

Dual Fuzzy Soft Topology and Dual Fuzzy Soft Algebra

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Abstract In this search we will introduce a dual fuzzy soft set and study operations on dual fuzzy soft set with examples, define the dual fuzzy soft algebra , dual fuzzy soft topology , make a comparison between them with examples .

Keywords : dual fuzzy soft set ; dual fuzzy soft algebra ; dual fuzzy soft topology .

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1- Introduction

In 1965 [6] Zadeh L. A. introduce a fuzzy sets , in 1999[3] Molodtsov D. define soft sets , fuzzy soft set and fuzzy soft topology was defined in 2013[1] ,2014[2] , later in recent years multi fuzzy soft sets was introduced [5] (for a given soft set each soft element associated to a membership belongs to $[0,1]$)which is different in our definition . In this search will define a dual fuzzy soft set (for a given soft set each soft element and each soft member associates to a membership belongs to $[0,1]$) , define a dual fuzzy soft element and construct two spaces (dual fuzzy soft algebra and dual fuzzy soft topology) make a comparison between them with examples and counter examples .

2- Construction Dual Fuzzy Soft Sets

In this part we will define a dual fuzzy soft set and operations on it , the set of all dual fuzzy soft set over a universal set X will be denoted by $DFS(X)$, the set of all soft sets $S(X)$.

The dual fuzzy soft elements will be denoted by d, d_1, d_2, \dots , and the dual fuzzy soft sets will be denoted by $(df, E), (dg, E), (dk, E) \dots$.

Definition 2.1.

Let X be a non- empty set and E be a non-empty set of parameters , (F,E) be soft set in $S(X)$.Then for each $e \in E$ then each $(e, F(e))$ called soft member of the soft set (F,E) .

Example 2.2.

Let $X = \{k_1, k_2\}$ be a set of two books , $G = \{p_1, p_2\}$ the set of two persons let $(F,G) = \{(p_1, \{k_1, k_2\}), (p_2, k_1)\}$ represents each person choice for the types of books that they prefer, the soft elements of (F,E) are: $(p_1, \{k_1\}), (p_1, \{k_2\}), (p_2, k_1)$, the soft members of the soft set (F,E) are $(p_1, \{k_1, k_2\})$ and (p_2, k_1) .

Definition 2.3.

Let X be a universal set , E be a set of parameters , (F,E) be a soft set . If each soft element in (F,E) associated to arbitrary number $\eta \in [0,1]$ and each soft member in (F,E) associated to arbitrary number $\mu \in [0,1]$ then resulting set is called a dual fuzzy soft set (simply d - set) .

Definition 2.4.

The dual fuzzy soft element d (simply d - element) is a soft element $\tilde{x} = (e, \{h\})$ associated to arbitrary two numbers $\eta, \mu \in [0,1]$ represented as follow: $d = (e, \{h^\eta\})^\mu$.

Definition 2.5.

The fact that d be an d - element of (df, E) will be denoted by $d \in (df, E)$.

Remark 2.6.

Two d - element $d_1 = (e_\alpha, \{h_\beta^\eta\})^\chi$, $d_2 = (e_\gamma, \{h_\kappa^\nu\})^\xi$ are d - equal if $e_\alpha = e_\gamma$, $h_\beta = h_\kappa$, $\eta = \nu$, $\chi = \xi$ otherwise they said to be d - not equal .

Example 2.7.

Let $X = \{R = \text{read}, G = \text{green}, B = \text{blue}\}$ represent the set of three colors , $E = \{h_1, h_2\}$ represent the set of two houses , let $(F,E) = \{(h_1, \{R\}), (h_2, \{R, G\})\}$ be a soft set representing houses and colours that are used to paint these houses $(df, E) = \{(h_1, \{R^{0.5}, G^{0.0}\})^{0.6}, (h_2, \{R^{0.5}, G^{0.3}\})^{0.7}\}$ be d - set representing two houses h_1, h_2 and their colours which are read and green with the percentage of use of each colour for each house and the totally percentage of

cost of paint for each house , for $(h_1, \{R^{0.5}, G^{0.0}\})^{0.6}$ means the first house use the red color with percentage 50% and green color with percentage 0% , the percentage of totally cost of paint for the first house h_1 60% (notice that each high percentage associated with high expensive)

$$d_1 = (h_1, \{R^{0.5}\})^{0.6} \overline{\in} (df, E) , d_2 = (h_1, \{G^{0.0}\})^{0.6} \overline{\in} (df, E) ,$$

$$d_1 \neq d_2 \text{ notice that for } (h_1, \{R\}) \cong (F, E) ,$$

$$\text{but the associated } d\text{- element } d_3 = (h_1, \{R^{0.7}\})^{0.5} \overline{\notin} (df, E) .$$

Definition 2.8.

The d - complement of the d - set (df, E) denoted by

$$(df, E)^c = \{d^c = (e_i, \{h_j^{1-\eta_{ij}}\})^{1-\mu_i} = \forall i, j \in \lambda , \eta_{ij}, \mu_i \in [0, 1] \} .$$

Definition 2.9.

The d -set (df, E) generated by a soft set is called the null d - set if each soft element and each soft member in the soft set associated to 0 (denoted by $\overline{\Phi}$) .

Definition 2.10.

The d -set (df, E) generated by a soft set is called the universal d - set if each soft element and each soft member in the soft set associated to 1 (denoted by \overline{X}) .

Definition 2.11.

$$\text{Let } d\text{- sets } (df, E) = \{d_1: d_1 = (e_i, \{h_j^{\eta_{ij}}\})^{\mu_i}\}, (dg, E) = \{d_2: d_2 = (e_i, \{h_j^{\delta_{ij}}\})^{\xi_i}\}$$

generated by the same soft set then (df, E) is said to be a d - subset of (dg, E) if for each $d_1 = (e_i, \{h_j^{\eta_{ij}}\})^{\mu_i} \overline{\in} (df, E)$ and $d_2 = (e_i, \{h_j^{\delta_{ij}}\})^{\xi_i} \overline{\in} (dg, E)$, $\eta_{ij} \leq \delta_{ij}$ and $\mu_i \leq \xi_i$, $\forall i, j \in \lambda$ (denoted by $(df, E) \overline{\subseteq} (dg, E)$) .

Definition 2.12.

$$\text{Let } d\text{- sets } (df, E) = \{d_1: d_1 = (e_i, \{h_j^{\eta_{ij}}\})^{\mu_i}\}, (dg, E) = \{d_2: d_2 = (e_i, \{h_j^{\delta_{ij}}\})^{\xi_i}\}$$

generated by the same soft set then (df, E) is said to be a d - equal sets of (dg, E) if for each $d_1 = (e_i, \{h_j^{\eta_{ij}}\})^{\mu_i} \overline{\in} (df, E)$ and $d_2 = (e_i, \{h_j^{\delta_{ij}}\})^{\xi_i} \overline{\in} (dg, E)$, $\eta_{ij} = \delta_{ij}$ and $\mu_i = \xi_i$, $\forall i, j \in \lambda$, (denoted by $(df, E) \overline{=} (dg, E)$) .

Remark 2.13.

From the previous definition the d - set (df, E) is said to be not d - equal to the d - set (dg, E) if $\eta_{ij} \neq \delta_{ij}$ or $\mu_i \neq \xi_i$ or both for some $\eta_{ij}, \delta_{ij}, \mu_i, \xi_i \forall i, j \in \lambda$

$$\text{(denoted by } (df, E) \overline{\neq} (dg, E) \text{) .}$$

Definition 2.14.

The d - union of two d - sets (df, E) , (dg, E) generated by the same soft set such that

$$(df, E) = \{d_1: d_1 = (e_i, \{h_j^{\eta_{ij}}\})^{\mu_i}\}, (dg, E) = \{d_2: d_2 = (e_i, \{h_j^{\delta_{ij}}\})^{\xi_i}\}$$

$$\text{is the } d\text{- set } (dk, E) = (df, E) \cup (dg, E) = \{d_3: d_3 = (e_i, \{h_j^{\nu_{ij}}\})^{\kappa_i}\},$$

$$\nu_{ij} = \max\{\eta_{ij}, \delta_{ij}\} , \kappa_i = \max\{\mu_i, \xi_i\} \forall i, j \in \lambda .$$

Definition 2.15.

The d - intersection of two d - sets (df, E) , (dg, E) generated by the same soft set such that $(df, E) = \{d_1: d_1 = (e_i, \{h_j^{\eta_{ij}}\})^{\mu_i}\}, (dg, E) = \{d_2: d_2 = (e_i, \{h_j^{\delta_{ij}}\})^{\xi_i}\}$

$$\text{is the } d\text{- set } (dk, E) = (df, E) \cap (dg, E) = \{d_3: d_3 = (e_i, \{h_j^{\nu_{ij}}\})^{\kappa_i}\},$$

$$\nu_{ij} = \min\{\eta_{ij}, \delta_{ij}\} , \kappa_i = \min\{\mu_i, \xi_i\} \forall i, j \in \lambda .$$

Example 2.16.

$$\text{Let } (df, E) = \{ (d_1, \{c_1^{0.5}, c_2^{0.0}\})^{0.3}, (d_2, \{c_1^{0.3}, c_2^{0.3}\})^{0.4} \} \text{ and}$$

$$(dg, E) = \{ (d_1, \{c_1^{0.1}, c_2^{0.0}\})^{0.1}, (d_2, \{c_1^{0.7}, c_2^{0.1}\})^{0.2} \} \text{ then}$$

$$(df, E) \cup (dg, E) = (hf, E) = \{ (d_1, \{c_1^{0.5}, c_2^{0.0}\})^{0.3}, (d_2, \{c_1^{0.7}, c_2^{0.3}\})^{0.4} \}$$

$$(df, E) \cap (dg, E) = (kf, E) = \{ (d_1, \{c_1^{0.1}, c_2^{0.0}\})^{0.1}, (d_2, \{c_1^{0.3}, c_2^{0.1}\})^{0.2} \} .$$

Example 2.17.

$$\text{Let } X = \{h_1, h_2\} , E = \{e_1, e_2\} ,$$

$$(df, E) = \left\{ (e_1, \{h_1^{0.5}, h_2^{0.0}\})^{0.1}, (e_2, \{h_1^{0.0}, h_2^{0.3}\})^{0.2} \right\}$$

$$(dg, E) = \left\{ (e_1, \{h_1^{0.9}, h_2^{0.0}\})^{0.3}, (e_2, \{h_1^{0.0}, h_2^{0.7}\})^{0.4} \right\} \text{ be two } d\text{- sets}$$

since $(df, E) \overline{\subseteq} (dg, E)$ then

$$(df, E) \cup (dg, E) = \left\{ (e_1, \{h_1^{0.9}, h_2^{0.0}\})^{0.3}, (e_2, \{h_1^{0.0}, h_2^{0.7}\})^{0.4} \right\} = (dg, E)$$

$$(df, E) \cap (dg, E) = \left\{ (e_1, \{h_1^{0.5}, h_2^{0.0}\})^{0.1}, (e_2, \{h_1^{0.0}, h_2^{0.3}\})^{0.2} \right\} = (df, E) .$$

Remarks 2.18.

$$(1) (df, E) \cap (df, E)^c \neq \overline{\Phi} . (2) (df, E) \cup (df, E)^c \neq \overline{X} .$$

Example 2.19.

$$\text{Let } X = \{h_1, h_2\} , E = \{e_1, e_2\}$$

$$\begin{aligned}
 (df, E) &= \left\{ (e_1, \{h_1^{0.5}, h_2^{0.0}\})^{0.9}, (e_2, \{h_1^{0.0}, h_2^{0.3}\})^{0.2} \right\} \\
 (df, E)^c &= \left\{ (e_1, \{h_1^{0.5}, h_2^{1.0}\})^{0.1}, (e_2, \{h_1^{1.0}, h_2^{0.7}\})^{0.8} \right\} \text{ then} \\
 (df, E) \bar{\cap} (df, E)^c &= \left\{ (e_1, \{h_1^{0.5}, h_2^{0.0}\})^{0.1}, (e_2, \{h_1^{0.0}, h_2^{0.3}\})^{0.2} \right\} \neq \bar{\Phi} \\
 (df, E) \bar{\cup} (df, E)^c &= \left\{ (e_1, \{h_1^{0.5}, h_2^{1.0}\})^{0.9}, (e_2, \{h_1^{1.0}, h_2^{0.7}\})^{0.8} \right\} \neq \bar{X}
 \end{aligned}$$

Definition 2.20.

The d^* - set is d^* - finite if its d^* - elements are finite and the d^* - set is d^* - infinite if its d^* - elements are infinite .

Definition 2.21.

More generally, for a family of d^* - sets , $\{(df, E)_\lambda : \lambda \in \Lambda$, where Λ is an infinite index set}, the d^* -union is defined by:

$(dh, E) = \bar{\cup}_\lambda (df, E)_\lambda = \{ d^* : d^* = (e_i, \{h_j^{kij}\})^{\xi_i} , kij = \text{Sup}_{\lambda \in \Lambda} \{ \eta_{ij} : \eta_{ij} \text{ are the memberships of each soft element of the } d^* \text{- sets } (df, E)_\lambda \text{ and } \xi_{ij} = \text{Sup}_{\lambda \in \Lambda} \{ \mu_{ij} : \mu_{ij} \text{ are the memberships of each soft member of the } d^* \text{- sets } (df, E)_\lambda \} , i, j \in \mathcal{X} , \text{ where } \mathcal{X} \text{ is an infinite index set .}$

the d^* -intersection is defined by:

$(dh, E) = \bar{\cap}_\lambda (df, E)_\lambda = \{ d^* : d^* = (e_i, \{h_j^{kij}\})^{\xi_i} , kij = \text{Inf}_{\lambda \in \Lambda} \{ \eta_{ij} : \eta_{ij} \text{ are the memberships of each soft element of the } d^* \text{- sets } (df, E)_\lambda \} \text{ and } \xi_{ij} = \text{Inf}_{\lambda \in \Lambda} \{ \mu_{ij} : \mu_{ij} \text{ are the memberships of each soft member of the } d^* \text{- sets } (df, E)_\lambda \} , i, j \in \mathcal{X} , \text{ where } \mathcal{X} \text{ is an infinite index set .}$

3- Generating d^* - Topological Space

In this part we will construct a new concept d^* - Topological Space which is a generalization of general topology that provides a tool to construct a models in many real life example .

Definition 3.1.

Let X be non-empty set , E be a set of parameters , let \bar{T} be the collection of d^* - sets generated by the soft set \bar{X} , if \bar{T} satisfies the following axioms :

- (1) $\bar{\Phi}, \bar{X}$ are in \bar{T} .
- (2) The d^* -union of any members of d^* -sets in \bar{T} belongs to \bar{T} .
- (3) The d^* -intersection of any two d^* -sets in \bar{T} belong to \bar{T} .

Then \bar{T} is called (dual fuzzy soft topology) (simply d^* - Topology) .

The triple (X, \bar{T}, E) is called dual fuzzy soft topological space over X

(simply d^* - Topological space) , the d^* - sets of \bar{T} are called d^* - open sets their complements are called d^* - closed sets .

Examples 3.2.

(1) Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ then $\bar{T} = \{ \bar{\Phi}, \bar{X} \}$ is the d^* -Indiscrete topology .

(2) Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$,
 $(df, E) = \left\{ (e_1, \{h_1^{0.7}, h_2^{0.9}, h_3^{0.9}\})^{0.5}, (e_2, \{h_1^{0.5}, h_2^{0.3}, h_3^{0.0}\})^{0.7} \right\}$
 then $\bar{T} = \{ \bar{\Phi}, (df, E), \bar{X} \}$ be d^* - Topology .

(3) The previous example give us an abstract vision of a universe that we live which contain a matters \bar{X} (with its elements and their ratios) , an empty space $\bar{\Phi}$ (which contains no elements) and a sub matter (df, E) as a part of that universe that contain a specific elements but not others, and the anti-matter $(df, E)^c$ which is not belong to \bar{T} , where \bar{T} represent the whole universe with some of its possibilities $\bar{\Phi}, (df, E), \bar{X}$.

Definition 3.3.

Let (X, \bar{T}, E) be d^* - Topological space , $d^* \bar{\in} (dg, E)$ then (dg, E) is said to be d^* - neighborhood of d^* if there exist an d^* - open set (df, E) such that $d^* \bar{\in} (df, E) \bar{\subseteq} (dg, E)$.

Remark 3.4.

Every d^* - open set is d^* - neighbourhood but the converse is not necessary true .

Example 3.5.

Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$,
 $(df, E) = \left\{ (e_1, \{h_1^{0.7}, h_2^{0.5}, h_3^{0.1}\})^{0.5}, (e_2, \{h_1^{0.5}, h_2^{0.3}, h_3^{0.5}\})^{0.7} \right\}$

then $\bar{T} = \{ \bar{\Phi}, (df, E), \bar{X} \}$ be an d -Topology on \bar{X} , (df, E) is an d -open set it is also a d -neighborhood of $d = (e_1, \{h_2^{0.5}\})^{0.5}$, while $(dg, E) = \{ (e_1, \{h_1^{0.8}, h_2^{0.5}, h_3^{0.9}\})^{0.6}, (e_2, \{h_1^{0.5}, h_2^{0.7}, h_3^{1.0}\})^{0.8} \}$ is d -neighbourhood of $d = (e_1, \{h_2^{0.5}\})^{0.6}$ but not d -open set .

4- Generating Dual Fuzzy Soft Algebras (d - Algebra)

The aim to constructing d - Algebra is to construct a collection that contains a sets and their complements which is not satisfied in d - topology .

Definition 4.1.

A collection A of subsets of $MFS(X)$ is dual fuzzy soft algebra over X (simply d - Algebra), if it satisfies the following properties:

1. $\bar{\Phi}, \bar{X} \in A$.
2. For all $(df, E) \in A$, $(df, E)^c \in A$.
3. For all $(df, E), (dg, E) \in A$, $(df, E) \bar{\cup} (dg, E) \in A$.

Examples 4.2.

The collection $P(\bar{X})$ of all subsets of \bar{X} is dual fuzzy soft algebra .

Proposition 4.3.

Let $(df, E), (dg, E)$ are two dual fuzzy soft sets in $MFS(X)$ that belongs to A , then,

1. $(df, E) \bar{\cap} (dg, E) \in A$
 2. $(df, E) \bar{\cap} (dg, E) \in A$
 3. $(df, E) \bar{\Delta} (dg, E) \in A$
- [where $(df, E) \bar{\Delta} (dg, E) = [(df, E) \bar{\cap} (dg, E)] \bar{\cup} ((dg, E) \bar{\cap} (df, E))$]

Proof :

1. Since $(df, E), (dg, E) \in A$, and A is d - Algebra, $((df, E)^c \bar{\cup} (dg, E)^c)^c \in A$.
The result follows from the fact that $((df, E)^c \bar{\cup} (dg, E)^c)^c = (df, E) \bar{\cap} (dg, E)$.
2. It follows from proposition 4.3.(1) and definition 4.1.(2) and the fact that $(df, E) \bar{\cap} (dg, E) = (df, E) \bar{\cap} (dg, E)^c$.
3. It follows from proposition 4.3.(2) and definition 4.1.(3) and the fact that $(df, E) \bar{\Delta} (dg, E) = ((df, E) \bar{\cap} (dg, E)) \bar{\cup} ((dg, E) \bar{\cap} (df, E))$.

Proposition 4.4.

Given $MFS(X)$, if $(df, E)_1, (df, E)_2, \dots, (df, E)_n \in A$, then

1. $\bar{\cup}_{i=1}^n (df, E)_i \in A$
2. $\bar{\cap}_{i=1}^n (df, E)_i \in A$

Proof: By induction .

5- A Comparative View Between Dual Fuzzy Soft Topology and Dual Fuzzy Soft Algebra

In this section we will introduce a comparative between two structures dual fuzzy soft topology and dual fuzzy soft algebra with additional theorems of finite intersection and finite union on these structures .

Proposition 5.1.

Every d - Algebra is d - topology .

Proof: directly from definition (3.1.) (4.1.) .

Remark 5.2.

The converse of proposition (5.1.) is not necessary true .

Examples 5.3.

Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$

$$(df, E) = \{ (e_1, \{h_1^{0.7}, h_2^{0.9}, h_3^{0.9}\})^{0.1}, (e_2, \{h_1^{0.5}, h_2^{0.3}, h_3^{0.0}\})^{0.2} \}$$

then $\bar{T} = \{ \bar{\Phi}, (df, E), \bar{X} \}$ be a d - Topology but not d - Algebra since

$$(df, E)^c = \{ (e_1, \{h_1^{0.3}, h_2^{0.1}, h_3^{0.1}\})^{0.9}, (e_2, \{h_1^{0.5}, h_2^{0.7}, h_3^{1}\})^{0.8} \} \notin \bar{T} .$$

Theorem 5.4.

The finite d - intersection of d - algebra is d - algebra .

Proof : Let α, β are two d' -algebras

1. $\bar{\Phi}, \bar{X} \in \alpha$ and $\bar{\Phi}, \bar{X} \in \beta$ then $\bar{\Phi} \in \alpha \cap \beta, \bar{X} \in \alpha \cap \beta$.
2. Let $(df, E) \in \alpha \cap \beta \Rightarrow (df, E) \in \alpha \wedge (df, E) \in \beta$ then $(df, E)^c \in \alpha \wedge (df, E)^c \in \beta \Rightarrow (df, E)^c \in \alpha \cap \beta$.
3. Let $(df, E), (dg, E) \in \alpha \cap \beta \Rightarrow (df, E), (dg, E) \in \alpha \wedge (df, E), (dg, E) \in \beta \Rightarrow (df, E) \bar{\cap} (dg, E) \in \alpha \wedge (df, E) \bar{\cap} (dg, E) \in \beta \Rightarrow (df, E) \bar{\cap} (dg, E) \in \alpha \cap \beta$.

Theorem 5.5.

The finite d' -union of d' -algebra is d' -algebra.

Proof : Let α, β are two d' -algebras

1. $\bar{\Phi}, \bar{X} \in \alpha$ and $\bar{\Phi}, \bar{X} \in \beta$ then $\bar{\Phi} \in \alpha \cup \beta, \bar{X} \in \alpha \cup \beta$.
2. Let $(df, E) \in \alpha \cup \beta \Rightarrow (df, E) \in \alpha \vee (df, E) \in \beta$ then $(df, E)^c \in \alpha \vee (df, E)^c \in \beta \Rightarrow (df, E)^c \in \alpha \cup \beta$.
3. Let $(df, E), (dg, E) \in \alpha \cup \beta \Rightarrow (df, E), (dg, E) \in \alpha \vee (df, E), (dg, E) \in \beta \Rightarrow (df, E)^c, (dg, E)^c \in \alpha \vee (df, E)^c, (dg, E)^c \in \beta \Rightarrow (df, E)^c \bar{\cup} (dg, E)^c \in \alpha \vee (df, E)^c \bar{\cup} (dg, E)^c \in \beta \Rightarrow (df, E) \bar{\cap} (dg, E) \in \alpha \vee (df, E) \bar{\cap} (dg, E) \in \beta \Rightarrow (df, E) \bar{\cap} (dg, E) \in \alpha \cup \beta$.

Theorem 5.6.

The finite d' -intersection of d' -topology is d' -topology.

Proof : Let $\bar{T}_\alpha, \bar{T}_\beta$ are two d' -topologies

1. $\bar{\Phi}, \bar{X} \in \bar{T}_\alpha$ and $\bar{\Phi}, \bar{X} \in \bar{T}_\beta$ then $\bar{\Phi} \in \bar{T}_\alpha \cap \bar{T}_\beta, \bar{X} \in \bar{T}_\alpha \cap \bar{T}_\beta$.
2. Let $(df, E), (dg, E) \in \bar{T}_\alpha \cap \bar{T}_\beta$ then $(df, E), (dg, E) \in \bar{T}_\alpha \wedge (df, E), (dg, E) \in \bar{T}_\beta \Rightarrow (df, E) \bar{\cap} (dg, E) \in \bar{T}_\alpha \wedge (df, E) \bar{\cap} (dg, E) \in \bar{T}_\beta \Rightarrow (df, E) \bar{\cap} (dg, E) \in \bar{T}_\alpha \cap \bar{T}_\beta$.
3. Let $(df, E)_\lambda \in \bar{T}_\alpha \cap \bar{T}_\beta$ then $(df, E)_\lambda \in \bar{T}_\alpha \wedge (df, E)_\lambda \in \bar{T}_\beta \Rightarrow \cup_\lambda (df, E)_\lambda \in \bar{T}_\alpha \wedge \cup_\lambda (df, E)_\lambda \in \bar{T}_\beta \Rightarrow \cup_\lambda (df, E)_\lambda \in \bar{T}_\alpha \cap \bar{T}_\beta$.

Theorem 5.7.

The finite d' -union of d' -topology not necessary d' -topology.

Example 5.8.

Let $X = \{h_1, h_2, h_3\}, E = \{e_1, e_2\}$,

$$(df, E) = \left\{ (e_1, \{h_1^{0.7}, h_2^{0.9}, h_3^{0.9}\})^{0.5}, (e_2, \{h_1^{0.5}, h_2^{0.3}, h_3^{0.0}\})^{0.7} \right\}$$

$$(dg, E) = \left\{ (e_1, \{h_1^{0.7}, h_2^{0.9}, h_3^{0.9}\})^{0.6}, (e_2, \{h_1^{0.5}, h_2^{0.3}, h_3^{0.0}\})^{0.8} \right\}$$

then $\bar{T}_\alpha = \{ \bar{\Phi}, (df, E), \bar{X} \}$ and $\bar{T}_\beta = \{ \bar{\Phi}, (dg, E), \bar{X} \}$ are d' -Topologies but

$\bar{T}_\alpha \cup \bar{T}_\beta = \{ \bar{\Phi}, (df, E), (dg, E), \bar{X} \}$ is not d' -Topology.

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