Inverse Domination And Inverse Total Domination In Digraph

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Abstract — *In this paper, we discussed about the various properties like domination in graph , Inverse domination in graph, Inverse total domination in graph, domination in digraph. Domination in graphs has been studied extensively. In contrast, there has been relatively little research involving domination in digraphs. In a digraph D, a vertex v openly (or 1-step) out-dominates every vertex to which v is adjacent and openly in-dominates every vertex from which v is adjacent. Let $D = (V, A)$ be a digraph. A subset S of V is called a dominating set of D if for every vertex v in $V - S$, there exists a vertex u in S such that $(u, v) \in A$. A subset S of V is called a total dominating set of D if S is a dominating set of D and the induced sub digraph $\langle S \rangle$ has no isolated vertices. The inverse domination number g'(G) of G is the order of a smallest inverse dominating set of G. The exact values of g'(G) for some standard graphs and also establish some general results on parameter. The study of inverse total domination in graphs and present some bounds and some exact values for $\gamma_t'(G)$. Also, some relationships between for $\gamma_t'(G)$ and other domination parameters are established. The inverse domination parameters corresponding to domination and total domination in digraphs and establish some results on these parameters. Also we introduce the disjoint domination parameters corresponding to domination and total domination in digraphs.*

Keywords: *domination in graph , Inverse domination in graph, Inverse total domination in graph, domination in digraph, total domination in digraphs.*

I. INTRODUCTION

The study of domination set in graphs was begun by V.R. Kulli and Janakiram [3]. Learned the applications of domination in graph theory in [3,4,5]. Attained the grap of Inverse domination in graphs in [7]. Acquired the domination number g*(G)* of *G* is the order of a smallest dominating set in *G*. Domination in Digraphs were known in [1,2,9,11,12]. Gain the knowledge of Application of inverse total domination in [6,8,10].

II. PRELIMINARIES

A. Definition

The dominating graph $D(G)$ of a graph $G = (V, E)$ is a graph with $V(D(G)) = V(G) \cup S(G)$ where $S(G)$ is the set of all minimal dominating sets of G, with $u, v \in V(D(G))$ adjacent if $u \in V(G)$ and v is a minimal dominating set of G containing u.

B. Definition

The **minimal dominating graph** $\boldsymbol{MD(G)}$ of G is the intersection graph on the minimal dominating sets of vertices in G.

C. Definition

If a graph G is said to be a **vertex minimal dominating graph** $M_vD(G)$ of G if the graph having $V(M_vD(G)) = V(G) \cup S(G)$, where $S(G)$ is the set of all minimal dominating sets of G and two vertices u and v adjacent if they are adjacent in G or $v = D$ is a minimal dominating set containing u.

D. Definition

Let *D* be a minimum dominating set of *G*. If *V* – *D* contains a dominating set say *D'*, then *D'* is called an **Inverse Dominating Set** with respect to *D.*

E. Definition

The inverse domination number $\gamma'(G)$ of *G* is the order of a smallest inverse dominating set in *G.*

F. Definition

A set $D \subseteq V$ is a **Total Dominating set** of G if every vertex in V is djecent to some vertex in D.

G. Definition

The **Total Dominating Number** $\gamma_t(G)$ is the minimum cardinality of a total dominating set of G.

H. Definition

Let $D \subseteq V$ be a minimum total dominating Set of G. If $V - D$ contains a total dominating set D' of G, then D' is called an **Inverse Total Dominating Set** with respect to D.

I. Definition

The **Inverse Total Domination Number** $\gamma_t{}'(G)$ of G is the minimum cardinality of an inverse total dominating set of G.

J. Definition

Let *D* be a **Digraph** and let v be a vertex of *D.* The *(open) out-neighborhood* of v is $N^+(v) = \{u \in V(D) \mid (v,u) \in E(D)\}$ and the *(open)* in-neighborhood *of* v is $N^-(v) = \{u \in V(D) \mid (u,v) \in E(D)\}$. For a set *S* of vertices, the out-neighborhood of S is $N^+(S) = \bigcup_{v \in S} N^+(v)$ and the in-neighborhood of S is $N^-(S) = \bigcup_{v \in S} N^-(v)$.

K. Definition

The **minimum indegree** and **minimum outdegree** of a digraph D are defined, respectively, as $\delta^-(D) = \min_{v \in V(D)}\{|N^-(v)|\}$ and $\delta^+(D) = \min_{v \in (D)}\{|N^+(v)|\}$. The maximum indegree and maximum outdegree of D are defined in $\Delta^-(D)$ and $\Delta^+(D)$.

L. Definition

A vertex v in a digraph D **dominate**s itself and all vertices of $N^+(v)$, while v *openly dominates* only the vertices belonging to $N^+(v)$.

The **domination number** $\gamma(D)$ is the minimum cardinality of a dominating set of D, and the **open domination number** $\rho_1(D)$ is the minimum cardinality of an open dominating set of D. For the digraph D of Figure 6.1, it is straightforward to show that $[u, w]$ is a minimum dominating set of D and $\{u, w, x\}$ is a minimum open dominating set of D. Therefore, $\gamma(D) = 2$ and $\rho_1(D)$= 3.

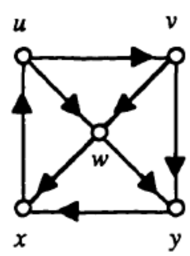

Figure: A Digraph

M. Definition

Two well-known digraphs are the **directed n-cycle** $\overrightarrow{C_n}$ of order n and the directed path P_n of order n. The digraphs C_6 and P_5 are shown in Figure 6.2. It is straightforward to show that $\gamma(\overrightarrow{C_n}) = \left\lceil \frac{n}{2} \right\rceil$, $\gamma(\overrightarrow{P_n}) = \left\lceil \frac{n}{2} \right\rceil$, $\rho_1(\overrightarrow{C_n}) = n$ and $\rho_1(\overrightarrow{P_n})$ is not defined.

III. INVERSE DOMINATION AND INVERSE TOTAL DOMINATION IN DIGRAPHS

Definition : 3.1

Let $D = (V, A)$ be a digraph. Let S be a minimum dominating set in a digraph D. If $V - S$ contains a dominating set S' of D, then S' is called an inverse dominating set with respect to S. The minimum cardinality of an **Inverse Dominating Set Of A Digraph** D is called the inverse domination number of D and is denoted by $\sqrt{-}^{1}(D)$.

Remark : 3.2

We note that not all digraphs have inverse dominating sets.

Example : 3.3

For the digraph D shown in Figure 1, $S = \{4, 5\}$ is a minimum dominating set and $V - S = \{1, 2, 3, 6, 7\}$ is not a dominating set. Thus $V - S$ does not contain a dominating set. Hence the digraph D has no an inverse dominating set.

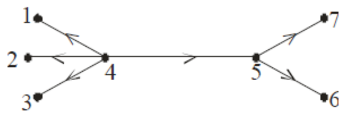

Figure 1

Definition: 3.4

The upper inverse domination number $\lceil -^{1}(D)$. of a digraph D is the maximum cardinality of an inverse dominating set of D.

Example : 3.5

Let D be the digraph as in Figure 2. The minimum dominating sets of D are $\{1, 3\}, \{2, 5\}$ and the corresponding inverse dominating sets are $\{2, 5\}, \{1, 3\}$ respectively. Thus $\gamma(D) = 2$ and $\gamma^{-1}(D)$ $=\lceil -^{1}(D) = 2$. Hence $\gamma(D) = \gamma^{-1}(D)$.

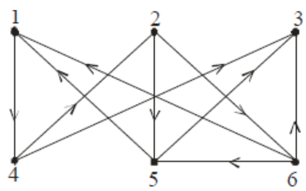

Figure 2

Proposition: 3.6
For any directed cycle C_{2p}, $p \geq 2, \sqrt{-}^{1}(C_{2p}) = p.$
A $\sqrt{-}^{1}$- set is a minimum inverse dominating set of a digraph D.

Proposition: 3.7
If a digraph D has a $\sqrt{-}^{1}$-set, then $\gamma(D) \leq \sqrt{-}^{1}(D)$ and this bound is sharp. (1)

Proof:
Clearly every inverse dominating set of a digraph is a dominating set. Thus (1) holds.
The directed cycles $C_{2p}, p \geq 2$ achieve this bound.

Proposition: 3.8
If a digraph D has a $\sqrt{-}^{1}$-set, then
$\gamma(D) + \sqrt{-}^{1}(D) \leq p$ and this bound is sharp. (2)

Proof
(2) follows from the definition of $\sqrt{-}^{1}(D)$.
The directed cycles $C_{2p}, p \geq 2$ achieve this bound.

Definition: 3.9
Let $D = (V, A)$ be a digraph in which $id(v) + od(v) > 0$ for all $v \in V$. Let S be a minimum total dominating set in a digraph D. If $V - S$ contains a total dominating set S' of D then S' is called an inverse total dominating set with respect to S. The inverse total domination number of D is the minimum cardinality of an inverse total dominating set of D.

Definition: 3.10
The upper inverse total domination number of a digraph D is the maximum cardinality of an inverse total dominating set of D.

Example: 3.11
Let D be a digraph as in Figure 3. The minimum total dominating sets of D are
$\{1, 2, 5\}$ and $\{3, 4, 6\}$ and the corresponding inverse total dominating sets are
$\{3, 4, 6\}$ and $\{1, 2, 5\}$ respectively.
Therefore $\gamma_t(D) = \gamma_t^{-1`}(G) = \lceil_t^{-1}(D) = 3.$

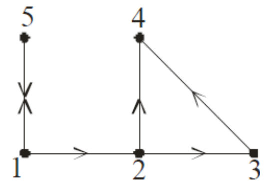

Figure 3

A $\gamma_t^{-1`}$-set is a minimum inverse total dominating set of a digraph D. Not all di-graphs without isolated vertices have a total dominating set. We also note that not all digraphs without isolated vertices have an inverse total dominating set. For example, the directed cycle C_4 has a total dominating set, but has no an inverse total dominating set.

Proposition: 3.12
If a digraph D has a -set, then $\gamma_t(D) \leq \gamma_t^{-1`}(\mathrm{D})$ and this bound is sharp. (3)

Proof

Clearly, every inverse total dominating set is a total dominating set. Thus (3) holds. The digraph D of Figure 7.3 achieves this bound.

Proposition: 3.13

If a digraph D has a $\gamma_t^{-1`}$-set,
then $\gamma_t(D) \leq \gamma_t^{-1`}(D) \leq p$ and this bound is sharp. (4)

Proof

(4) follows from the definition of .
The digraph D of Figure 7.3 achieves this bound.

Proposition: 3.14

Let S be a γ_t-set of a connected digraph D. If a -set exists in D, then D has at least 4 vertices.

Proof

Let S be a γ_t-set of D. Since D has no isolated vertices, $\gamma_t(D) = |S| \geq 2$. If a $\gamma_t^{-1`}$-set exists, then $V - S$ contains a total dominating set with respect to D. Thus $|V - S| \geq 2$. Thus D has at least 4 vertices.

Theorem: 3.15

If a digraph D has a γ_t-set, then $2 \leq \sqrt{}^{-1t}(D) \leq p - 2$.

Proof

By definition, $\gamma_t(D) \geq 2$ and by Proposition 7.4, $\gamma_t(D) \leq \gamma_t^{-1`}(D)$.
Thus $2 \leq \gamma_t^{-1`}(D)$.
By Proposition,
$\gamma_t^{-1`}(D) \leq p - \gamma_t(D) \leq p - 2$, since $2 \leq \gamma_t(D)$.
We establish a Nordhaus-Gaddum type result.

NOTE: 3.16

Let D be a digraph such that both D and have no isolated vertices.
Then

$$4 \leq g_t^{-1`}(D) + g_t^{-1`}(\bar{D}) \leq 2(p-2),$$

$$4 \leq g_t^{-1`}(D).g_t^{-1`}(\bar{D}) \leq (p-2)^2.$$

CONCLUSION

In this paper entitled "**INVERSE DOMINATION AND INVERSE TOTAL DOMINATION IN DIGRAPH**" we make an in-depth study in the domination theory for graph and digraph and its related works. We discussed about the Graph theory, Domination in Graph, Domination in digraph, inverse domination in graph and application of inverse total domination graph theory and history of these graph. We dealt with the basic definitions related to the graph theory We dealt with the definitions, examples and theorems of the inverse domination and inverse total domination in digraph.

REFERENCES

[1] Arumugam, .S, Jacob .K and Volkmann L., "Total and connected domination in digraphs", Australasian J. Combin., 2007, 39, 283-292.

[2] Chartand . G and Lesniak L., "Graphs and Digraphs", CRC, 2004.

[3] Kulli. V. R. and Janakiram B., "The dominating graph," Graph Theory Notes of New York XLVI, 2004, 5-8.

[4] Kulli. V. R and Janakiram. B. "The Minimal Dominating Graph", Graph Theory Notes of New York, XXVII:3, New York Academy of Sciences, 1995, 12-15.

[5] Meera Paulson and Lilly T.I., "Domination in Graph Theory" ISSN 2319-5770, e- ISSN 2394-1138, VISTAS, 2015 Vol. 4, 70-73.

[6] Kulli V. R and Radha Rajamani Iyer, “Inverse total domination in graphs ” Journal of Discrete Mathematical Sciences and Cryptography, 2007, 613-619.

[7] Kulli V. R and Sigarkanti S.C.,“Inverse domination in graphs”,Nat. Acad.Sci.Lett., 1991, Vol 14, 473-475.

[8] Kulli V. R and Patwari D.K.,“The Total Bondage Number of a Graph” in Advances in Graph Theory, 1991, 227-235.

[9] Kulli V. R., “Inverse Domination and Inverse total domination in Digraphs ”, Journal of Advanced Research in Computer Science & Technology International, 2004, Vol 2, 106-109.

[10] Kulli V. R. and Iyer R.R., “Inverse total domination in graphs”, J. Discrete Mathematical Sciences and Cryptography, 2007, 10(5), 613-62. Lee L., “On the Domination Number of a Digraph”, Ph.D. Dissertation, Michigan State University, 1994.

[11] Lisa Hansen , “Domination in Digraphs”, Western Michigan University, 1997, Vol.6.