$(\mathbf{Q^j_{m,n}})$ .Using this iterative formula we consider the polynomial for SD $(\mathbf{Q_{m,n}}, \mathbf{x})$= $\sum_{j=1}^{m+n-1}\left[\binom{m+n-1}{j}-\binom{m+n-2}{j}\right]x^{j+1}$ Also we have determine several properties of polynomials on Queen crown graphs.

Keywords — *Strong dominating set , Strong domination polynomial , Queen crown graph*

I. INTRODUCTION

Let G = (V, E) be a bipartite graph of order |V| = m+n A set S ⊆V $is called a dominating set if every vertex v ∈ V is either a member of S or adjacent to a member of S. A set S ⊆ V is a Strong dominating set of G if for every v ∈ V – S , there u∈ S such that uv ∈ E and deg (u) ≥ deg (v). The minimum cardinality of Strong dominating set is called minimum Strong domination number and is denoted by $\gamma_{sd}(G)$. Alkhani and Peng[1][2] found the dominating sets and domination polynomial of cycles and certain graphs.Gehet, Khalf and Hasni found the dominating set and domination polynomial of stars and wheels[3][4]..Angelin and Robinson found the weak dominating sets and weak domination polynoimal of complete graphs [5].Let H_m be a graph with order m and let H^j_m be the family of dominating sets of a graph H_m with the number of elements in the set j and let d (H_m, j) =$|H_j|$.We call the polynomial D$(H_m$,x)= $\sum_{j=\gamma(G)}^{n} \mathbf{d(H_m, j)x^j}$ the domination polynomial of graph G[2].Let $\mathbf{Q_{m,n}}$ be a Queen crown graph which is obtained from two null graphs of order zero and taking one copy of null graph G_1 with m vetices , $m \geq 3$ and another copy of null graph G_2 with n=2 vertices (that should be fixed) then joining the vertex of $G_1$with an edge to every vertex of G_2.Let Sd$(Q^j_{m,n})$ be the family of strong dominating set of Queen crown graph with number of elements in the set j and let Sd$(\mathbf{Q_{m,n}}, j)$=|Sd($Q^j_{m,n}$)|. We call the polynomial *SD* $(\mathbf{Q_{m,n}}, x)$= $\sum_{j=1}^{m+n-1}\left[\binom{m+n-1}{j}-\binom{m+n-2}{j}\right]x^{j+1}$ the strong domination polynomial of Queen crown graph. In the next section we establish the families of strong dominating sets of $\mathbf{Q_{m,n}}$ with the number of elements in the set j by the families of strong dominating sets of $Q_{m-1,n}$ with number of elements j and j-1. We explore the strong domination polynomial of Queen crown graphs in section 3.As usual we use $\binom{n}{i}$or nC_i for the combination n to i and we denote the set {1, 2, …, n} simply by [n], and we denote deg (u) to degree of the vertex u and let

$\Delta(G) = \max\{\deg(u) : \forall u \in V(G)\}$ and

$\delta(G) = \min\{\deg(u) : \forall u \in V(G)\}$

II. STRONG DOMINATING SETS OF QUEEN CROWN GRAPH

Let $Q_{m,n}$, m ≥ 3$ and n=2 be the Queen Crown graph with (m+2) vertices, V[$Q_{m,n}$]=[m+n] and E[$Q_{m,n}$]={(u,v) : forall u ∈ G_1 and v ∈G_2}.Let $(Q^j_{m,n})$ be the family of strong dominating sets of $Q_{m,n}$ with the number of elements 'j'. We shall explore the strong dominatings sets of Queen crown graph :.

Lemma 2.1.

The following properties hold for all Queen crown graph $G_q = Q_{m,n}$, where q = m+n.

(i) $|G_q^q|=1$

(ii) $|G_q^{q-1}|=q-2$

(iii) $|G_q^r|=0$ if r > q

(iv) $|G_q^0|=0$

Proof

Let $G = (V, E)$ be a Queen crown graph of order $m + n$.Then,

(i) $G_q^q=\{G\}$

$\therefore$ $|G_q^q|=1$

(ii) $G_q^{q-1} = \{u : u \in G_1\}$

Since in $Q_{m,n}$ n is fixed and the set $\{a, b\}$ in G_2 has a degree m where $m \geq 3$, the set {a, b} is a minimal strong dominating set for $Q_{m,n}$ forall $m \geq 3$

(iii) We can't find any subset K of G such that $|V(K)| > |V(G)|$

$\therefore |G_q^r| = 0$ if r > q

(iv) We can't find any subset K of G such that $|V(K)| = 0$, φ is not a strong dominating set of G.

$\therefore |G_q^0|=0$

Theorem-2.2

Let $Q_{m,n}$ be a Queen crown graph with order (m+n) then $Sd(Q_{m,n}, j+1) = \left[\binom{m+n-1}{j} - \binom{m+n-2}{j}\right]$ forall $(m+n) \in Z^+$ and j=1,2,…,(m+n-1)

Proof

Let $Q_{m,n}$ be a Queen crown graph with order $(m + n)$

$\Rightarrow$ there are two null graphs G_1 of order $m \geq 3$ and G_2 of order n where n=2 is fixed. Let ' a ' and ' b ' be two vertices of G_2.

W.K.T., Both the vertices ' a ' and ' b ' are adjacent to exactly ' m ' vertices in $Q_{m,n}$.

$\Rightarrow$ deg(a) = deg(b) = m, $m \geq 3$

$\Rightarrow$ every strong dominating set must contain both verices ' a ' and ' b '. Also, every vertices of G_1 has exactly degree 2.

Hence $\deg(u) < \deg(a) = \deg(b)$ $\forall$ $u \in G_1$

$\Rightarrow$ Every vertex subset of $Q_{m,n}$ that contains both ' a ' and ' b ' with number of elements of the set j+1 where j = 1, 2, . . . , (m + n − 1) is a strong dominating set of $Q_{m,n}$

$\therefore$ $Sd(Q_{m,n}, j+1) = \left[\binom{m+n-1}{j} - \binom{m+n-2}{j}\right]$ forall $(m+n) \in Z^+$ and j=1,2,…,(m+n-1)

Theorem-2.3

Let $Q_{m,n}$ be a Queen crown graph with order (m + n) then $Sd(Q_{m,n}, j + 1) = Sd(Q_{m-1,n}, j + 1) + Sd(Q_{m-1,n}, j)$.

Proof

W.K.T $Sd(Q_{m,n}, j+1) = \left[\binom{m+n-1}{j} - \binom{m+n-2}{j}\right]$

Similarly $Sd(Q_{m-1,n}, j+1) = \left[\binom{m+n-2}{j} - \binom{m+n-3}{j}\right]$ and $Sd(Q_{m-1,n}, j) = \left[\binom{m+n-2}{j-1} - \binom{m+n-3}{j-1}\right]$

Now,

$$Sd(Q_{m-1,n}, j+1) + Sd(Q_{m-1,n}, j) = \left[\binom{m+n-2}{j} - \binom{m+n-3}{j}\right] + \left[\binom{m+n-2}{j-1} - \binom{m+n-3}{j-1}\right]$$

$$= \left[\binom{m+n-2}{j} + \binom{m+n-2}{j-1}\right] - \left[\binom{m+n-3}{j} + \binom{m+n-3}{j-1}\right]$$

$$= \left[\binom{m+n-1}{j} - \binom{m+n-2}{j}\right] \left[Since\ \binom{m}{j} = \binom{m-1}{j} + \binom{m-1}{j-1}\right]$$

$$= Sd(Q_{m,n}, j+1)$$

Theorem-2.4

The following characteristics hold for co-efficient $Sd(Q_{m,n}, x)\ \forall\ (m+n) \in Z^+$

(i) $Sd(Q_{m,n}, 2) = 1$

(ii) $Sd(Q_{m,n}, 3) = m$

(iii) $\gamma_{sd}(Q_{m,n}) = 2$

(iv) $Sd(Q_{m,n}, m+1) = m$

(v) $Sd(Q_{m,n}, m+n) = 1$

Proof

(i)

$$Sd(Q_{m,n}, 2) = \left[\binom{m+n-1}{1} - \binom{m+n-2}{1}\right]$$

$= (m + n - 1) - (m + n - 2)$
$= m + n - 1 - m - n + 2$
$= 1$

(ii)

$$Sd(Q_{m,n}, 3) = \left[\binom{m+n-1}{2} - \binom{m+n-2}{2}\right]$$

$= \frac{(m+n-1)(m+n-2)}{2} - \frac{(m+n-2)(m+n-3)}{2}$
$= m + n - 2$

Since n=2 in $Q_{m,n}$, $Sd(Q_{m,n}, 3) = m$

(iii) W.K.T the two vertices namely 'a ' and 'b' in G_2 has degree m. Also,all the vertices in G_1 has adjacent to only these vertices a and b in $Q_{m,n}$

$\Rightarrow deg(u) = 2$ forall $u \in G_1$

And

$deg(a) = deg(b) = m$, $m \geq 3$

Hence the set {a,b} is a minimal strong dominating set of $Q_{m,n}$.

$\gamma_{sd}(Q_{m,n}) = 2$

(iv)

$$Sd(Q_{m,n}, m+1) = \left[\binom{m+n-1}{m} - \binom{m+n-2}{m}\right]$$

$$= \frac{(m+n-1)!}{m!(m+n-1-m)!} - \frac{(m+n-2)!}{m!(m+n-2-m)!}$$

$$= \frac{(m+1)!}{m!} - \frac{(m)!}{m!} \quad \text{[Put n=2]}$$

$$= m$$

(v) The only set in $Q_{m,n}$ of order m+n is itself

Hence $Sd(Q_{m,n}, m+n) = 1$

Using theorem 2.3 and 2.4, We obtain the coefficient of $SD(Q_{m,n}.x)$ for $5 \leq m+n \leq 20$ in Table1 .Let $SD(Q_{m,n}, x) = |Q^j_{m,n}|$

III. STRONG DOMINATION POLYNOMIALS OF QUEEN CROWN GRAPH

In this section we introduce and establish the Strong domination polynomial of Queen crown graph.

Let $Q^j_{m,n}$ be the family of strong dominating sets of a Queen crown $Q_{m,n}$ with cardinality j and let $Sd(Q_{m,n},j) = |Q^j_{m,n}|$ and since $\gamma_{sd}(Q_{m,n}) = 2$. Then the strong domination polynomial $SD(Q_{m,n},x)$ of $Q_{m,n}$ is defined as $SD(Q_{m,n},x) = {}^{m+n-1}\sum_{j=1} Sd(Q_{m,n},j+1)\, x^{j+1}$

Theorem 3.1

The following characteristics hold for all $SD(Q_{m,n},x)$ forall $m \geq 3$

(i) $SD(Q_{m,n},x)=SD(Q_{m-1,n},x)+xSD(Q_{m-1,n},x)$

(ii) $SD(Q_{m,n},x)= \sum_{j=1}^{m+n-1}\left[\binom{m+n-1}{j} - \binom{m+n-2}{j}\right] x^{j+1}$

Proof

(i) W.K.T $SD(Q_{m,n},x) = {}^{m+n-1}\sum_{j=1} Sd(Q_{m,n},j+1)\, x^{j+1}$

Now,

$$SD(Q_{m,n},x) = \sum_{j=1}^{m+n-1} [Sd(Q_{m-1,n}, j+1) + Sd(Q_{m-1,n}, j)]\, x^{j+1}$$

$$= \sum_{j=1}^{m+n-1} [Sd(Q_{m-1,n}, j+1)]x^{j+1} + \sum_{j=1}^{m+n-1} [Sd(Q_{m-1,n}, j)]\, x^{j+1}$$

$$= \sum_{j=1}^{m+n-1} [Sd(Q_{m-1,n}, j+1)]x^{j+1} + x \sum_{j=2}^{m+n-1} [Sd(Q_{m-1,n}, j)]x^{j}$$

$$= SD(Q_{m-1,n}\, x) + x\, SD(Q_{m-1,n}, x)$$

(ii)

$SD(Q_{m,n},x) = {}^{m+n-1}\sum_{j=1} Sd(Q_{m,n},j+1)\, x^{j+1}$

$$= \sum_{j=1}^{m+n-1}\left[\binom{m+n-1}{j} - \binom{m+n-2}{j}\right] x^{j+1}$$

Example-3.2

Let $Q_{4,2}$ be a Queen crown graph with order 6 then $\gamma_{sd}(Q_{4,2})=2$ and $SD(Q_{4,2},x)=$

$x^2+4x^3+6x^4+4x^5+x^6$ (See Fig.1)

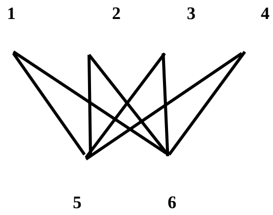

Fig. 1 : H = $Q_{4,2}$ has $\binom{5}{j}-\binom{4}{j}$ Strong dominating set with cardinality j.

Table-1($5 \leq m+n \leq 20$)

J	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
5	1	3	3	1															
6	1	4	6	4	1														
7	1	5	10	10	5	1													
8	1	6	15	20	15	6	1												
9	1	7	21	35	35	21	7	1											
10	1	8	28	56	70	56	28	8	1										
11	1	9	36	84	126	126	84	36	9	1									
12	1	10	45	120	210	252	210	120	45	10	1								
13	1	11	55	165	330	462	462	330	165	55	11	1							
14	1	12	66	220	495	792	924	792	495	220	66	12	1						
15	1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1					
16	1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1				
17	1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1			
18	1	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	560	120	16	1		
19	1	17	136	680	2380	6188	12376	19448	24310	24310	19448	12376	6188	2380	680	136	17	1	
20	1	18	153	816	3060	8568	18564	31824	43758	48620	43758	31824	18564	8568	3060	816	153	18	1

Conclusions

In this paper we concluded that for all $m \geq 3$ and n=2 we can find the Strong domination polynomials of Queen crown graph using the recursive formula

$$SD(Q_{m,n},x)=\sum_{j=1}^{m+n-1}\left[\binom{m+n-1}{j}-\binom{m+n-2}{j}\right]x^{j+1}$$

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