The Travelling Purchaser Problem with Budget Constraint

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Abstract — *The traveling purchaser problem (TPP) is an NP-hard problem in the family of combinatorial optimization. The purchaser needs to buy several items with variable demands which are available at different marketplaces. The cost of travelling between different marketplaces and a list of available items together with the price of such item at each marketplace is known. The objective of the TPP is to design an optimal tour such that the purchaser tour starts and ends at a home point/ domicile point, purchases all the required items on travelling through a subset of marketplaces exactly once and which satisfy the budget constraint i.e. the total purchasing cost of the items should not surpass the pre-defined budget threshold. The tour may not necessary contain all the marketplaces. This problem finds interesting applications in machine scheduling, transportation logistics among others. The problem is nicely designed with zero-one integer programming. In order to find optimum solution for this problem, an exact algorithm called Lexi-search algorithm is proposed, it searches feasible solutions systematically and with effective bounding and backtracking strategies quickly moves towards the optimal solution.*

Keywords - *Travelling purchaser problem, Lexi-search Algorithm, Budget threshold, Backtracking.*

I. INTRODUCTION

During recent times, the Travelling Purchaser Problem (TPP) has received the maximum attention of several researchers, most of whom have proposed the heuristics for its solution. Because of its dual nature of the procurement and as well as transportation, the TPP has significantly attracted the attention of the scientific researchers in combinatorial optimization [15] and also the practitioners in the recent decades [6]. The TPP has been professionally used to model several application contexts and has been computationally quite challenging. It could be observed that the researchers have developed a great interest around the TPP problem due to the fact that it would ideally combine three different aspects viz., (i) the suppliers selection (ii) the routing construction aspects/decisions of the purchaser and the (iii) optimization of the product purchase planning.

The present emphasized TPP problem could be considered as a generalization of the traditional Travelling Salesman Problem (TSP). The generalization of the Travelling Salesman Problem as a Travelling Purchaser Problem was first discussed by Ramesh [19]. Under this new type of thinking, we presume that there has been a set of 'm' markets and a set of 'n' products that would be purchased from the market. Each product must be physically available with different quantities or levels in a subset of markets. The unit cost of each product would actually depend on the market where the product is available. The demand for each product and also the traveling cost, are generally, known in advance. From the purchaser point of view, the main objective of the purchaser would be (i) to buy according to the complete demand of the products, (ii) departing and also returning to a domicile, with the objective of minimizing the sum of the cost of the travel and the purchase cost. It should be noted that, as a basic rule, we need not include all the markets in the tour that would be undertaken.

TPP has gained significant attention from the scientific research workers of the Operations Research (OR) group mainly due to the reinterpretation and reconsideration of TPP as a vehicle routing problem [19]. The TPP has also found its applications in several contexts. Job Scheduling on a Multi-purpose Production Line [3], Decorator's Problem [4], warehousing operations [24], the network design [21], the forest management problem [18], the inter-modal transportation problem [10], waste collection management [9].

A. The TPP with upper bound restrictions (TPP-B)

One of the most observed variants of the TPP is with different budget constraints (TPP-B). It is a problem where budget B is specified and represents a threshold on the total amount which can be spent on purchasing products. It comprises a selection of a subset of markets to visit with a tour starting and ending at the depot such that the demand d_k for each product $k \in K$ is satisfied at a global purchasing cost without surpassing the given budget threshold B with minimization of the total traveling cost. Two types of costs are involved in the TPP-B. The traveling cost present in the objective function and the purchasing cost present in the constraints. Most of the previous studies on similar problems have restricted the analysis to the case where the two cost measures were both in the objective function.

The TPP-B was found to be highly popular with so many real-time applications under the telecommunications network design. It should be noted that some research workers have used this problem as an intermediate step for solving the bi-objective TPP.

Kyle Booth et al.,[12] described that most of the TPP variants fall under two dimensions, viz., (i) Capacitated vs. Uncapacitated, and (ii) Symmetric vs. Asymmetric.

Quite interestingly, the combinatorial structure of the TPP has appeared for the first time in its 'unrestricted' form with the research work that was carried out by Burstall [3]. This is with reference to the relation of the scheduling of different jobs on a multi-purpose production line.

Based on a survey, Daniele Manerba et al.,[5] observed that TPP is a procurement/routing problem that aims at selecting purchasing plan of set of products from subset of suppliers, and corresponding visiting tour, in order to satisfy pre-defined products demand, apart from minimizing overall purchase and travel costs

Jorge Riera-Ledesma and Juan JoséSalazar-González [11] have shown that the weak LP-relaxation as induced by the 'budget constraints' has produced the branching trees with many conspicuous number of nodes under their Branch and-Cut algorithm.

Singh and van Oudheusden [24] suggested that the TPP could be used in the warehousing operations for dispatching a vehicle to pick up all the ordered items that have been stored in the different picking locations, and properly transport them to the shipping area. The other routing application of TPP is the tour needed for a school bus to pick-up the students from different locations.

A wide variety of solution methods have been proposed to achieve optimal or near-optimal solutions. Although computational classification of the problem predicts poor results when assuring optimality, few authors proposed exact algorithms. Considering all the details of the combinatorial structure and also the hardness of the problem, many heuristics were proposed. It is observed that the meta-heuristic related contributions have been quite limited, but would explore about the known frame works. Importance of having simple heuristic algorithm is confirmed by the fact that quasi-totality of existing exact methods for TPP include some heuristic components. In turn, mathematical/structural properties extracted to obtain exact methods led to basic ideas on which heuristic algorithms were developed in subsequent works. Exact and heuristic approaches have their own relevance, and complement each other for enriching the knowledge and its tractability.

Till now, only three exact algorithms are available, which are (i) the Lexicographic algorithm proposed by Ramesh (1981) [19] (ii) the Branch-and-Bound algorithm proposed by Singh and van Oudheusden (1997) [24] and (iii) the Branch-and-Cut algorithm proposed by Laporte et al., (2003) [13]. The very first research work where the TPP has been introduced as it is presently known to the scientific world was due to Ramesh (1981). He defined about the problem of '1TPP' under a routing context. In his research paper, he has clearly discussed about (i) an exact algorithm and (ii) a heuristic algorithm. The 'exact' algorithm method was developed based on a Lexicographic search, which would handle the different instances with $m \leq 12$ and $n \leq 10$. The 'heuristic' algorithm approach would be a version of the nearest insertion algorithm for the Travelling Salesman Problem (TSP) as described by Bentley [1].

Some of the heuristic procedures proposed to solve TPP include 'Generalized Savings Heuristic' proposed by Golden et al. [8], 'Tour Reduction Heuristic' proposed by Ong [16], 'Commodity Adding Heuristic' given by Pearn and Chien [17], Tabu Search proposed by Voß [26], 'Market Adding Heuristic' given by Laporte et al. [13], 'Perturbation heuristics' by Renaud et al. [23].

Goldbarg et al. [7] have proposed 'Transgenetic algorithm' (TA) inspired on two major evolutionary driving forces, viz., (i) horizontal gene transfer (acquisition of foreign genes by organisms); and (ii) endosymbiosis.

Renata Mansini et al. [22] have first proposed the heuristics for the bounded version of the TPP problem. They have proposed two different algorithms to solve the TPP-B. i) an enhanced local-search heuristic and ii) a variable neighborhood search (VNS) approach. These two algorithms were used to solve both the capacitated and the un-capacitated versions of the TPP-B.

In a recent research paper, Raquel Bernardino and Ana Paias [20] have discussed about the TPP, an NP-hard problem that would generalize the TSP. 'Meta-heuristics' that would combine the 'Genetic Algorithms' and the 'Local search' have been described by them.

The present paper is organized as follows. Section 2 presents the Materials and Methods where problem description and the concept of Lexi-search algorithm are explained. Section 3 presents the solution of a numerical example using Lexi-search algorithm. Also the recursive algorithm for searching the feasibility of a partial word and then improving the feasible solution towards the optimality is presented in this section. Conclusion and the remarks are presented in section 4. Section 5 presents the references.

II. MATERIALS AND METHODS

A. PROBLEM DESCRIPTION

The TPP with budget constraint can be stated as follows: Let $M = \{m_1, m_2, m_3, \ldots, m_m\}$ and $N = \{p_1, p_2, p_3, \ldots, p_n\}$ respectively, denote a set of $m-$marketplaces with m_1 as domicile and $n-$items to be purchased. $R = \{d_1, d_2, d_3, \ldots, d_n\}$ be the quantity of items required. The time of travel between a pair of marketplaces (i, j) is given by T_{ij} . A_{kj} and C_{kj}, respectively represent the available quantity and the unit cost of k^{th} item at market j. B Indicates the maximum budget that is allocated to purchase the required items. A purchaser starts from his domicile m_1, travels through a subset of $m-$markets and purchases each of $n-$items from one or more of markets of which he visits, and finally returns to starting point. The objective is to trace a tour for the purchaser such that the total travel time is minimum and the total purchase cost should not exceed the budget limit. The following assumptions made on this problem:

- $G(M, E)$ is a complete directed network with $m-$markets as nodes and are connected with edges in the edge set E.
- The entries in T_{ij} can be asymmetric.
- Each item is available in at least one market.
- Purchaser visits each market exactly once.
- No multiple tours for purchasing the required items.
- Only one purchaser is permitted to purchase all the items at the instant of purchase time.
- $d_k \leq \sum_{j=1}^{n} A_{kj}$, the total availability of k^{th} item is greater than or equal to its demand.
- An item can be purchased from more than one market.
- If a purchaser visits two or more markets, he purchases the item(s) with least cost wherever it is possible along the tour.
- In order to make a tour length as least, a purchaser can visit one or more markets without any purchase of items at those markets.
- An item can be purchased at more than one market to meet the demand.
- If an item k is not available at some market j, then the corresponding cost value C_{kj} is set to a high value.
- The values d_k, A_{kj} and C_{kj} against the domicile are assumed to be zero/neglected.

The proposed TPP with budget constraint is mathematically expressed using the zero-one linear programming, as follows:

Mathematical Formulation:

$Minimize\ Z = \sum_{i=1}^{m} \sum_{j=1}^{m} T_{ij} X_{ij}$	
Subject to the constraints:	
$\sum_{j=1}^{m} X_{1j} = 1, j \neq 1$	………………………………………..…….. (1)
$\sum_{i=1}^{m} X_{i1} = 1, i \neq 1$	………………………………………..…….. (2)
$\sum_{j=1}^{m} X_{ij} \leq 1, \forall i, i \neq j$	………………………………………..…….. (3)

$\sum_{i=1}^{m} X_{ij} \leq 1, \forall j, j \neq i$	……………………………………..……. (4)
$\sum_{i=1}^{m} X_{ij} - \sum_{k=1}^{m} X_{jk} = 0$	……………………………………..……. (5)
$Q_{kj}X_{ij} \leq A_{kj}, \forall i,j,k$	……………………………………..……. (6)
$\sum_{k=1}^{n}\sum_{i=1}^{m}\sum_{j=1}^{m} Q_{kj}C_{kj}X_{ij} \leq B, \forall j, j \neq i$	……………………………………..……. (7)
$\sum_{i=1}^{m}\sum_{j=1}^{m} Q_{kj}X_{ij} = d_k \leq \sum_{j=1}^{m} A_{kj}, \forall k$	……………………………………..……. (8)
$X_{1j} = 0\ (or)\ 1$	……………………………………..……. (9)

The objective function represents the total travel time (which includes the purchase time at each market) of the purchaser. The constraints (1) and (2) respectively denote that the purchaser should start at a domicile point 1 and returns to the same point. The purchaser enters and departs each market at most once, given by the inequalities (3) and (4). Since these two inequalities cannot guarantee the continuity of the tour, thus the constraint (5) is enforced to hold it. The quantity of an item k at market j purchased is denoted by Q_{kj} and it should not exceed the amount of availability at that market, which is expressed in (6). The inequality (7) is added to hold the budget constraint i.e. the total purchase cost of all the required items do not surpass the pre defined budget limit B. The constraint (8) represents the sum of quantum purchase of an item k from all the markets must equals to its demand d_k and also this sum cannot exceed the total amount of availability of that item. Finally, if a purchaser ever visits market j from i, then X_{ij} assigned with a value 1, otherwise X_{ij} assumes a value 0, which is expressed in (9).

B. LEXI-SEARCH ALGORITHM

In 1962, Pandit proposed an implicit search procedure called Lexi-Search method or Lexicographic Algorithm. Here, each solution is represented as a proper sequence of symbols where searching for an optimal solution would be quite analogous to the search for a specific word's location in the entire dictionary. The solutions are generated starting from a 'partial word' in some hierarchy which would reflect an 'analogous order' in their values. Each partial word would clearly define a 'block of solutions', and for each block of solutions, a 'lower bound' would be computed. Suppose, if this lower bound surpasses the value of the best -known solution, the entered block of words would be rejected because it will not lead to any of the favorable solutions. Subsequently, the next block of words would be explored for finding a solution.

It is very effective, efficient, faster, and can be easily adapted to solve the combinatorial programming problems by specifying simple rules for branching, bounding and termination. Hence the name is given as the Lexi-Search. This orderly search saves a lot of time and memory.

III. NUMERICAL EXAMPLE

To make possible understanding of the proposed TPP model and the concepts introduced in the Lexi-search algorithm, a suitable numerical example is considered in this section from Goldbarg et al. [7], with $m = 7$ and $n = 4$. The Table I provides the time matrix between 7 markets in which market 1 act as domicile point. For example if the purchaser wants to purchase 4 items from different markets within the budget limit of 60 units of purchase cost, the quantity of an item k required, the availability of an item k at the market j i.e. availability matrix and the unit purchase cost of each item k at market j i.e. cost matrix is given in Table II. The symbol ‘‘–’’ against certain cell entries of A_{kj} and C_{kj} respectively, denote that the particular item k is not available at the respective market j and the cost also neglected or imposed a high value.

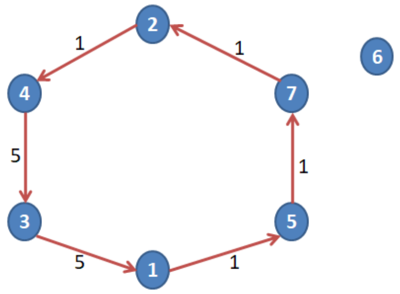
Table I: The time matrix (T_{ij})

	1	**2**	**3**	**4**	**5**	**6**	**7**
1	—	4	5	5	1	1	4
2	4	—	2	1	4	9	1
3	5	2	—	5	9	7	2
4	5	1	5	—	3	8	6
5	1	4	9	3	—	2	1
6	1	9	7	8	2	—	2
7	4	1	2	6	1	2	—

Table II

Item (k)	**Demand (d_k)**	**Availability of item-k at market-j (A_{kj})**						**Unit cost of the item-k at market-j (C_{kj})**					
		2	**3**	**4**	**5**	**6**	**7**	**2**	**3**	**4**	**5**	**6**	**7**
1	5	3	1	—	5	—	3	5	1	—	1	—	8
2	6	2	1	5	5	7	4	3	4	6	5	2	1
3	4	7	7	3	4	7	4	3	7	6	3	2	1
4	3	8	3	4	4	2	3	10	6	1	8	10	8

Given a set of $7-$markets with known travel time between each pair of markets, the TPP involves in finding a closed tour of minimum total length by visiting a subset of markets such that each market along the tour covers exactly once and the total purchasing cost of $4-$items is within the predefined budget limit. Typically, a tour of TPP can be expressed as an ordered permeation or a sequence of markets visits. One of the feasible solutions which is obtained using Lexi-search algorithm, represented by the ordered sequence of market indices is shown in Fig. 1. The directed lines indicate the direction of moment of the purchaser and the values along the directed lines gives units of travel time between the markets.



Ordered sequence: $1 \to 5 \to 7 \to 2 \to 4 \to 3 \to 1$

Fig. 1 Feasible tour of TPP

From the Fig. 1, the initial total tour length is observed as 14 units of time and the purchase cost of all items is 22 units which is clearly explained in Table 3. Here, the purchaser starts from domicile **1**, traverses through the markets **5**, **7**, **2**, **4** and **3**, and returns to domicile point.

Table III

Item (k)	Demand (d_k)	Quantity purchased (Q_{kj}) from the markets					Cost(C_{kj})					$\sum_j Q_{kj}C_{kj}$
		5	**7**	**2**	**4**	**3**	**5**	**7**	**2**	**4**	**3**	
1	5	5	0	0	0	0	1	8	5	—	1	5
2	6	0	4	2	0	0	5	1	3	6	4	4*1+2*3=10
3	4	0	4	0	0	0	3	1	3	6	7	4
4	3	0	0	0	3	0	8	8	10	1	6	3

Table III illustrates that the item 1 with demand 5 units is purchased in market 5 or in market 3 at the unit cost 1 unit. The item 2 purchased in market 7 and market 2 with the quantities 4 and 2 respectively at cost 1 and 3, the required quantity of this item is satisfied. Similarly, the items 3 and 4 respectively with demands 4 and 3 purchased in market 7 and market at cost 1 and 1. Although the purchaser visits markets 3 and 5 along the tour but he may purchase the item 1 in one of the markets only and no item is purchased in the other market. Therefore, the total cost of purchase of all items in different markets results as 22 units, which is lower than the predefined budget limit.

A. CONCEPTS AND DEFINITIONS

An indicator two dimensional array $\mathrm{X} = [\mathrm{X}_{\mathrm{ij}}]$ which is associated with the cell entries zeros and ones is called a "pattern". A pattern is said to be feasible if X is a feasible solution. The value of the pattern X is computed using the equation (10), provides the total time of travel represented by it.

$$\boldsymbol{V}(\boldsymbol{X}) = \sum_{i=1}^{m}\sum_{j=1}^{m} T_{ij}X_{ij} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

In the algorithm, which is developed in the sequel, a search is made for a feasible pattern with the least value. Each pattern of the solution X is represented by the set of ordered pairs $[(i,j)]$ for which $X_{ij} = 1$ with understanding that the other X_{ij}'s are zeros. For instance the feasible solution represented by Fig. 1 can be written as a set of ordered pairs as $\{(1,5),(7,2),(2,4),\ (4,3),(3,1)\}$. In order to find the appropriate set of ordered pairs which produce the optimal solution, the search technique depends on the construction of a suitable alphabet table.

B. ALPHABET TABLE

There are $m^2 (= m \times m)$ ordered pairs in the two-dimensional array T of the time matrix between the markets. For convenience these are arranged in an ascending order of their corresponding time values and are indexed from $1\ to\ m^2$ (Sundara Murthy-1979) [25]. Let $SN = [1,2,\ldots,m^2]$ be an array of the set of m^2 indices. The alphabet table consists of Time values in increasing order, cumulative time values along with the respective row and column indices, given in Table IV. Let $L_k = \{l_1, l_2, \ldots, l_k\} \in SN$ be an ordered sequence of k indices from SN. The pattern represented by the ordered pairs whose indices are given by L_k is independent of the order of l_i, in the sequence. Hence for uniqueness the indices are arranged in the increasing order such that $l_i < l_{i+1}, i = 1,2,\ldots,k-1$. Any letter l_i can occupy the prime position in L_k. L_k is said to be feasible if the corresponding pattern X is feasible, otherwise it is infeasible.

Table IV: Alphabet Table				
SN.	T	CT	R	C
1	1	1	1	5
2	1	2	1	6
3	1	3	2	4
4	1	4	2	7
5	1	5	4	2
6	1	6	5	1
7	1	7	5	7
8	1	8	6	1
9	1	9	7	2
10	1	10	7	5
11	2	12	2	3
12	2	14	3	2
13	2	16	3	7
14	2	18	5	6
15	2	20	6	5
16	2	22	6	7
17	2	24	7	3
18	2	26	7	6
19	3	29	4	5
20	3	32	5	4
21	4	36	1	2
22	4	40	1	7
23	4	44	2	1
24	4	48	2	5
25	4	52	5	2
26	4	56	7	1
27	5	61	1	3
28	5	66	1	4
29	5	71	3	1
30	5	76	3	4
31	5	81	4	1
32	5	86	4	3
33	6	92	4	7
34	6	98	7	4
35	7	105	3	6
36	7	112	6	3
37	8	120	4	6
38	8	128	6	4
39	9	137	2	6
40	9	146	3	5
41	9	155	5	3
42	9	164	6	2
43	999	1163	1	1

44	999	2162	2	2
45	999	3161	3	3
46	999	4160	4	4
47	999	5159	5	5
48	999	6158	6	6
49	999	7157	7	7

C. LOWER BOUND OF A PARTIAL WORD $(LB(L_k))$:

The lower bound of a partial word L_k, is computed using the equation (11)

$$LB(L_k) = V(L_k) + CT(l_k + \alpha - k) - CT(l_k) \quad \ldots\ldots\ldots\ldots\ldots (11)$$

Where, $V(L_k) = V(L_{k-1}) + T(l_k)$ and $V(L_1) = T(l_1)$ and α be the minimum number of distinct markets required to make a closed tour including domicile appear in L_{k-1}. Initially α is considered as 2, since for a closed tour at least we have 2 distinct places (nodes). The recursive algorithm for searching the feasibility of a partial word L_k and then improving the feasible solution towards the optimality is given in Fig. 2.

Step 1 : Initialization

$m \leftarrow number\ of\ markets, n \leftarrow number\ of\ items$

$B \leftarrow Budget\ limit;\ T = [t_{ij}] \leftarrow time\ matrix;$
$d = [d_k] \leftarrow demandof\ items;\ A = [a_{ij}] \leftarrow Availability\ matrix,$
$C = [c_{ij}] \leftarrow cost\ matrix;\ VT = \infty \leftarrow upper\ bound\ (Trail\ value)$

Step 2 : Construct the Appropriate alphabet table as explained earlier. Go to step 3

Step 3 : The algorithm begins with a partial word $L_k = (l_k) = (1)$, where the length of the partial word L_k is one i.e. k = 1. Go to step 4

Step 4 : Determine $LB(L_k)$ using equation (11).
If $LB(L_k) < VT$, then goto step 5; else dismiss all the partial words of order k that succeeds L_k. Go to step 9.

Step 5 : If L_k is a partial feasible word, go to step 6, else i.e. L_k is infeasible then select next partial word with the same order by considering another letter that follows l_k in its k^{th}position , go to step 4.

Step 6 : If L_k is a full length feasible tour (for instance Fig.a), go to step 7; else go to step 8

Step 7 : $VT = LB(L_k)$, Record L_k then go to step 10

Step 8 : $L_{k+1} = L_k * (l_{k+1})$, where * refers the concatenation operation. Go to step 4.

Step 9 : If length of L_k is one then go to step11, else go to step 10

Step 10 : Select the next immediate letter of the partial word of order $k - 1$, repeat the steps from 4 to 10 until it enters in to step 11.

Step 11 : Stop and print VT and the corresponding L_k.

Fig. 2 The steps in Lexi search algorithm

D. SEARCH TABLE

In Table V, SN indicates the serial number, the subsequent columns numbered from 1 to 6 respectively denote the position of a letter in the partial word L_k, the next four successive columns provides the value of L_k, lower bound of L_k, row and column index of the letter k in L_k and finally the last column contains the remarks regarding the partial word L_k with leader k is accepted (A) or rejected (R).

Table V: Search Table

SN.	1	2	3	4	5	6	V	LB	R	C	Remarks
1	1						1	2	1	5	A
2		2					2	2	1	6	R
3		3					2	2	2	4	A
4			4				3	4	2	7	R
5			5				3	4	4	2	R
6			6				3	4	5	1	R
7			7				3	4	5	7	A
8				8			4	5	6	1	R
9				9			4	5	7	2	A
10					10		5	5	7	5	R
11					11		6	6	2	3	R
12					12		6	6	3	2	R
13					13		6	6	3	7	R
14					14		6	6	5	6	R
15					15		6	6	6	5	R
16					16		6	6	6	7	R
17					17		6	6	7	3	R
18					18		6	6	7	6	R
19					19		7	7	4	5	R
20					20		7	7	5	4	R
21					21		8	8	1	2	R
22					22		8	8	1	7	R
23					23		8	8	2	1	R
24					24		8	8	2	5	R
25					25		8	8	5	2	R
26					26		8	8	7	1	R
27					27		9	9	1	3	R
28					28		9	9	1	4	R
29					29		9	9	3	1	A
30						30	14	14	3	4	R
31						31	14	14	4	1	R
32						32	14	VT=14	4	3	A
33					30		9	9	3	4	R
34					31		9	VT=9	4	1	A
35				10			4	6	7	5	R
36	-	-	-	-	-	-	-	-	-	-	-
287		6					2	3	5	1	R

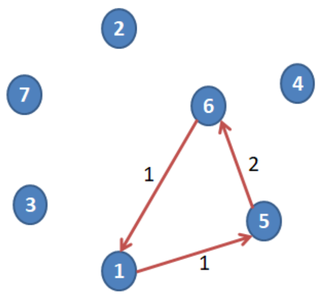
288		7					2	3	5	7	A
289			8				3	3	6	1	A
290				9			4	4	7	2	A
291					10		5	5	7	5	R
292					11		6	6	2	3	A
293	-	-	-	-	-	-	-	-	-	-	-
322				13			5	5	3	7	R
323				14			5	5	5	6	R
324				15			5	5	6	5	R
325				16			5	5	6	7	R
326				17			5	5	7	3	A
327					18		7	7	7	6	R
328					19		8	8	4	5	R
329					20		8	8	5	4	R
330					21		9	9=VT			R
331				18			5	VT=5	7	6	A
332	-	-	-	-	-	-	-	-	-	-	-
357				13			6	8>VT			R
358			13				4	4	3	7	A
359				14			6	8>VT			R
360			14				4	VT=4	5	6	A
361		9					2	2	7	2	A
362	-	-	-	-	-	-	-	-	-	-	-
498		19					4	4=VT			R
499	9						1	2	7	2	A
500		10					2	4=VT			R
501	10						1	3	7	5	A
502		11					3	5>VT			R
503	11						2	4=VT			R
The search is over											

Further the initial tour length is recursively improved using the proposed Lexi-search algorithm with effective backtracking, the successive improved solutions are 9 units of time (34^{th} row in Table V with the ordered sequence: $\mathbf{1 \to 5 \to 7 \to 2 \to 4 \to 1}$), 5 units of time ($331^{st}$ row in Table V with the ordered sequence: $\mathbf{1 \to 5 \to 7 \to 6 \to 1}$) and finally ended the search with an optimal solution 4 units of time (360^{th} row in Table V with the ordered sequence: $\mathbf{1 \to 5 \to 6 \to 1}$) to purchase the required items from the different markets. The optimal tour is shown in Fig.3. The schedule of purchase of each item is given in Table VI.

Table VI

Item (k)	**Demand (d_k)**	**Quantity purchased (Q_{kj}) from the markets**		**Cost(C_{kj})**		$\sum_j Q_{kj}C_{kj}$
		5	**6**	**5**	**6**	
1	5	5	0	1	—	5*1=5
2	6	0	6	5	2	6*2=12
3	4	0	4	3	2	4*2=8
4	3	3	0	8	10	3*8=24

Table VI demonstrates that the items 1 and 4 respectively, with demand 5 units and 3 units are purchased in market 5 at the unit cost 1 and 8. The rest of the items 2 and 3 respectively, with demand 6 units and 4 units are purchased in market 6 at the unit cost 2, Therefore, the total cost of purchase of all items in different markets results as 49 units, which is lower than the predefined budget limit.



Ordered sequence: $1 \rightarrow 5 \rightarrow 6 \rightarrow 1$

Fig. 3 Optimal tour of TPP

IV. CONCLUSIONS

In this paper we have introduced a variant of the traveling purchaser problem i.e TPP-B where the objective function looks for a tour with minimum time, visiting a subset of markets such that all the required products are bought and the total purchasing cost does not surpass a given budget threshold. The problem has been designed with zero-one integer programming. In order to find optimum solution for this problem, an exact algorithm called Lexi-search algorithm has been used. A numerical example proposed by M.C. Goldbarg et al.,(2009) has been solved using the lexi-search algorithm. The feasible solutions are searched systematically and with effective bounding and backtracking strategies using the Lexi-search algorithm which helped in moving towards the optimal solution very quickly and effectively.

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