α_1 −Near Subtraction Semigroups

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Abstract

In this paper we introduce the concept of α_1- near subtraction semigroups and we have to establish α_1-near subtraction semigroups and investigate some of their characteristics.

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I. Introduction

The theory of near subtraction semigroups is a fast going branch of abstract algebra. **In[1],** Dheena.P defined that A non-empty subset X together with two binary operations "–" and "." is said to be **Subtraction semigroup** If **(i)** (X,–) is a subtraction algebra **(ii)** (X, .) is a semigroup**(iii)** $x(y–z) = xy–xz\ and\ (x–y)z = xz–yz\quad for\ every\ x, y, z \in X.$ and also defined A **Near Subtraction Semigroup** is an algebraic system consisting of a set X and two binary operations '–' and '**.**' Such that (X,–) is subtraction algebra, (X, **.**) is a semigroup and $(a–b).c\ =\ a.c–b.c\ \ for\ all\ a, b, c \in X.$

The notion of near subtraction semigroup are fundamental notions of subtraction algebra. Many researchers have investigated the basic properties of near subtraction semigroup. For basic definition one may refer to Pillz[2]. Motivated by these concept, we establish a Structure in near subtraction semigroup and also discuss various types of near subtraction algebra and obtained equivalent conditions for regularity. More precisely the above near subtraction semigroup is a right near subtraction semigroup..

The purpose of the present paper is to make a systematic study of regularities in structure of near subtraction semigroup. In this Paper challenges of a new type of Regularities in α_1 −Near Subtraction Semigroup are discussed. We discuss some properties of $S\ \alpha_1$ − near subtraction semigroup. We find some characterisation of α_1-near subtraction semigroup without non-zero zero divisors. Finally, We introduce some properties of $S\ \alpha_1$ −near subtraction semigroup with property (α) and their generalizations

II. Preliminaries

Definition: 2.1

A subtraction semigroup X is said to have **Insertion of Factors Property**

(IFP) if for $a, b\ \ in\ X, ab\ =\ 0\ \Rightarrow axb = 0\ for\ all\ x\ \in\ X.$

Definition: 2.2

A near subtraction semigroup X is called an **S - near subtraction semigroup**. If $a \in Xa\ for\ all\ a\ in\ X.$

Definition: 2.3

A near subtraction semigroup X has **(* , IFP)** if (i) X has IFP and (ii) for $a, b \in X, ab = 0 \Rightarrow ba = 0$.

Definition: 2.4

A near subtraction semigroup X has **strong IFP** if for all ideals I of X and $for\ all\ a, b, n\ in\ X, ab\epsilon I \Rightarrow anb \in I$.

Definition:2.5

A near subtraction semigroup X is said to be **Von-Neumann regular** $if\ for\ every\ a \in X, there\ exists\ x\ \in\ X\ such\ that\ a\ =\ axa$.

Definition: 2.6

A near subtraction semigroup X is said to be **Boolean Near Subtraction Semigroup** if $a^2 = a\ for\ all\ a\ in\ X$.

Definition: 2.7

A near subtraction semigroup X is said to have **property P_4** if $for\ all\ ideals\ I\ of\ X\ and\ for\ all\ x, y\ in\ X, xy \in I\ \Rightarrow yx \in I$.

Definition: 2.8

A near subtraction semigroup X is said to be **weak commutative Near Subtraction Semigroup** if $abc =\ acb\ for\ all\ a, b, c \in\ X$.

Definition: 2.9

A near subtraction semigroup X is called **strongly regular Near Subtraction Semigroup** if $axa = xa^2\ for\ each\ a\ in\ X$.

3.α_1- Near subtraction semigroup

Definition: 3.1

A near subtraction semigroup X is said to be **α_1- near subtraction semigroup** if for every a ϵ X there exists x ϵ X such that a = xax.

Example: 3.2

Let X = {0, a, b, c} in which '**-**' and'**.**' Are defined by

-	**0**	**a**	**b**	**c**
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

.	**0**	**a**	**b**	**c**
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

X is a α_1-near subtraction semigroup.

Proposition:3.3

Let X be an α_1- near subtraction semigroup. for every a in X there exists some x in X such that

(i) $a^2x = xa^2$

(ii) $a = x^n a\, x^n$ $for\ all\ n \geq 1$

Proof: Since X be an α_1 near subtraction semigroup and a ϵ X. Then there exists $x\ in\ X$ such that $a = xax$. (i) $xa^2 = (xa)a = xa(xax) = (xax)ax = a(ax) = a^2x$. Hence $x\,a^2 = a^2x$ for every $a\ in\ X$. (ii) $xax = x(xax)x = x^2a\,x^2 = x^2(xax)\,x^2 = \dots. = x^n a\, x^n$ for all $n \geq 1$

Proposition: 3.4

Let X be a regular near subtraction semigroup. If X is weak commutative then X is an α_1 –near subtraction semigroup.

Proof: Since X is regular for every $a\,\epsilon\, X$ there exists $b\,\epsilon\,X$ such that $aba = a$. Let $x = ab$ then $xax = (ab)a(ab) = (aba)ab = a(ab)aba$ .Consequently$xax\ a\ for\ every\ a\ in\ x$. Thus X becomes an α_1 - near subtraction semigroup.

Proposition:3.5

Let X be a zero symmetric weak commutative α_1-near subtraction semigroup. Then for any $a, b\ in\ X$,$ab = 0\ implies.ba = 0$

Proof: Suppose $ab = 0\ for\ a, b\epsilon X$, Since x is an α_1 – near subtraction semigroup there exists

$x, y\,\epsilon\, X$ Such that $xax = a$ and $yby = b$, .Also, sinceX is weak commutative Now, $ba = (yby)(xax) = yb(yxa)x = yb(yax)x = y(bay)x^2 = (yab)yx^2 = 0$.

Proposition:3.6

Let X be a near subtraction semigroup. Then X is an α_1- near subtraction semigroupiff X is zero symmetric.

Proof: For the only if part, We take $x\,\epsilon\, X$ Since X is an α_1- near subtraction semigroup there exists $a\,\epsilon\, X\ such\ that\ xax = a$ --------(1)We shall prove that $x^k ax^k = a$------(2) for all positive integers k.Equation (1) demands that (2) is true for $k = 1$,We assume that the result is true for $k = s - 1$,$If\ k = s\ then\ x^s ax^s = x^{s-1}(xax)\,x^{s-1} = x^{s-1}(x)\,x^{s-1} = \cdots. = xax = a$for $x^k ax^k = 0$,For the if part, let $a\,\epsilon\, X$,If $x = 0$ therefore $x\,a\,x = x\,a(0) = 0$. Thus X is an α_1-near subtraction semigroup.

Proposition:3.7

Let X be a zero symmetric α_1- near subtraction semigroup. If X has no non-zero zero-divisors then every X- system and every ideal of X is an α_1- near subtraction semigroup in its own right.

Proof: Let A be an X-system of X and a ϵ A.Let $n = xa\,\epsilon\, X\,A\ implies\ that\ n\epsilon A$.Since X is an α_1- near subtraction semigroup there exists $x\,\epsilon\, X$ Such that $axa = x$.We take $n = a\epsilon A$ Since A is an X-system of $x, n\,\epsilon\, A$. since X has no non-zero zero-divisors,$n \neq 0$.Now,$ana = a(xa)a = (axa)a = xa = n$.If $a = 0$, since X is zero symmetric, $ana = n$ for any $n\epsilon A$.Thus A is an α_1- near subtraction semigroup.Next, Let I be an ideal of X and let $a\,\epsilon\, I$. If a=0 then ana=n for any nϵI. Suppose a≠0.Since X is an α_1- near subtraction semigroup there exists $x\,\epsilon\, X$ such that $axa = a$. If $i = a\,\epsilon I$. Since I is an ideal of x we get i ϵ I our hypothesis demands that $i \neq 0$.Now, $aia = a(ax)a = a(axa) = a(x) = ax = i$. Thus I is an α_1- near subtraction semigroup.

Theorem:3.8

Let X be a commutative zero symmetric s-α_1-bi near subtraction semigroup. Then the following are equivalent.

(i) X has IFP
(ii) X has (*,IFP)
(iii) X has property P_4
(iv) X has strong IFP

Proof : Since X is a α_1 near subtraction semigroup for every $a\epsilon X$ there exists b in x such that $aba = b$ and $bab = a$.

(i) =>(ii)

Assume that X has IFP Now, If $ab = 0$ and $ba = xbax = xabx = ab = 0$. Thus X has (*,IFP)

(ii)=>(iii)

Assume that X has (*,IFP) Let $a, b\epsilon X$.Let I be an ideal of X. suppose abϵI. Now, $ba = xbax = xabx = ab\epsilon I$, Thus X has property P_4

(iii)=>(iv)

Assume that X has propery P_4. Now, $axb = (bab)xb = ba(bxb) = bax = (ab)x$ =x(a b)ϵ X I $\subset$ I . Thus $axb \epsilon I$. Thus X has strong IFP.

(iv)=(i)

Assume that X has strong IFP. Let $ab = 0$. To prove $ba = 0$ Now, $ba = (aba)(bab) = ab(aba)b = a(bbb) = ab = 0$. Hence X has IFP.

Theorem:3.9

Let X be a zero symmetric commutative $\alpha_1$near subtraction semigroup.Then X is regular if and only if strongly regular.

Proof: Assume that X is α_1 near subtraction semigroup for every $a\epsilon X$ there exists and element $x\epsilon X$ such that $axa = x$. since X is regular. Implies $a = (ax)a = (xa)a = xa^2$. Thus X is strongly regular. conversely, Assume that X is strongly regular. Implies $a = xa^2 = (xa)a = (ax)a$. Thus X is regular.

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