Inverse Domination of Some Special Graphs

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Abstract :
Let G = (V,E) be a simple, finite, undirected and connected graph. A non-empty subset D$\subseteq$V is a dominating set of G if every vertex in V-D is adjacent to atleast one vertex in D. The domination number $\gamma(G)$ is the minimum cardinality taken over all the minimal dominating sets of G. Let D be the minimum dominating set of G. If V-D contains a dominating set D' of G, then D' is an inverse dominating set with respect to D. The inverse domination number $\gamma'(G)$ is the minimum cardinality of a minimal inverse dominating set of G. In this paper, the values of $\gamma'(G)$ is obtained for some special graphs.

Keywords :
Dominating set, Inverse dominating set, Domination number, Inverse domination number.

Introduction :
Let G = (V,E) be a simple, finite, undirected and connected graph with |V| = n and |E| = m.
A non-empty subset D$\subseteq$V is a dominating set of G if every vertex in V-D is adjacent to atleast one vertex in D. The domination number $\gamma(G)$ is the minimum cardinality taken over all the minimal dominating sets of G. Let D be the minimum dominating set of G. If V-D contains a dominating set D' of G, then D' is an inverse dominating set with respect to D. The inverse domination number $\gamma'(G)$ is the minimum cardinality of a minimal inverse dominating set of G.
The purpose of this paper is to determine the values of $\gamma'(G)$ for some special graphs and its relationship with other domination parameters.

Definitions of some named graphs :
Mongolian tent graph :
A graph with 7 vertices and 9 edges is called as Mongolian tent graph.

Unit distance graph :
The Peterson graph is a unit distance graph. It is a 3 regular graph with 10 vertices and 15 edges.

Pancyclic graph :
It is a graph with 5 vertices and 7 edges.

Distance hereditary graph :
It is also called as completely separable graph. Distance hereditary graph was named and first studied by Howarka in 1977. It is a graph with 10 vertices and 17 edges.

Truncated Tetrahedral graph :
A Truncated tetrahedral graph has 4 regular hexagonal faces, 4 equilateral triangle faces, 12 vertices and 18 edges.

Tietze graph :
Tietze graph is an undirected cubic graph with 12 vertices and 18 edges. It is named after Heinrich Franz Friedrich Tietze who showed in 1910.

Frucht graph :
The Frucht graph is polyhedral and Hamiltonian. It is a graph with 12 vertices and 18 edges. It is named after Robert Frucht.

Lollipop graph :

It is a graph with 8 vertices and 17 edges.

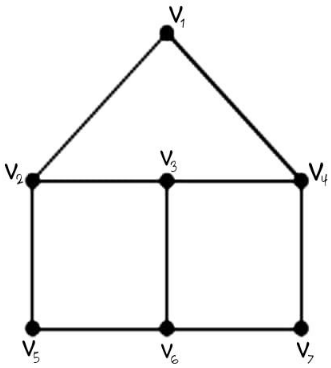
Prism graph :
It is a graph with 12 vertices and 18 edges.

Cocktail party graph :
Cocktail party graph is a graph with 6 vertices and 12 edges.

Dominating set :
A non empty subset $D \subseteq V$ in a graph G = (V,E) is a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set of G.

Inverse dominating set :
Let D be the minimum dominating set of G. If V-D contains a dominating set D' of G, then D' is an inverse dominating set w.r.to D. The inverse domination number $\gamma'(G)$ is the minimum cardinality of a minimal inverse dominating set of G.

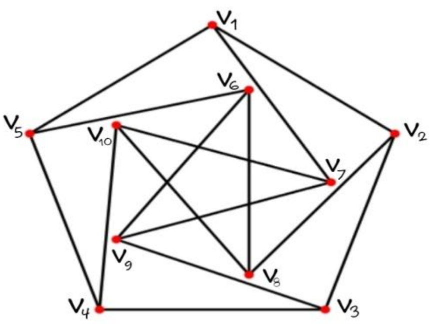
Results on Inverse domination on some special graphs :
Mongolian Tent graph :



Let G be the Mongolian tent graph with 7 vertices and 9 edges. Let V={v_1, v_2...v_7} be the set of vertices of G. Let {v_1, v_2, v_4, v_5, v_6, v_7} be the outer vertices and {v_3} be the inner vertex. Let D be the dominating set. The minimum dominating set is {v_1, v_6} which dominates the entire vertices of V. Also there is an another set D'= {v_2, v_7} which dominates the remaining vertices. Thus D' is the inverse dominating set of G.

$$\gamma(G) = \gamma'(G) = 2$$

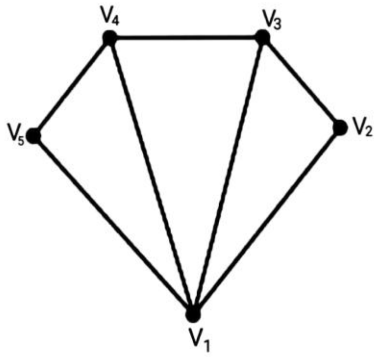
Unit distance graph :



Let G=(V,E) be the unit distance graph. Let V={v_1, v_2...v_{10}} be the vertex set of G. The vertex v_2 is adjacent to the vertices {v_1, v_3, v_8} and the vertex v_5 is adjacent to {v_1, v_4, v_6} and the vertex v_7 is adjacent to the vertices {v_1, v_9, v_{10}}. The minimum dominating set is D = {v_2, v_5, v_7}. Also there is an inverse dominating set D' = {v_1, v_4, v_6}.

$$\gamma(G) = \gamma'(G) = 3$$

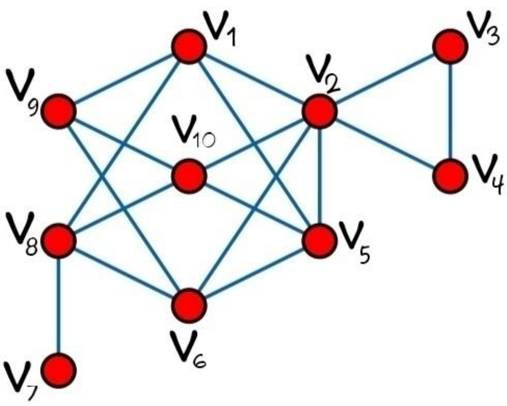
Pancyclic graph :



Let G be the pancyclic graph with 5 vertices and 7 edges. The minimum dominating set is $\{v_1\}$ and $\{v_3, v_4\}$ is the inverse dominating set which dominates the entire vertex set V.

$$1 = \gamma(G) < \gamma'(G) = 2$$

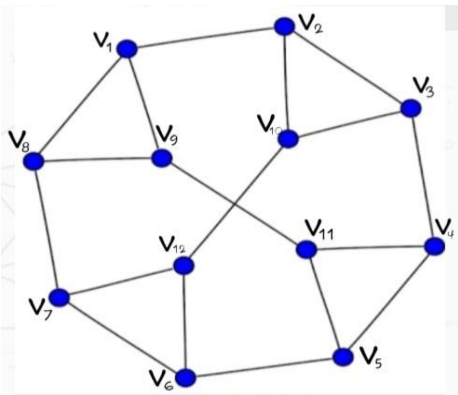
Distance hereditary graph :



Let G be the distance hereditary graph with 10 vertices and 17 edges. Let V=$\{v_1, v_2 \ldots v_{10}\}$ be the set of vertices. The minimum dominating set is $\{v_1, v_2, v_8\}$. The inverse dominating set D' which dominates the remaining vertices is $\{v_3, v_5, v_7, v_9\}$.

$$3 = \gamma(G) < \gamma'(G) = 4$$

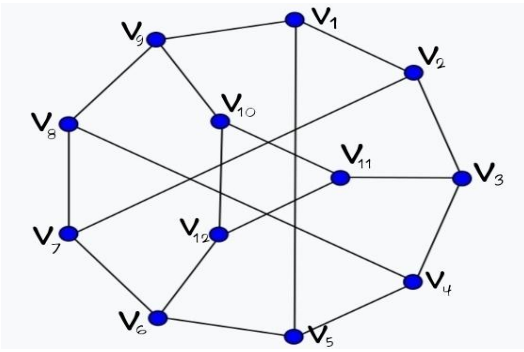
Truncated tetrahedral graph:



Let G=(V,E) be the truncated tetrahedral graph with vertices $\{v_1, v_2 \ldots v_{12}\}$. The vertices $\{v_1, v_2 \ldots v_8\}$ be the outer vertices and $\{v_9, v_{10} \ldots v_{12}\}$ be the inner vertices. Let D be the dominating set . The minimum dominating set is $\{v_1, v_4, v_{11}\}$ then there is an inverse dominating set D' such that D'= $\{v_3, v_6, v_9\}$ dominates the entire vertex set V.

$$\gamma(G) = \gamma'(G) = 3$$

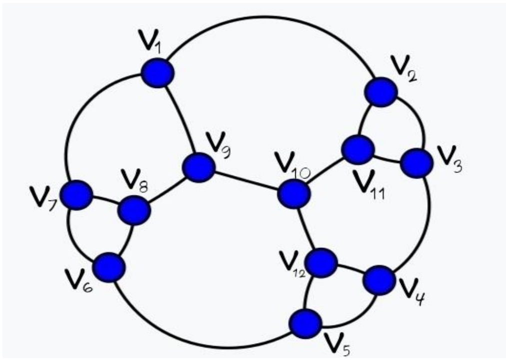
Tietze graph :



Let G be the Tietze graph. Let V={v_1, v_2...v_{12}} be the vertex set of G. The vertex v_3 is adjacent to {v_2, v_4, v_{11}} and v_6 is adjacent to {v_5, v_7, v_{12}} and v_9 is adjacent to {v_1, v_8, v_{10}} then the dominating set is {v_3, v_6, v_9} which is minimum. The inverse dominating set is {v_1, v_3, v_8, v_{12}}. Thus $\gamma'(G)$= 4.

$$3 = \gamma(G) < \gamma'(G) = 4$$

Frucht graph :



Let G be the Frucht graph with 12 vertices and 18 edges. Let V={v_1, v_2...v_{12}} be the set of vertices of G. The vertex set V is partitioned into 2 vertex subsets V_1 and V_2 such that

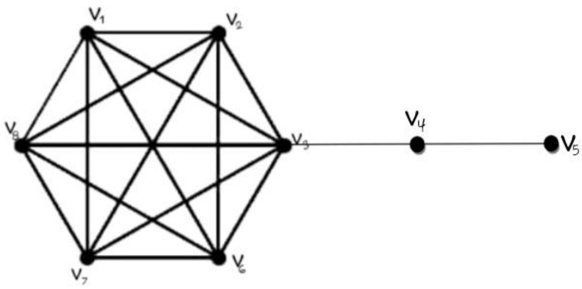
V_1 = {v_1, v_2...v_7}

V_2 = {v_8, v_9...v_{12}}

Where V_1 is the set of outer vertices and V_2 is the set of inner vertices. The minimum dominating set D = {v_2, v_8, v_{12}}. Also there exists an inverse dominating set D'= {v_1, v_3, v_6, v_{10}} which dominates the whole vertex set V.

$$3 = \gamma(G) < \gamma'(G) = 4$$

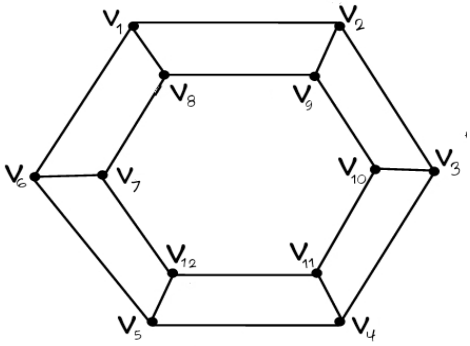
Lollipop graph :



Let G=(V,E) be the lollipop graph with 8 vertices and 17 edges. Let V={v_1, v_2...v_8} be the set of vertices. The minimum dominating set is {v_3, v_4}. The minimum inverse dominating set is {v_2, v_5}.

$$\gamma(G) = \gamma'(G) = 2$$

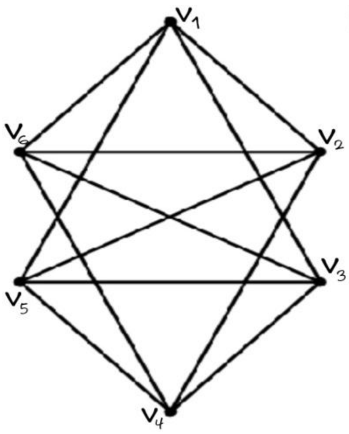
Prism graph :



Let G be the Prism graph. Let $\{v_1, v_2...v_6\}$ be the outer vertices and $\{v_7, v_8...v_{12}\}$ be the inner vertices. Let $V=\{v_1, v_2...v_{12}\}$ be the vertex set of G. The dominating set is $\{v_3, v_6, v_8, v_{11}\}$ which dominates the entire set. The minimum inverse dominating set is $\{v_1, v_5, v_7, v_{10}\}$.

$$\gamma(G) = \gamma'(G) = 4$$

Cocktail party graph :



Let G be the Cocktail party graph. Let $V=\{v_1, v_2...v_6\}$ be the set of vertices. The minimum dominating set D = $\{v_1, v_4\}$ and the minimum inverse dominating set D'= $\{v_2, v_3\}$ where v_2 is adjacent to the vertices $\{v_1, v_4, v_5, v_6\}$ and v_3 is adjacent to $\{v_1, v_4, v_5, v_6\}$.

$$\gamma(G) = \gamma'(G) = 4$$

Conclusion :

In this paper, we obtained the inverse domination numbers of 10 different named graphs. The relationship between $\gamma(G)$ and $\gamma'(G)$ is also analysed.

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