Calculation of thin, isotropic circular Plates subject to constant loading by the Generalized Equations of Finite Difference Method

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Abstract

For calculation of circular plates under bending, we used generalized equations of Finite difference method. The algorithm allows taking into consideration finite breaks of required function, her first derivative and the right part of the differential equation without using surrounding points and a special condensation of grid. The shown examples here illustrate the simplicity of the algorithm because the results obtained are satisfactory; compared to those of other researchers, the error is less than ***5%.***

Keywords: *circular Plate, generalized equations, Finite difference method, edge conditions, discontinuity, finite breaks, surroundings points, convergence, constant loading.*

I. INTRODUCTION

Sails, raft, are generally constitute with plates of circular form. Tank scan be considered as hollow ring plate whose thickness varies according to purpose and formatting. In machine elements, washers, toothed gear, piston head, turbine discs, gear box are also hollow plates having circular form.

During the process of their exploitation, these elements are subjected under the action of transversal load (statics and dynamics). The calculation of such elements must be precise, easy to execute and necessary to set up modeling tools for more sophisticated mechanical behavior, and taking into account the specificities of these structures. Despite the practical importance of elements of this type raised in certain number of work**[1, 8-10],**many questions linked to their calculation still remain topicality.

The behavior of those structures is governed by the equations of 2nd order partial derivative which cannot be solved using analytical methods **[1-3].** Hence necessity to resort to Numerical methods, simple, less onerous and reliable. Among the numerical method, the finite element method is the most used but presents a certain number of difficulties such as the formation of rigidity matrix and the problem of required function discontinuity**[4], [5] [7].**

New numerical methods that are simpler and which give more precise solutions are developed by other researchers **[4], [5], [10].**

In this survey, we will focus be put on circular plates of thin constant thickness and isotropic supporting a uniform loading, perpendicular to the plate area and having diverse conditions at the supports. Diametrical or in-plane loading is not taken to account.

For that, we are going to use the generalized equation of the finites differences method, because of their precision and simplicity, that in the aim of their popularization.

II. METHODOLOGY

The resolution methodology is as follows:

- ✓ Transformation of the partial derivatives 4th order deformation equation of a circular plate of thin constant thickness into a system of two differential equations of 2nd order partial derivative.
- ✓ Introduction of new dimensionless parameters in the system of equations obtained and in the equation describing the boundary conditions;
- ✓ Substitution of new differential equations by the generalized equations of the finite difference method, these permits a system of algebraic equation to be obtain.
- ✓ Substitution of boundary conditions
- ✓ Elaboration of a calculation algorithms
- ✓ Resolution of the system of algebraic equation in order to obtain the bending moment and the maximum displacement.

2-1)Differential deformation equation of a circular plate on elastic base

In the following paragraphs, only circular plates will be analysed, so, it is convenient to express the governing differential equation of the circular plate in polar coordinates. **Figure 1.1** illustrates the equilibrium of a circular plate element[**11**]:

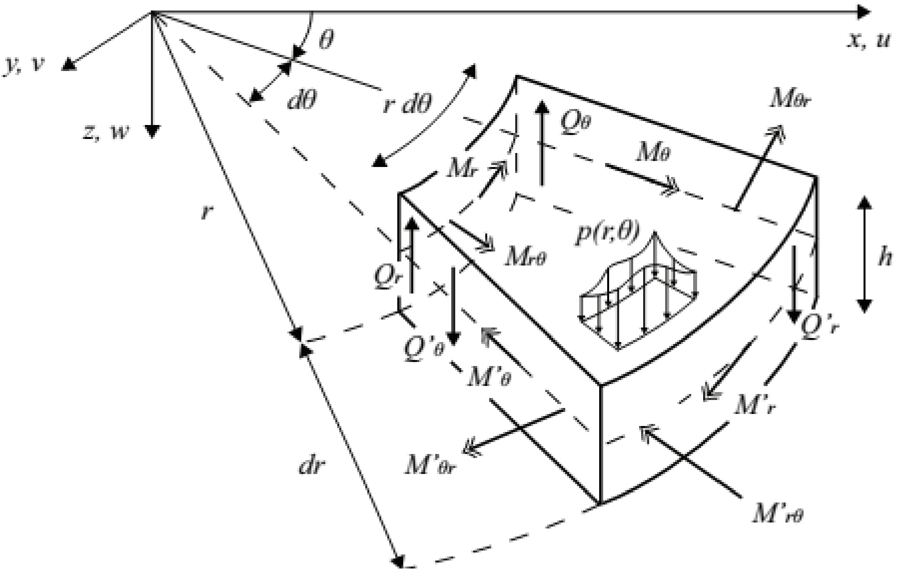


Fig. 1.1: Equilibrium of a circular plate element**[11]**

The deformation equation of an isotropic circular plate, thin and having a constant rigidity is written in polar coordinate **[2]**as:

$$\frac{\partial^4 w}{\partial r^4}+\frac{2}{r}\frac{\partial^3 w}{\partial r^3}-\frac{1}{r^2}\frac{\partial^2 w}{\partial r^2}+\frac{1}{r^3}\frac{\partial w}{\partial r}+\frac{4}{r^4}\frac{\partial^2 w}{\partial \theta^2}-\frac{2}{r^3}\frac{\partial^3 w}{\partial \theta^2 \partial r}+\frac{2}{r^2}\frac{\partial^4 w}{\partial \theta^2 \partial r^2}+\frac{1}{r^4}\frac{\partial^4 w}{\partial \theta^4}=-\frac{P(r,\theta)}{D} \quad \textbf{(1)}$$

where:

- *(r, θ)*: polar coordinates
- *W (r,θ)*: transversal deformation of the plate (searched function)
- *p(r, θ)*: load intensity arbitrary distributed;
- $D=\frac{Eh^3}{12(1-\nu^2)}$: cylindrical rigidity of a constant thickness plate

- ***v**:* Poisson coefficient; **E-** Young's Modulus; **h-** plate thickness.

The equation **(1)** can be bringing to a system of two 2nd order partial derivative equations:

$$\begin{cases} \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} = -\frac{M}{D} \\ \frac{\partial^2 M}{\partial r^2} + \frac{1}{r}\frac{\partial M}{\partial r} + \frac{1}{r^2}\frac{\partial^2 M}{\partial \theta^2} = -q \end{cases} \quad \textbf{(2)}$$

where:

$$M = \frac{M_r + M_\theta}{1+\nu} \quad \textbf{(3a)}$$

with:

$$M_r = -D[\frac{\partial^2 w}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)] \;;\textbf{(3b)}$$

$$M_\theta = -D\;[\frac{1}{r}\frac{\partial w}{\partial r} + \nu(\frac{\partial^2 w}{\partial r^2}) + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\;];\;\textbf{(3c)}$$

M_r: bending moment in (e_r) direction;

M_θ: bending moment in (e_θ) direction;

Equations **(2)** are solving using dimensionless parameters and the boundary conditions.

2-2) Introduction of dimensionless parameters

Rewriting the system **(2)** using dimensionless parameters:

- Let define the dimensionless parameters**[8], [10]** :

$$\xi = \frac{X}{a};\eta = \frac{Y}{a};\; P = \frac{q(x,y)}{q_0};\; m = \frac{M}{P_0 a^2}\;\;;\omega = \frac{WD}{P_0 a^4};m^{(\xi)} = \frac{M_X}{P_0 a^2};m^{(\eta)} = \frac{M_y}{P_0 a^2}\textbf{(4)}$$

$$\theta = \frac{\eta}{\rho}; \qquad r = \rho a\textbf{(5)}$$

where:

ξ, η – Cartesian coordinates without units; ***a***-radius of the plate; ***q_0***- a fixed value of load q.

Introducing the parameter of **(4)** and **(5)** in the system of equations **(2),** we obtain:

$$\begin{cases} \frac{\partial^2 \omega}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \omega}{\partial \rho} + \frac{\partial^2 \omega}{\partial \eta^2} = -m \\ \frac{\partial^2 m}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial m}{\partial \rho} + \frac{\partial^2 m}{\partial \eta^2} = -P \end{cases} \quad \textbf{(6)}$$

with:

$$m \;\; = \frac{m^{(\xi)} + m^{(\eta)}}{1+\mu}\textbf{(7)}$$

where:

$$m^{(\xi)} = -\left(\omega^{\xi\xi} + \mu\omega^{\eta\eta}\right);$$

$$m^{(\eta)} = -\left(\omega^{\eta\eta} + \mu\omega^{\xi\xi}\right);\quad \textbf{(8)}$$

$$m^{(\xi\eta)} = -m^{(\eta\xi)} = (1-\nu)\omega^{\xi\eta}$$

$$\omega^{\eta\xi} = \frac{\partial^2 \omega}{\partial \xi \partial \eta}\;;\quad \omega^{\xi\xi} = \frac{\partial^2 \omega}{\partial \xi^2}\;; \qquad \omega^{\eta\eta} = \frac{\partial^2 \omega}{\partial \eta^2}\;;\textbf{(9)}$$

$$m^{(\xi\eta)} = \frac{M_{XY}}{P_0 a^2}. \tag{10}$$

The systems of 2nd order differential equations **(6),** valid for the intermediate points of the mesh, will be substituted by the generalized equation of the finite difference method to obtain a system of algebraic equation taking account the boundary conditions.

2-3- Boundary conditions

2-3-1fixed edges

A circular plate in section with recessed edges is shown schematically as follows:

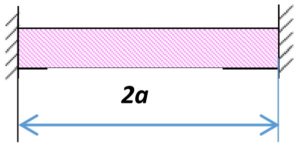


Fig2.1: fixed circular plate

As the edgesare embedded, the maximum displacement and the rotation on this edge are zero. Then:

$(W)_{r=a}$ **=0 (11)**

$(\frac{\partial W}{\partial r})_{r=a} = 0.$ **(12)**

2-3-2) simply supported edge

Figure 2.2 illustrates a circular plate in section with simply supported edges:

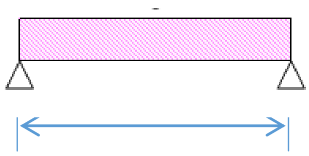


Fig2.2:simply supported circular plate

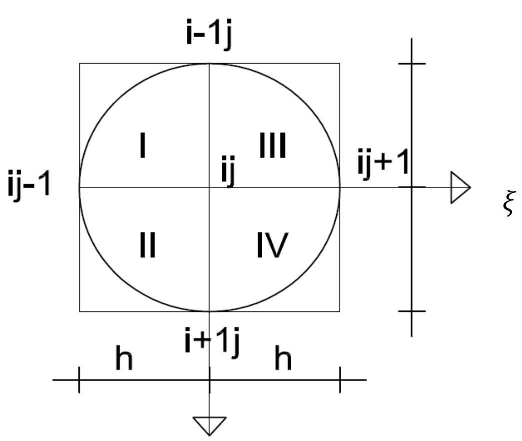
In this case, maximums displacement and moment are both zero, hence:

$(W_r)_{r=a}$ **=0**and $(M_r)_{r=a} = 0.$ **(13)**

III. SUBSTITUTION OF DIFFERENTIAL EQUATIONS BY THE GENERALIZED EQUATION OF THE FINITE DIFFERENCE METHOD

*3-1- **Substitution of equation (6)***

To simplify the equations, we take a circular plate of radius ***r=h*** inscribed in a square of side ***2h***; we choose a coordinate system (ξ,η) as indicated in ***Figure 1***



η

Fig.3.1:circular plate

Hypothesis and data:

- We accept that $\boldsymbol{w}$ and $\boldsymbol{\varphi}$ present a discontinuity at the bound of each element as well as their first derivative and varies along the boundary of each element following a parabolic law (spline);
- The digits I, II, III and IV indicates the elements issues which have common node i, j;
- We choose the origin of the coordinate system to be the point *(i, j);*
- Equations **(6)** is a particular case ofthe generalized equation of the finite difference method defines in **[10]**,with$\boldsymbol{\alpha} = \boldsymbol{\beta} = \mathbf{1}\,, \boldsymbol{\delta} = \frac{\mathbf{1}}{\boldsymbol{\rho}}$ *and* $\boldsymbol{\sigma} = \boldsymbol{\gamma} = \mathbf{0}$then **p = -m** andor$\boldsymbol{m} = \boldsymbol{\varphi}$**.**
- Suppose thatω and$\boldsymbol{m}$are continuous together with their first and second derivative;
- As $\boldsymbol{m}$ is known, so does its first and second derivative, the variation of the derivatives is zero and $\boldsymbol{p}$ is constant;
- In the case of regular mesh, $\boldsymbol{h_i} = \boldsymbol{h_{i+1}} = \boldsymbol{h} = \boldsymbol{\tau_j} = \boldsymbol{\tau_{j+1}} = \boldsymbol{\tau}$; $\xi = \rho$; **(14)**

Considering above hypothesis, the first equation of **(6)**give:

$$\left(1-\frac{h}{2\rho_{ij}}\right)m_{i-1,j}+\frac{h^2}{\tau^2}m_{i,j-1}-2\left(1+\frac{h^2}{\tau^2}\right)m_{ij}+\frac{h^2}{\tau^2}m_{i,j+1}+\left(1+\frac{h}{2\rho_{ij}}\right)m_{i+1,j}=-h^2P_{ij}\,.\mathbf{(15a)}$$

The second equation of**(6)** is obtained in replacing $\boldsymbol{m}$ by ω in equation **(15a):**

$$\left(1-\frac{h}{2\rho_{ij}}\right)\omega_{i-1,j}+\frac{h^2}{\tau^2}\omega_{i,j-1}-2\left(1+\frac{h^2}{\tau^2}\right)\omega_{ij}+\frac{h^2}{\tau^2}\omega_{i,j+1}+\left(1+\frac{h}{2\rho_{ij}}\right)\omega_{i+1,j}=-h^2m_{ij}\,.\mathbf{(15b)}$$

Considering**(14)** and after simplification the equations **(15a)** and **(15b)** becomes:

$$\begin{cases} m_{ij}=\dfrac{1}{4}\left[(1-\dfrac{h}{2\rho_{ij}})m_{i-1j}+m_{ij-1}+h^2P_{ij}+m_{ij+1}+(1+\dfrac{h}{2\rho_{ij}})m_{i+1j}\right] \\ \omega_{ij}=\dfrac{1}{4}\left[(1-\dfrac{h}{2\rho_{ij}})\omega_{i-1j}+\omega_{ij-1}+h^2m_{ij}+\omega_{ij+1}+(1+\dfrac{h}{2\rho_{ij}})\omega_{i+1j}\right] \end{cases}\mathbf{(16)}$$

with : $\rho_{ij} = \frac{r}{a} = \sqrt{\xi^2+\eta^2}$.

Equation **(16)** is called the generalized equation of the finite difference method for a circular plate, which substitute's equation **(6).**

3-2-substitutionof the Boundary conditions by the generalized equation of the finite difference method

3-1-1-fixed edges

The first equation of system (6), substituted by the generalized equation of the finite difference method of **[13]**,and considering that $\boldsymbol{\alpha} = \boldsymbol{\beta} = \mathbf{1}\,, \boldsymbol{\delta} = \frac{\mathbf{1}}{\boldsymbol{\rho}}$as well as $\boldsymbol{\sigma} = \boldsymbol{\gamma} = \mathbf{0}$**,** we obtain:

- **For the lower edge** $(\eta = -1)$ **:**

$$2\frac{h}{\tau}\omega_{\,i-1,j}+\frac{h}{\tau}\omega_{\,i,j-1}-2\left(\frac{h}{\tau}+\frac{h}{\tau}\right)\omega_{\,i,j}+\frac{h}{\tau}\omega_{\,i,j+1}=-\tau h m_{ij} \qquad \mathbf{(17)}$$

Considering **(14), (17)** becomes:

$$2\omega_{i-1,j}+\omega_{ij-1}-4\omega_{i,j}+\omega_{i,j+1}=-h^2m_{ij}\,\mathbf{(17')}$$

Knowing that $\omega_{i,j-1} = \omega_{i,j} = \omega_{i,j+1} = 0$ for all points on the this edge (fixed edges), it becomes:

$$\omega_{i-1j} = -\frac{h^2}{2} m_{ij} \quad (18)$$

- **For the upper edge** ($\eta = 1$)

Proceeding in the same way like previously, and knowing that $\omega_{i,j+1} = \omega_{i,j-1} = \omega_{i,j} = 0$, for this edge we obtain:

$$\omega_{i+1,j} = \frac{h^2}{-2} m_{i,j} (19)$$

- **For the right edge ($\xi = 1$) :**

According to(17')and considering that, $\omega_{i-1,j} = \omega_{i,j} = \omega_{i+1,j} = 0$ we obtain:

$$\omega_{ij-1} = -\frac{h^2}{2} m_{ij} \; and m_{i,\eta} = \frac{-2\omega_{i,\eta-1}}{h^2} (20)$$

- **For the left edge** ($\xi = -1$)

We have: $2\omega_{ij+1} + \omega_{i-1j} - 4\omega_{ij} + \omega_{i+1j} = -h^2 m_{ij}$ with :

$\omega_{i-1j} = 4\omega_{ij} = \omega_{i+1j} = 0$, we obtained:

$$\omega_{ij+1} = -\frac{h^2}{2} m_{ij} \text{ and } m_{\eta,j} = \frac{-2\omega_{\eta,j+1}}{h^2} (21)$$

3-1- 2)Simply supported edge

According to **(13),** moments and arrows are zero for this contour; it comes down to determining the coefficients of the moments and arrows at the central point(**$\xi = \eta = 0$**):

Taking into account equations **(3a),** we obtain:

$$\frac{\partial^2 W}{\partial r^2}(1-\upsilon) + \frac{M}{D}\upsilon = 0 (22)$$

Introducing the dimensionless parameters define in (4) and (5), we obtain:

$$\frac{\partial^2 \omega}{\partial \rho^2} = -m \frac{\upsilon}{(1-\upsilon)} \quad (23)$$

Introducing equation**(25)** into **(6)** gives the following result:

$$\begin{cases} \omega_{i-1j} - 2\omega_{ij} - 2\omega_{i+1j} = -h^2 m \dfrac{\upsilon}{1-\upsilon} \\ m_{i-1j} - 2m_{ij} - 2m_{i+1j} = -h^2 \rho \dfrac{\upsilon}{1-\upsilon} \end{cases} .(24)$$

Equation**(2)** is solved taking into consideration equations **(18), (19), (20), (21)** and **(23)**

IV. RESULTS AND DISCUSSIONS

4-1-*Circular Plate Fix Around Its Entire Circumference*

In this application, we will examine a circular plate of constant rigidity submitted under a load concentrated in its center and perpendicular to the surface of the plate; it is embedded on all edges. Its characteristics are follows:

P=-5 KN (load intensity)

e=L=10mm (plate thickness)

v=0.3 (Poisson coefficient)

E= 200 00 MPa (Young'smodulus)

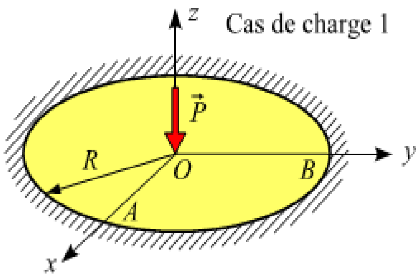


Fig4-1: circular plate recessed around its entire circumference [6]

The calculations will be consisting in the determination of the maximum displacement coefficient and the moment coefficient at the center and edges of the plate, following the mesh.

To do this, we are going to use the system of equation **(16)** for the intermediate points except the points where (i≠j). Equation**(21)** for left boundary, **(20)** for right boundary, **(19)** for upper boundary, **(18)** for lower boundary.

The points where the index i=j will be substituted in the equation **(16)** by letting$\boldsymbol{\omega = v or \omega = m}$.

Let us examine first of all the case of ¼ and **1/6** meshes:

<u>EXAMPLE 1</u>: mesh at 1/4

The plate being symmetrical, we will be interested for the calculation of the coefficients of moments and arrows to the quarter of the circular plate and assign the results to other points.

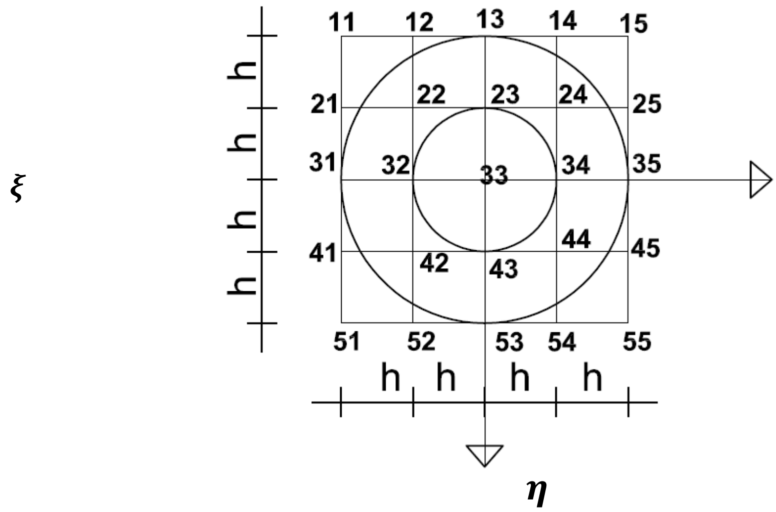


Fig.4.2: mesh at **1/4** of a quarter of the circular plate

Let us consider the right portion of the plate and putting our choice on the points: **22, 23, 24, 31, 33, 34, 35, 43,** and **53**.
The symmetry of the plate allows us to write:

$\boldsymbol{m_{13} = m_{31} = m_{35} = m_{53}}$

$m_{22}=m_{24}= m_{44}= m_{42}$
$m_{32}=m_{23}= m_{34}= m_{43}$
$\omega_{22}= \omega_{24}= \omega_{44}=\omega_{42}$
$\omega_{32}= \omega_{23}= \omega_{34}=\omega_{43}$
$\omega_{13}= \omega_{31} = \omega_{35} = \omega_{53}$

The boundary conditions allow us to pose: $\omega_{13}= \omega_{31}=_{35}= \omega_{53}=0$

Applying the **equations** (16) **, (21),(20)(19) and (18)**at point**35(middle of the edge)** and at point **33(center of the plate),** we obtain the following results:

Summary table 4.1:*values of the coefficients of* **v**and **m***at the central points and the edge of the circular plate meshes1/4*

	***P=-5KN and* ν= 0.3**			
Mesh size	***points***	***Coefficients of the moments in the middle of the edge***	***Coefficients of the moments in the center***	***Coefficients of arrows in the center***
1/4	***35***	***0.1688***	***/***	***0,0000***
	33	***/***	***-0,7528***	***- 0,2321***

The margin of error here is about **1.9%** compared to the values obtained in **[12]**using a numerical program using 199 members of the Fourier series proposed by **[11].**

EXAMPLE 2: mesh at 1/6

Let now consider the mesh 1/6 of a quarter of the circular plate as shown in Fig4.3:

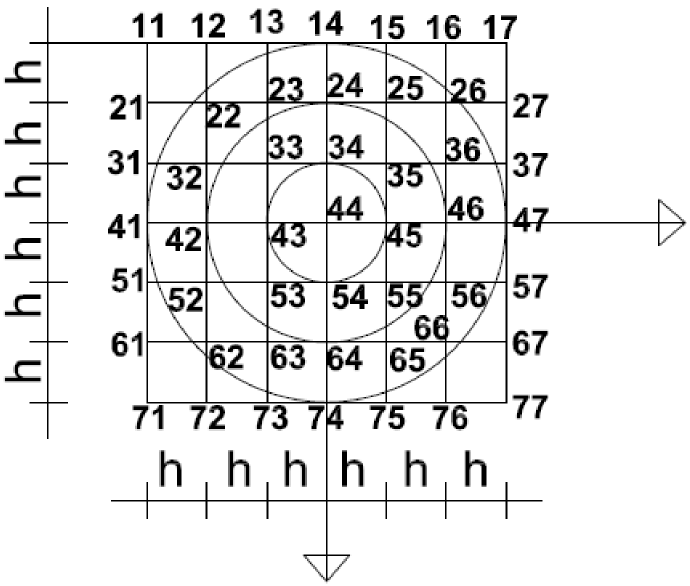


Fig.4.3:mesh at 1/6 of a quarter of the circular plate

Let us consider the right portion of the plate and putting our choice on the points **14, 24, 25, 26, 34, 35, 36, 44, 45, 46, 47.**

The symmetry of our plate permits us to write:

$m_{23}= m_{25} = m_{36} =m_{56}= m_{65} = m_{63} =m_{52}= m_{32}$

$\omega_{23} = \omega_{25} = \omega_{36} = \omega_{56} = \omega_{65} = \omega_{63} = \omega_{52} = \omega_{32}$

The boundary conditions **(fix joint)** permit us to pose: $\omega_{14}=\omega_{47} = \omega_{74} = \omega_{41} = 0$

Applying the **equations (16) , (21),(20)(19) et (18)**at point **47(middle of the edge)** and at point **44 (center of the plate),** we obtain the following results:

Summary table 4.2:*values of the coefficients of v at the central points and the edge of the circular plate meshes 1/6*

		P=-5KNand ν***= 0.3***		
Mesh size	***points***	***Coefficients of the moments in the middle of the edge***	***Coefficients of the moments in the center***	***Coefficients of arrows in the center***
1/6	***44***	/	***-0,5496***	***-0,1645***
	47	***0,1566***	/	***0.000***

The margin of error is about **1.7 %** compared to the values obtained in **[12]** using a numerical program using 199 members of the Fourier series proposed by **[11]**

Let now evaluate of the convergence of moment and maximum displacement coefficients for the meshes *1/8; 1/16; 1/24; 1/32; 1/40; 1/48:*

To evaluate the convergence of the values of the moment and arrows coefficients for the central node, as well as for the values of the moment in the middle, at the edges of the plate, we have the following **summary table 4.3**and **4.4**

Summary table 4.3:*values of the coefficients of* ***v*** *at the central points and the edge of the circular plate according to the meshes 1/8; 1/16; 1/24; 1/32; 1/40; 1/48*

	generalized equation of the finite difference method						WO(cm) ***(Kirchhoff theory)***: Reference **[6]**	WO(cm) ***finite elements (DKT)*** **[6]**
Mesh size	***1/8***	***1/16***	***1/24***	***1/32***	***1/40***	***1/48***		***1/48***
w$_c$ (a^2P/D)	**-0.1214**	***-0.0463***	**-0.0232**	**-0.0138**	**-0.0061**	**-0.00560**	**-0.00543**	**-0.00544**

Summary table 4.4:*values of the coefficients of* ***m*** *at the central points and the edge of the circular plate according to the meshes 1/8; 1/16; 1/24; 1/32; 1/40; 1/48*

generalized equation of the finite difference method							Reference**[6]**
Mesh size	***1/8***	***1/16***	***1/24***	***1/32***	***1/40***	***1/48***	
m$_c$ (a^2P)	***-0.4237***	***-0.1510***	***-0.0696***	***-0.0394***	***-0.0253***	***-0.0176***	-
m$_b$ $(10^{-1}a^2P)$	***0.1599***	***0.1195***	***0.0883***	***0.0693***	***0.0570***	***0.0483***	***0.0460***

- The results of the arrow **w**$_c$ obtained on the meshes above are compared to those obtained in **[6]**(Kirchhoff theory and ***(DKT)***) for the mesh ***1/48*** and The margin of error is about

3.5% for Kirchhoff theory and for the ***DKT***.

- We can observe that as we refine the mesh, we get closer and closer to the result of the reference (**-0.00543).**
- The results of the moment $\mathbf{m_b}$ obtained on the meshes above are compared to those obtained in **[6]**for the mesh ***1/48*** and The margin of error is about ***4.7%***
- The refinement of the mesh shows the convergence of the moments $\mathbf{m_c}$ **et** $\mathbf{m_b}$ **;** and this can be better observed on the convergence curves (**Fig.4-4**, **Fig.4-5 et Fig.4-6)** below:

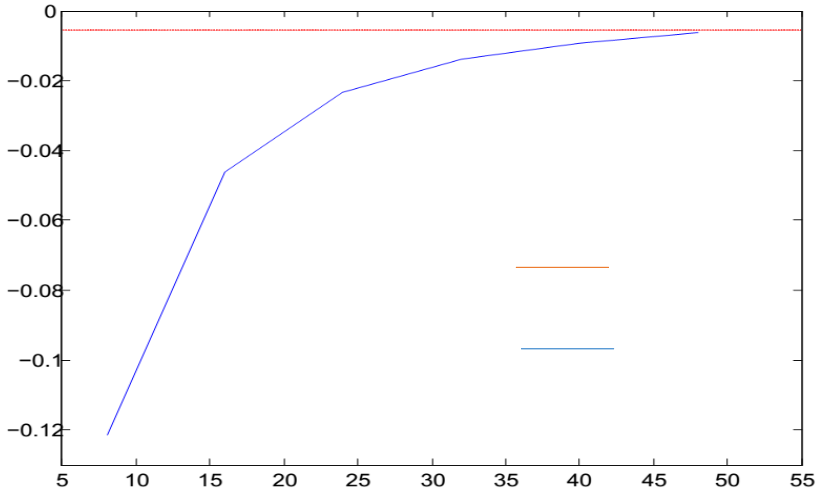


Fig 4-4*:*convergence curve of the arrows coefficients at the center of the circular plate for the meshes: 1/8; 1/16; 1/24; 1/32; 1/40; 1/4

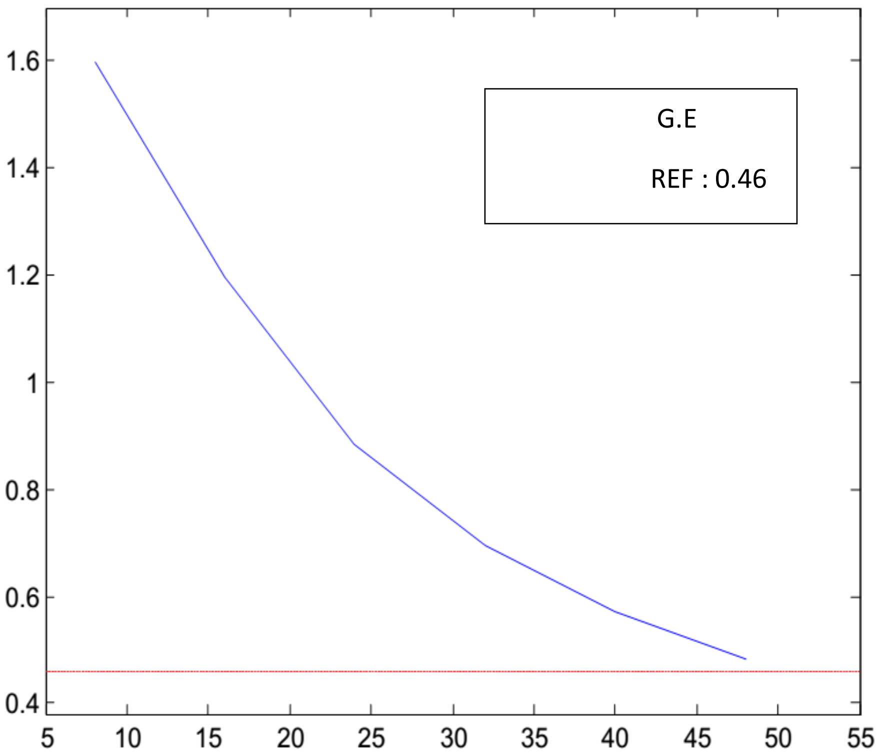


Fig4-5:convergence curve of the moments coefficients at the edge of the circular plate for the meshes 1/8; 1/16; 1/24; 1/32; 1/40; 1/48

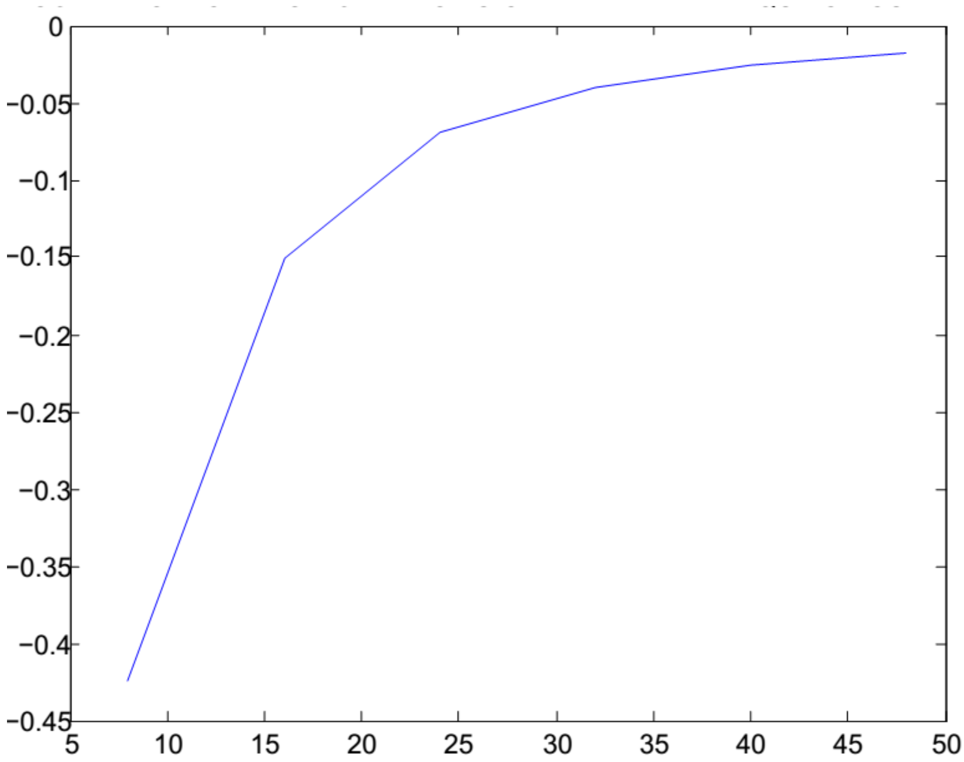


Fig4-6:convergence curve of the moments coefficients at the center of the circular plate for the meshes 1/8; 1/16; 1/24; 1/32; 1/40; 1/48

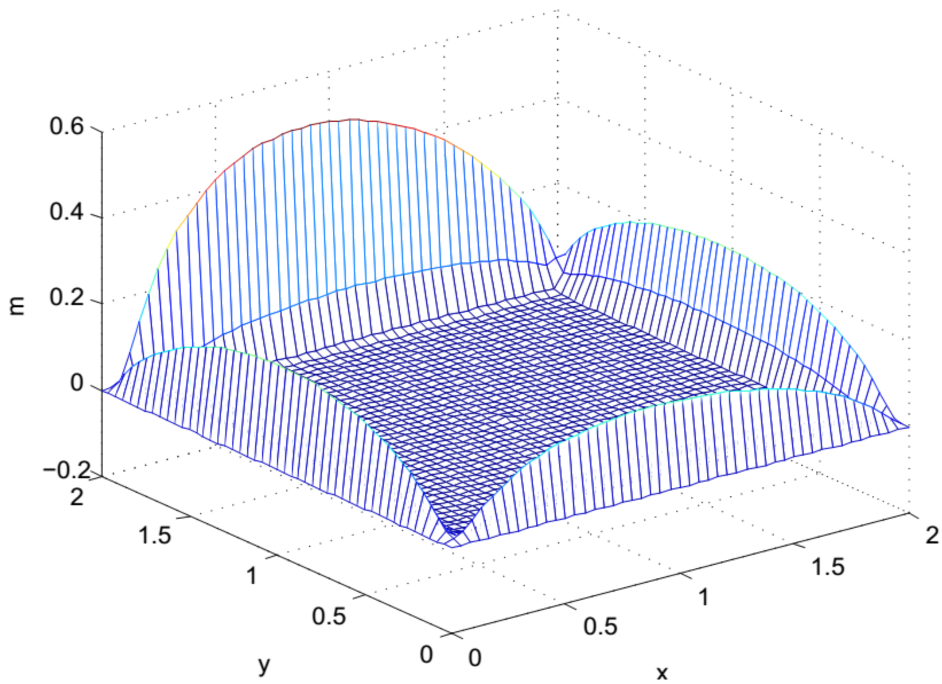


Fig 4-7:moment in the circular plate: case of mesh 1/48

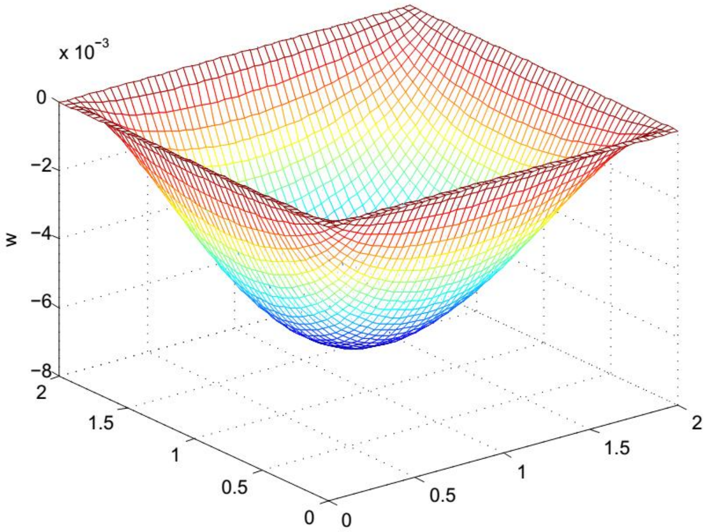


Fig 4-8:displacement in the circular plate: case of mesh 1/4

Thus, we have shown that with a coarse mesh, the generalized equations of the finite difference method give satisfactory results. Now consider the case of the ***plate simply pressed:***

4-2- plate simply pressed

Let us examine now the case of a circular plate of constant rigidity subjected to a constant loading and simply supported on all edges: these characteristics are identical to the previous case (embedded plate). The calculations will consist in determining the coefficients of the arrows and moments in the center of the plate using meshes *1/8; 1/16; 1/24; 1/32; 1/40; 1/48*

To do this, we are going to use the system of equation **(24)** for the intermediate points except the points where (i ≠j).The following **summary table 4.5**determined the values of m and v in the central node for the meshes:

Summary table 4.5:values of the coefficients of v at the central points of the circular plate accordingto the meshes

	Equations généralisées de la méthode des différences finis					
Pas du maillage	***1/8***	***1/16***	***1/24***	***1/32***	***1/40***	***1/48***
$\mathbf{m_c}$ (a^2P)	**-0.5361**	*-0.1542*	**-0.0697**	**-0.0394**	**-0.0253**	**-0.0176**
$\mathbf{w_c}$ $(\frac{a^2P}{D})$	**-0.1878**	*-0.0575*	**-0.0267**	**-0.0153**	**-0.0098**	**-0.0069**

As in the previous case, we can observe that as we fine tune the mesh, ***m*** and ***v*** grow, thus showing good convergence .We also note that from mesh 1/32, the moments and arrows are almost monotonous. All this can be better observed on the convergence curves **Fig4-9 a**nd **4-10** below**.**

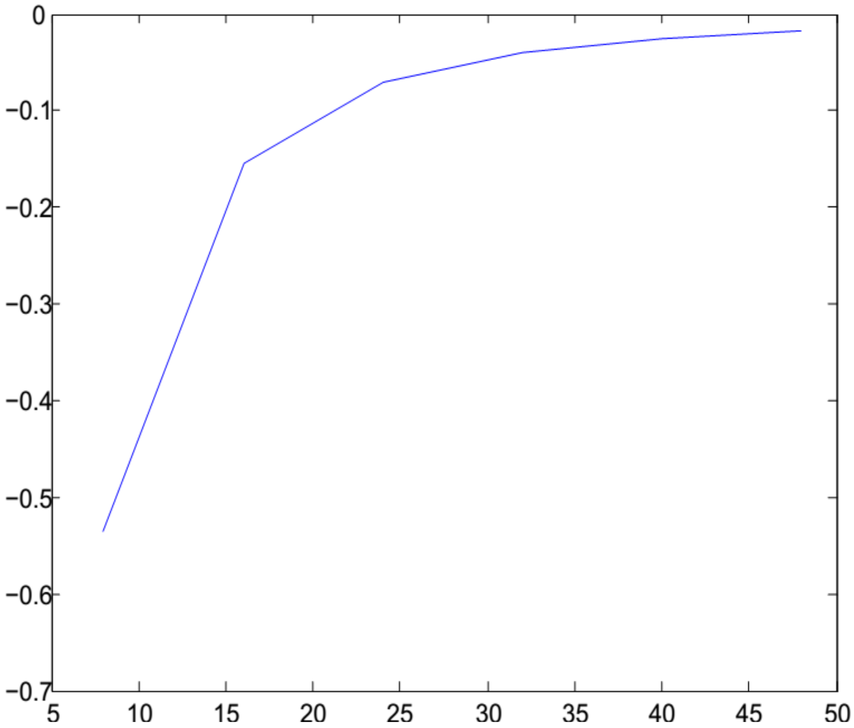


Fig4-9:convergence curve of the moments coefficients at the center of the circular plate for the meshes 1/8; 1/16; 1/24; 1/32; 1/40; 1/48

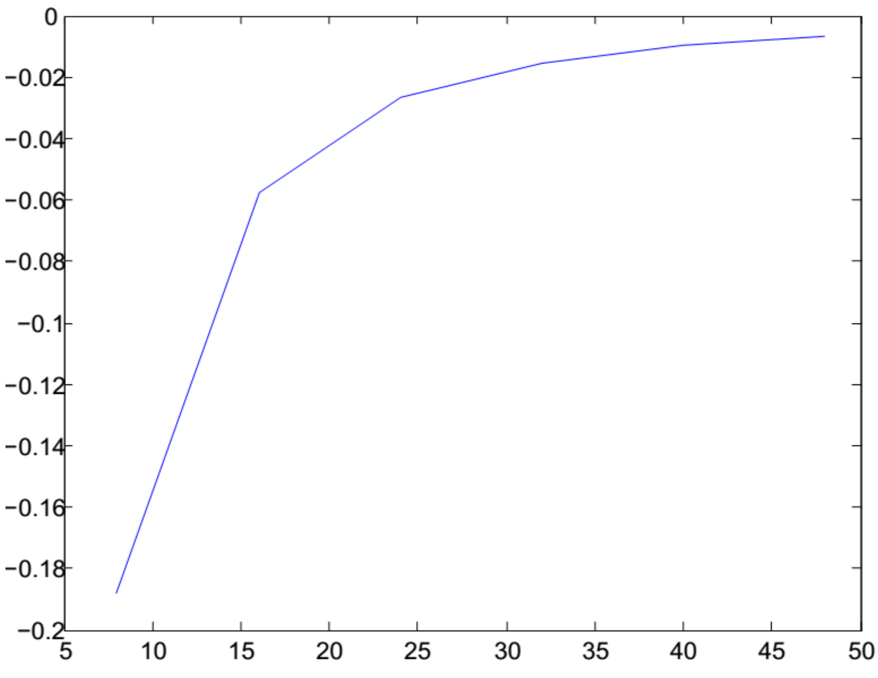


Fig4-10:convergence curve of the arrows coefficients at the center of the circular plate for the meshes 1/8; 1/16; 1/24; 1/32; 1/40; 1/48

V. CONCLUSION

At the end of this work, we have succeeded to transform the partial derivatives 4th order deformation equation of a circular plate into a system of two differential equations of 2nd order partial derivative. We then replaced the parameters of the system of equation, as well as the boundary conditions by dimensionless parameters. The equation obtain is then substituted into the generalized equation of the finite difference method. This permits us to obtain a system of algebraic equations hence, the resolution takes into account the boundary conditions and it was done using the iterative method of Gauss-Seidel.

This permits us to avoid the prior formation of matrix with unknowns. The algorithm develop here is simple and has permitted the calculation problem of an isotropic circular plate of thin constant thickness which is summited under a uniformly distributed loading and fixed on its entire edges to be solved. The results obtain in the different examples show a good convergence and the high accuracy of calculation of the order of **96%**.This shows as well the stability of the method.

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