On Weakly η- regular Closed Sets in Generalized Topological Spaces

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Abstract

In this paper, we introduce a weak form of η-regular closed sets namely weakly η-regular closed sets (briefly, wηr-closed) in generalized topological space. Also we discuss some of its properties. Further we introduce three types of continuous functions using wηr-closed sets and characterise them.

Keywords: *η-regular closed, wηg-sets. (η,δ)continuous functions*

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I. INTRODUCTION

In 2014, Ankit Gupta et el [1] given some decompositions of regular open sets and regular closed sets using PS-regular sets in topological spaces. In 2015, Syed Ali Fathima [6] introduced wηg-closed sets in GTS which is also the weak form the regular closed sets. The purpose of the present paper is to introduce weakly η regular closed sets and different types of continuous function using wηr- closed sets in GTS and discuss their basic properties.

II. PRELIMINARIES

Throughout this paper X,Y and Z means generalized topological space (X,η), (Y,δ) and (Z,Ɛ) respectively on which no separation axioms are assumed unless otherwise explicitly mentioned. The function f:X→Y denote the single valued function of space (X,η) into a space (Y,δ). We recall the following definitions.

Definition 2.1. A generalized topology or simply GT η [3] on a nonempty set X is a collection of subsets of X such that $\phi\in\mu$ and μ is closed under arbitrary union. Elements of η are called η-open sets. A subset A of X is said to be η-closed if A^c is η-open. The pair (X,η) is called a generalized topological space (GTS). If A is a subset of X, then $C_\eta(A)$ is the smallest η-closed set containing A and $i_\eta(A)$ is the largest η-open set contained in A. A space (X,η) is said to be strong if $X\in\eta$.

Definition 2.2. A subset A of X is called a η- generalized closed set [2](briefly ηg-closed set= g_η-closed[5]) iff $C_\eta(A)\subseteq U$ whenever $A\subseteq U$ where U is η-open in X.

Definition 2.3. Let (X,η) be a GTS and $A\subseteq X$. Then A is said to be η-regular closed [4] if $A=C_\eta i_\eta(A)$.

Definition 2.4. A subset A of X is called a PS – regular [1] if $A = i_\sigma C_\sigma(A)$

Definition 2.5. A function f:X→Y is said to be

(i) (η,δ)continuous functions [3] if $f^{-1}(U)$ is η-open in X for every δ-open set U of Y.

(ii) (η, Wηg-δ) continuous functions [8] if $f^{-1}(U)$ is η – open in X for every Wηg – open set U of Y.

(iii)(Wηg-η, δ) continuous functions [8] if $f^{-1}(U)$ is Wηg – open in X for every δ – open set U of Y.

(iv)Wηg-irresolute [8] if $f^{-1}(U)$ is Wηg – open in X for every Wηg – open set U of Y.

III. WEAKLY η - REGULAR CLOSED SETS

DEFINITION 3.1. Let (X,η) be a GTS. A subset A of X is called weakly η -regular closed set (or in short, Wηr-closed) if $C_\eta i_\eta(A) \subseteq U$; whenever $A \subseteq U$ and U is ηg -open. The complement of a Wηr-closed set is called a Wηr-open set and WηrO(X) denotes that set of all Wηr-opens sets in X.

EXAMPLE 3.2. Let X={p_1,p_2,p_3} with GT η={ Ø,X,{p_1},{p_1,p_2}}. Here Wηr-closed sets are {Ø,X,{p_2},{p_3} {p_1,p_2}, {p_2,p_3}}.

THEOREM 3.3. Every η closed set is Wηr – closed set.

PROOF: Let A be any η closed set and $A \subseteq U$ where U is ηg -open . Since A is η-closed set $C_\eta(A)=A$. Also $i_\eta(A) \subseteq A \subseteq C_\eta(A) = A$. Then $i_\eta(A) \subseteq A$. Thus $C_\eta i_\eta(A) \subseteq C_\eta(A) = A \subseteq U$. This implies $C_\eta i_\eta(A) \subseteq U$. Therefore A is Wηr - closed.

Converse of the theorem need not be true as seen from the following example.

EXAMPLE 3.4. Let X={p_1,p_2,p_3} with GT η = { Ø,X,{p_1},{p_2,p_3}{p_1,p_3}}.Now Wηr–closed sets={ Ø,X,{p_1},{p_2}, {p_3},{p_2,p_3}{p_1,p_2}}. It is easy to verify that {p_3} and {p_1,p_2} are Wηr –closed but not η- closed.

THEOREM 3.5. Every η - regular closed set is wηr – closed.

PROOF: Suppose A is any η - regular closed set in X. Then $C_\eta i_\eta(A) = A$;whenever $A \subseteq U$ and U is ηg -open. This implies $C_\eta i_\eta(A) = A \subseteq U$; Clearly $C_\eta i_\eta(A) \subseteq U$; whenever $A \subseteq U$ and U is ηg -open. Hence A is Wηr – closed.

Converse of the above theorem need not be true as seen from the following example.

EXAMPLE 3.6. Let X={p_1,p_2,p_3} with GT η = { Ø,X,{p_1},{p_2,p_3}{p_1,p_3}}; Then η regular closed ={{p_1},{$p_2$$p_3$}} and Wηr - closed ={ Ø,X,{p_1},{p_2},{p_3},{p_2,p_3}{p_1,p_2}}. Here easy to observe that {p_2}{p_3}{p_1,p_2} are Wηr – closed but not η regular closed set.

THEOREM 3.7. Every Wηr – closed set is Wηg- closed set.

PROOF: Let A be the Wηr – closed set and $A \subseteq U$ and U is η -open. We know that every η-open is ηg-open. Thus U is ηg - open and A is Wηr – closed. That is $C_\eta i_\eta(A) \subseteq U$. Hence A is Wηg – closed.

Converse of the above theorem false as seen from the following example.

EXAMPLE 3.8. Let X={p_1,p_2,p_3} with GT η={Ø,X,{p_1}}. Then (X,η). Here Wηr – closed ={Ø,X,{p_2},{p_3},{p_2,p_3}} and Wηg – closed = {Ø,X,{p_2},{p_3},{p_1,p_2}{p_1,p_3}{p_2,p_3}}. Clearly, {p_1,p_2} and {p_1,p_3} are Wηg – closed set but not Wηr – closed set.

THEOREM 3.9. A subset A of a generalized topological space is Wηr – closed then $C_\eta i_\eta(A) \setminus A$ does not contain any non – empty ηg - closed set.

PROOF: Suppose A is Wηr – closed set in X. We assume that U be a ηg - closed set such that $C_\eta i_\eta(A) \setminus A \supseteq U$ and U≠ Ø. Here $U \subseteq C_\eta i_\eta(A) \setminus A$. Then $A \subseteq X \setminus U$; As U is ηg - closed, X \ U is ηg-open. Since A is Wηr – closed set. Then by definition $C_\eta i_\eta(A) \subseteq X \setminus U$; Thus $U \subseteq X \setminus C_\eta i_\eta(A)$. Hence $U \subseteq (C_\eta i_\eta(A)) \cap (X \setminus C_\eta i_\eta(A))$. This implies U=Ø; this is a contradiction to our assumption. Hence $C_\eta i_\eta(A) \setminus A$ does not contain any non empty ηg - closed sets in X.

REMARK 3.10: The union of two Wηr – closed sets in GTS is generally not Wηr – closed set.

EXAMPLE 3.11. Let X={p_1,p_2,p_3} with η={Ø,X,{p_1,p_2}}.Here Wηr – closed ={Ø,X,{p_1},{p_2},{p_3},{p_1p_3}{p_2,p_3}}. Now consider A = {p_1} and B = {p_2}. Then A and B are Wηr – closed sets in X. But AUB ={p_1,p_2} is not Wηr – closed set in X.

THEOREM 3.12: If a Wηr – closed subset A of a GTS in X be such that $C_\eta i_\eta$(A) \ A is ηg - closed. Then A is η– regular closed.

PROOF: Let A be a Wηr – closed subset such that $C_\eta i_\eta$(A) \ A is ηg - closed. Then $C_\eta i_\eta$(A) \ A is ηg - closed subset of itself. Then by Theorem [3.9] $C_\eta i_\eta$(A) \ A =Ø. That implies $C_\eta i_\eta$(A) = A. Hence A is η- regular closed set.

Converse of the theorem is false as seen from the following example.

EXAMPLE 3.13: Let X={p_1,p_2,p_3} with GT η = { Ø,,{p_1},{p_1,p_2}}. Here η- regular closed set are {X,{p_3}} and Wηr – closed ={X,{p_3},{p_1,p3},{p_2,p_3}}. Let A = {p_3}; A is both Wηr - closed and η- regular closed set, but $C_\eta i_\eta$(A) \ A = Ø. Ø is not ηg - closed set.

COROLLARY 3.14: If a Wηr – closed subset A of a GTS (X,η) be such that $C_\eta i_\eta$(A) \ A is ηg- closed, then A is PS – regular.

PROOF: Every η- regular closed set is PS – regular [by theorem 16[1]]. Now by theorem [3.12] A is η- regular closed. This implies A is PS – regular.

THEOREM 3.15: For every point x of a strong GTS X. X \ {x} is Wηr – closed or ηg - open.

PROOF: Suppose X \ {x} is not ηg - open. Then X is the only ηg-open set containing X \ {x}. This implies C_η i_η(X \ {x}) ⊆ X. Hence X \ {x} is Wηr – closed.

REMARK 3.16: In GTS, ηg - closed sets and Wηr – closed sets are independent.

EXAMPLE 3.17: Let X={p_1,p_2,p_3} with GT η = { Ø,X,{p_1},{p_1,p_3}}. Then ηg - closed = {Ø,X,{p_2},{p_1,p_2},{p_2,p_3}} and Wηr – closed = {Ø,X,{p_2},{p_3},{p_1,p_2},{p_2,p_3}}. It is easy to verify that {p_3} is Wηr – closed but not ηg - closed set.

EXAMPLE3.18: Let, X={p_1,p_2,p_3}with GT η={Ø,X,{p_1}}. Then ηg-closed={Ø,X,{p_2},{p_3},{p_1,p_2},{p_2,p_3},{p_1,p_3}} and Wηr – closed = {Ø,X,{p_2},{p_3},{p_2,p_3}}. It is easy to see that {p_1,p_2} and {p_1,p_3} are ηg - closed but not Wηr – closed set.

THEOREM 3.19: If A is a Wηr – closed subset of GTS such that A ⊆ B ⊆ $C_\eta i_\eta$(A) then B is also Wηr – closed.

PROOF: Let U be a ηg - open set in (X,η) such that B ⊆ U; Since A is Wηr – closed, $C_\eta i_\eta$(A) ⊆ U. Now, B ⊆ $C_\eta i_\eta$(A) this implies $C_\eta i_\eta$(B) ⊆ $C_\eta i_\eta$($C_\eta i_\eta$(A)) ⊆ $C_\eta i_\eta$(A) ⊆ U. Thus $C_\eta i_\eta$(B) ⊆ U. Hence B is Wηr – closed.

Converse part of the theorem is not true as shown from the following example.

EXAMPLE 3.20: Let X={p_1,p_2,p_3,p_4} with GT η={Ø,{p_2},{p_1,p_2}}.Then Wηr-closed={X,{p_3,p_4}, {p_1,p_3,p_4}}. Now consider, A = {p_3,p_4} and B = {p_1,p_3,p_4}. Clearly A ⊆ B; A and B are Wηr – closed. But B is not a subset of $C_\eta i_\eta$(A).

THEOREM 3.21: A subset A of a GTS X is Wηr – open if and only if F ⊆ $i_\eta C_\eta$(A); whenever F⊆A and F is ηg - closed.

PROOF: Let A be any Wηr – open. Then A^c is Wηr – closed. Let F be ηg - closed set contained in A. Then F^c is a ηg - open set containing A^c. Since A^c is Wηr – closed; $C_\eta i_\eta(A^c)$ ⊆ F^c, This implies F⊆ $i_\eta C_\eta$ (A)

Conversely, Suppose that $F \subseteq i_\eta C_\eta(A)$; whenever $F \subseteq A$ and F is ηg - closed. Then F^c is ηg - open set containing A^c and $F \subseteq i_\eta C_\eta(A)$. Thus $C_\eta(i_\eta(A^c)) \subseteq F^c$. THerefore A^c is Wηr – closed. This implies A is Wηr – open.

THEOREM 3.22: If $A \subseteq X$ is Wηr – closed then $C_\eta i_\eta(A) \setminus A$ is Wηr – closed.

PROOF: Suppose A is Wηr – closed and $F \subseteq C_\eta i_\eta(A) \setminus A$; where F is ηg - closed subset of X. Then by theorem [3.9] $C_\eta i_\eta(A) \setminus A = Ø$,F =Ø. Hence F ⊆$[i_\eta C_\eta(A)(C_\eta i_\eta(A) \setminus A)]$. Then by theorem [3.19] $C_\eta i_\eta(A) \setminus A$ is Wηr – closed.

IV. WEAKLY η – REGULAR CONTINUITY

DEFINITION 4.1. Let (X,η) and (Y,δ) be generalized topological space's. Then a mapping f:X→Y is said to be

(i) (η, Wηr-δ) continuous if $f^{-1}(U)$ is η – open in X for every Wηr – open set U of Y.
(ii) (Wηr-η, δ) continuous if $f^{-1}(U)$ is Wηr – open in X for every δ – open set U of Y.
(iii) (Wηr-η, Wηr-δ) continuous if $f^{-1}(U)$ is Wηr – open in X for every Wηr – open set U of Y.

EXAMPLE 4.2: Let X = Y=$\{p_1,p_2,p_3\}$ with GT η = {Ø, X, $\{p_1\}$, $\{p_2,p_3\}$, $\{p_1,p_3\}$} and δ = { Ø, Y, $\{p_1\}$, $\{p_1,p_3\}$}. Then WηrO(Y) = {Ø, X, $\{p_1\}$, $\{p_3\}$, $\{p_1,p_3\}$, $\{p_1,p_2\}$}. A mapping f: (X,η)→ (Y,δ) is defined by $f(p_1) = p_3$, $f(p_2) = p_1$, $f(p_3) = p_1$. Then f is (η, Wηr-δ) continuous.

EXAMPLE 4.3: Let X = $\{p_1,p_2,p_3\}$ = Y with GT η = {Ø, X, $\{p_2,p_3\}$, $\{p_1,p_2\}$} and δ = { Ø, Y, $\{p_1\}$, $\{p_2\}$, $\{p_1,p_2\}$}. Then WηrO(X) = {Ø, X, $\{p_2\}$, $\{p_2,p_3\}$, $\{p_1,p_3\}$, $\{p_1,p_2\}$}. A mapping f: (X,η) → (Y,δ) is defined by $f(p_1) = p_2$, $f(p_2)=p_2$, $f(p_3) = p_2$. Then f is (Wηr-η,δ) continuous.

EXAMPLE 4.4: Let X = Y = $\{p_1,p_2,p_3\}$ with GT η = {Ø, X, $\{p_1\}$, $\{p_1,p_3\}$} and δ = { Ø, Y, $\{p_1\}$}. Then WηrO(X) = {Ø, X, $\{p_1\}$, $\{p_3\}$, $\{p_1,p_3\}$, $\{p_1,p_2\}$} and WηrO(Y) = {Ø, X, $\{p_1\}$, $\{p_1,p_3\}$, $\{p_1,p_2\}$}. A mapping f: (X,η)→(Y,δ) is defined by $f(p_1) = p_2$, $f(p_2) = p_2$, $f(p_3) = p_1$. Then f is (Wηr-η, Wηr-δ) continuous.

THEOREM 4.5: Every (η, Wηr-δ) continuous function is (η,δ) continuous.

PROOF : Let f: (X,η) → (Y,δ) be (η, Wηr-δ) continuous. Let A be a δ – open set in Y. We have every δ – open set is Wηr-δ – open. This implies A is Wηr- δ – open. Since f is (η, Wηr-δ) continuous, $f^{-1}(A)$ is η – open. Therefore f is (η,δ) continuous.

Converse of the theorem is false as seen from the following example.

EXAMPLE 4.6: Let X = Y = $\{p_1,p_2,p_3\}$ with GT η = {Ø, X, $\{p_1\}$, $\{p_2,p_3\}$, $\{p_1,p_3\}$} and δ = { Ø,Y,$\{p_1\}$, $\{p_1,p_3\}$}. Then WηrO(Y) = {Ø, X, $\{p_1\}$, $\{p_3\}$, $\{p_1,p_3\}$, $\{p_1,p_2\}$}. A mapping f: (X,η) → (Y,δ) is defined by $f(p_1) = p_1$, $f(p_2)=p_3$, $f(p_3) = p_1$. Then f is (η,δ) continuous but not (η, Wηr-δ) continuous.

THEOREM 4.7: Every (η,δ) continuous function is (Wηr-η,δ) continuous.

PROOF : Let f: (X,η) → (Y,δ) be (η,δ) continuous. Let A be a δ – open set in Y. Since f is (η,δ) continuous, $f^{-1}(A)$ is η – open and every η – open set is Wηr-η – open. This implies $f^{-1}(A)$ is Wηr-η – open. Hence f is (Wηr-η,δ) continuous.

Converse of the theorem is not true as seen from the following example.

EXAMPLE 4.8: Let X = Y = $\{p_1,p_2,p_3\}$ with GT η = {Ø, X, $\{p_1\}$, $\{p_1,p_3\}$} and δ = { Ø, Y, $\{p_1\}$}. Then WηrO(X) = {Ø, X, $\{p_1\}$, $\{p_3\}$, $\{p_1,p_3\}$, $\{p_1,p_2\}$}. A function f: X → Y is defined by $f(p_1) = p_2$, $f(p_2) = p_3$, $f(p_3) = p_1$. Then f is (Wηr-η,δ) continuous but not (η,δ) continuous.

THEOREM 4.9: Every (Wηr-η,δ) continuous function is (Wηg-η,δ) continuous.

PROOF : Let f: (X,η) → (Y,δ) be (Wηr-η,δ) continuous. Let A be a δ – open set in Y. Since f is (Wηr-η,δ) continuous. $f^{-1}(A)$ is Wηr-η – open. Also Every Wηr – open set is Wηg – open. This implies $f^{-1}(A)$ is Wηg-η – open. Therefore f is (Wηg-η,δ) continuous.

Converse of the above theorem is false as seen from the following example.

EXAMPLE 4.10: Let X = {p_1,p_2,p_3} = Y with GT η = {Ø, X, {p_1}} and δ = { Ø, Y, {p_1}, {p_1,p_3}}. Then WηrO(X) = {Ø, X, {p_1}, {p_1,p_3}, {p_1,p_2}} and WηgO(X) = {Ø, X, {p_1}, {p_3}, {p_2}, {p_1,p_3}, {p_1,p_2}}. A mapping f: X→Y is defined by f(p_1) = p_3, f(p_2) = p_1, f(p_3) = p_2. Then f is (Wηg-η,δ) continuous but not (Wηr-η,δ) continuous.

THEOREM 4.11: If f:(X,η) → (Y,δ) is (Wηr-η, Wηr-δ) continuous and g: (Y,δ) → (Z,Ɛ) is (δ, Wηr-Ɛ) continuous. Then gof : (X,η) → (Z,Ɛ) is (Wηr-η, Wηr-Ɛ) continuous.

PROOF: Let U be Wηr-Ɛ open in Z. Since g is (δ, Wηr-Ɛ) continuous, $g^{-1}(U)$ is δ- open in Y. Further, Every δ – open set is Wηr-δ – open. This implies $g^{-1}(U)$ is Wηr-δ – open in Y. Since f is (Wηr-η, Wηr-δ) continuous, $f^{-1}(g^{-1}(U))$ is Wηr-η – open in X. Hence gof is (Wηr-η, Wηr-Ɛ) continuous.

THEOREM 4.12: If f: X → Y is (Wηr-η,δ) continuous and g: Y → Z is (δ,Ɛ) continuous. Then gof : X → Z is (Wηr-η,Ɛ) continuous.

PROOF: Let V be Ɛ – open set in Z. Since g is (δ,Ɛ) continuous, $g^{-1}(V)$ is δ – open in Y. Since f is (Wηr-η, δ) continuous, $f^{-1}(g^{-1}(V))$ is Wηr-η – open in X. Hence gof is (Wηr-η,Ɛ) continuous.

THEOREM 4.13: If f: (X,η) → (Y,δ) is (η, Wηg-δ) continuous and g: (Y,δ) → (Z,Ɛ) is (Wηr-δ,Ɛ) continuous. Then gof : (X,η) → (Z,Ɛ) is (η,Ɛ) continuous.

PROOF: Let U be Ɛ – open in Z. Since g is (Wηr-δ, Ɛ) continuous, $g^{-1}(U)$ is Wηr-δ –open in Y. We have every Wηr-δ – open set is Wηg-δ – open. This implies $g^{-1}(U)$ is Wηg-δ – open in Y. Since f is (η, Wηg-δ) continuous, $f^{-1}(g^{-1}(U))$ is η – open in X. Therefore gof : (X,η)→ (Z,Ɛ) is (η,Ɛ) continuous.

THEOREM 4.14: If f: X → Y is (Wηr-η,Wηr-δ) continuous and g: Y → Z is (Wηr-δ,Wηr-Ɛ) continuous. Then gof:X→Z is (Wηr-η, Wηr-Ɛ) continuous.

PROOF: Let V be Wηr-Ɛ – open set in Z. Since g is (Wηr-δ, Wηr-Ɛ) continuous, $g^{-1}(V)$ is Wηr-δ – open in Y. Also f is (Wηr-η, Wηr-δ) continuous. This implies $f^{-1}(g^{-1}(V))$ is Wηr-η – open in X. Hence gof is (Wηr-η, Wηr-Ɛ) continuous.

REMARK 4.15 : If f: (X,η) → (Y,δ) is (Wηr-η, δ) continuous and g: (Y,δ) → (Z,Ɛ) is (Wηr-δ, Ɛ) continuous. Then gof : (X,η) → (Z,Ɛ) is not (Wηr-η,Ɛ) continuous.

EXAMPLE 4.16: Let X = Y = Z= {p_1,p_2,p_3} with GT η = {Ø, X, {p_1}, {p_2}, {p_1,p_2}} and δ = { Ø, Y, {p_1}, {p_1,p_3}} and Ɛ = {Ø, Z, {p_1}, {p_2}, {p_1,p_2}}. Then WηrO(X) = {Ø, X, {p_1}, {p_2}, {p_1,p_2}} and WηrO(Y) = {Ø, Y, {p_1}, {p_3}, {p_1,p_3}, {p_1,p_2}}. Mapping f: (X,η) → (Y,δ) is defined by f(p_1) = p_1, f(p_2) = p_3, f(p_3) = p_2 and g:(Y,δ) →(Z,Ɛ) is defined by g(p_1) = p_1, g(p_2) = p_1, g(p_3) = p_2. Then clearly f is (Wηr-η, δ) continuous and g is (Wηr-δ, Ɛ) continuous. But gof : (X,η) → (Z,Ɛ) is not (Wηr-η,Ɛ) continuous

THEOREM 4.17 . If f: X → Y is (η, Wηr-δ) continuous and g: Y → Z is (δ, Wηr-Ɛ) continuous then gof : X → Z is (η, Wηr-Ɛ) continuous.

PROOF: Let V be Wηr-Ɛ – open set in Z. Since g is (δ, Wηr-Ɛ) continuous, $g^{-1}(V)$ is δ – open in Y. Also every δ – open set is Wηr-δ – open. Therefore $g^{-1}(V)$ is Wηr-δ – open in Y Since f is (η, Wηr-δ) continuous, $f^{-1}(g^{-1}(V))$ is η – open in X. Hence gof is (η, Wηr-Ɛ) continuous.

THEOREM 4.18: If f: (X,η) → (Y,δ) is (Wηr-η, Wηg-δ) continuous and g: (Y,δ) → (Z,Ɛ) is (Wηr-δ, Wηg-Ɛ) continuous. Then gof : (X,η) → (Z,Ɛ) is (Wηr-η, Wηg-Ɛ) continuous.

PROOF: Let U be Wηg-Ɛ open in Z. Since g is (Wηr-δ, Wηg-Ɛ) continuous, $g^{-1}(U)$ is Wηr-δ – open in Y. We have every Wηr-δ – open set is Wηg-δ – open. This implies $g^{-1}(U)$ is Wηg-δ – open in Y. Since f is (Wηr-η, Wηg-δ) continuous, $f^{-1}(g^{-1}(U))$ is Wηr-η – open in X. Hence gof is (Wηr-η, Wηg-Ɛ) continuous.

THEOREM 4.19: If f: (X,η)→(Y,δ) is (η,δ) continuous and g:(Y,δ)→(Z,Ɛ) is (δ,Ɛ) continuous. Then gof:(X,η)→(Z,Ɛ) is (η,Ɛ) continuous.

PROOF: Let U be Ɛ – open in Z. Since g is (δ,Ɛ) continuous, $g^{-1}(U)$ is δ- open in Y. Since f is (η,δ) continuous, $f^{-1}(g^{-1}(U))$ is η – open in X. Hence gof is (η,Ɛ) continuous.

V. Conclusions

In this paper, we have introduced the new class of sets namely weakly η-regular closed sets in generalized topological spaces and discussed its basic properties. Further we have defined three types of continuous function using weakly η-regular closed set and characterized them.

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