The generalized Kudryshov method applied to the unstable nonlinear Schrodinger equation in mathematical physics

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Abstract

*This study examines the new different varieties of soliton structures of the unstable nonlinear Schrodinger equation through the generalized Kudryshov method. This model has valuable applications in mathematical physics***.** *The obtained new different types of soliton structures are represented in names of the rational, and exponential functions show that the considered approach is useful to investigate the nonlinear dispersive equations in mathematical physics***.**

Keywords: *The generalized Kudryshov method, The unstable nonlinear Schrodinger equation, Solitary wave solutions.*

I. INTRODUCTION

Nonlinear evolution equations (NLEEs) have been contained with paramount importance due to its diversity of uses in physics, mathematical sciences and engineerings such as electrostatics, fluids dynamics, elasticity, quantum mechanics and electrodynamics. Lately, numerous novel dominant processes have been recommended for attaining the exact solution of NLEEs such as modified (G'/G)-expansion method [1, 2], new generalized (G'/G)-expansion method [3, 4, 5], new generalized (G'/G)-expansion method [6], the exp-$\phi(\xi)$-expansion method [7, 8], finite series Jacobi elliptic cosine function ansatz [9], residual power series method [10], Variation of Parameters Method [11], Riccati-Bernoulli sub-ODE method [12], Extended mapping method [13], Extended direct algebraic sech method [14], extended modified mapping method [15], Sech-tanh method [16], the direct algebraic function method [17], Generalized unified method [18], the generalized exponential function [19], eneral bilinear form [20], modified Kudryashov method [21] and many more.

The paper applied the generalized Kudryshov method [23] to derive the different type of soliton structures for unstable Schrödinger equation [21, 22].

Let us consider that the general form of unstable Schrödinger equation:

$$iW_t + W_{xx} + 2p_1|w|^2 W - 2p_2 W = 0 \tag{1}$$

where, p_1 is a real number and $W(x, t)$ is a complex-valued function, is a special type of nonlinear evolution equations that arises in the vast areas of applied sciences, such as nonlinear optics, plasma physics, quantum mechanics, and so on.

II. Glimpse of the generalized Kudryshov method

Step 1: We consider that a NLEE for $W(x, t)$:

$$k(\frac{\partial W}{\partial t}, \frac{\partial W}{\partial x}, \frac{\partial^2 W}{\partial t^2}, ...) = 0 \tag{2}$$

where, K represents a polynomial in W.

To locate the transformation of equation 2:

$$W = W(x,t) = W(\xi), \xi = x - ct \tag{3}$$

From Equation (2) and equation (3),we locate the following ODE:

$$L(W, W', W'', \ldots\ldots\ldots) = 0 \tag{4}$$

Step 2: Calculate M and N through the balance rule on equation (4).

Step 3: Let us consider that:

$$W(\xi) = \frac{\sum_{i=0}^{N} A_i \psi^i}{\sum_{j=0}^{M} B_j \psi^j} \tag{5}$$

where, A_i and B_j are real constants, N and M are positive integers such that $A_N, B_M \neq 0$ and Ψ satisfies the following ODE:

$$\psi'(\xi) = \psi^2(\xi) - \psi(\xi) \tag{6}$$

The general solution of equation (6) is of the form:

$$\psi(\xi) = \frac{1}{1 + he^{\xi}} \tag{7}$$

where, h is any arbitrary constant.

Step 4: Determine the positive integers N and M in equation (5) by balancing the highest order derivative term with the nonlinear term of $W(\xi)$ in equation (2) or equation (4). Moreover, we define the degree of $W(\xi)$ as $D(W(\xi)) = N - M$, which gives rise to the degree of other expression as

$$D\left(\frac{d^q W}{d\xi^q}\right) = N - M + q, \; D\left(W^p \left(\frac{d^q W}{d\xi^q}\right)^s\right) = (N - M)p + s(N - M + q) \tag{8}$$

where, p, q, s are integer numbers. Thus, we can find the value of N and M in equation (5).

Step 5: Applying equation (5) into equation (4) and equation (8), collecting all terms with the same order of Φ together. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations which can be solved to find the values of $\Phi(\xi)$ with the help of MAPLE.

III. Solitons to the unstable nonlinear Schrodinger equation

Let us consider that the general form of unstable Schrdinger equation:

$$iW_t + W_{xx} + 2p_1|w|^2 W - 2p_2 W = 0 \tag{9}$$

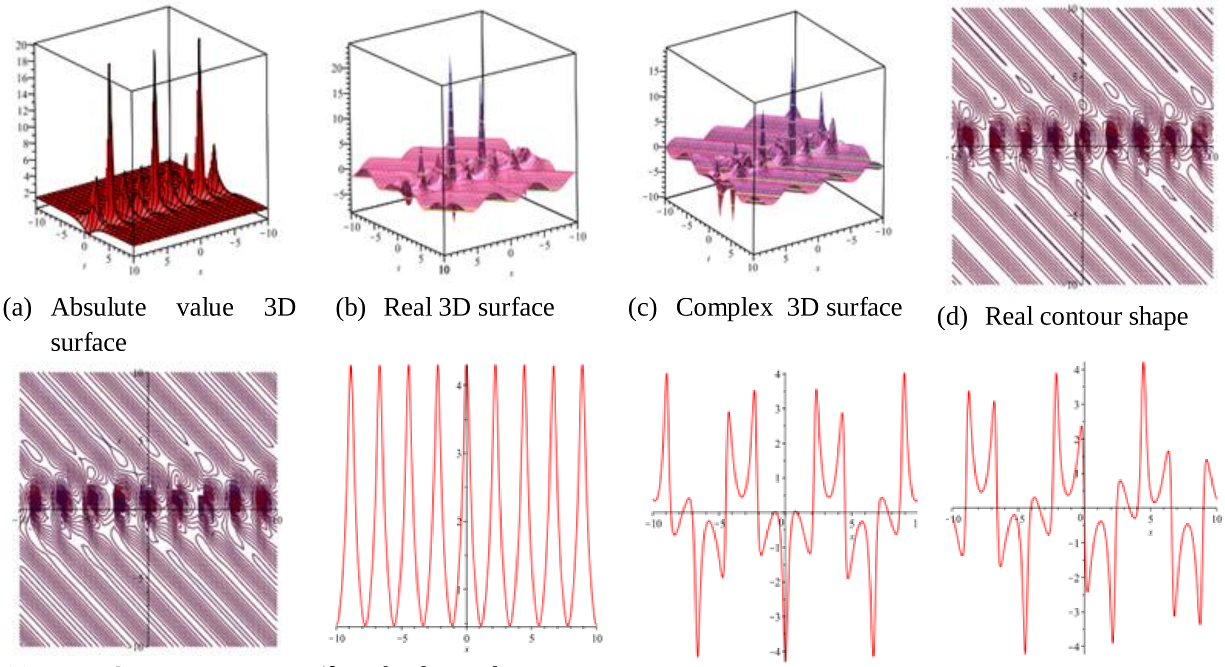
Using $W(x,t) = \Phi(\xi)e^{i\eta}$, where $\xi = px + qt$, $\eta = rx + st$, ten equation (9) converts into the following ODE:

$$p^2\Phi''(\xi) - (r^2 + s + 2p_2)\Phi(\xi) + 2p_1\Phi^3(\xi) = 0 \tag{10}$$

Applying the rule of homogeneous balance on equation (10), then we get:

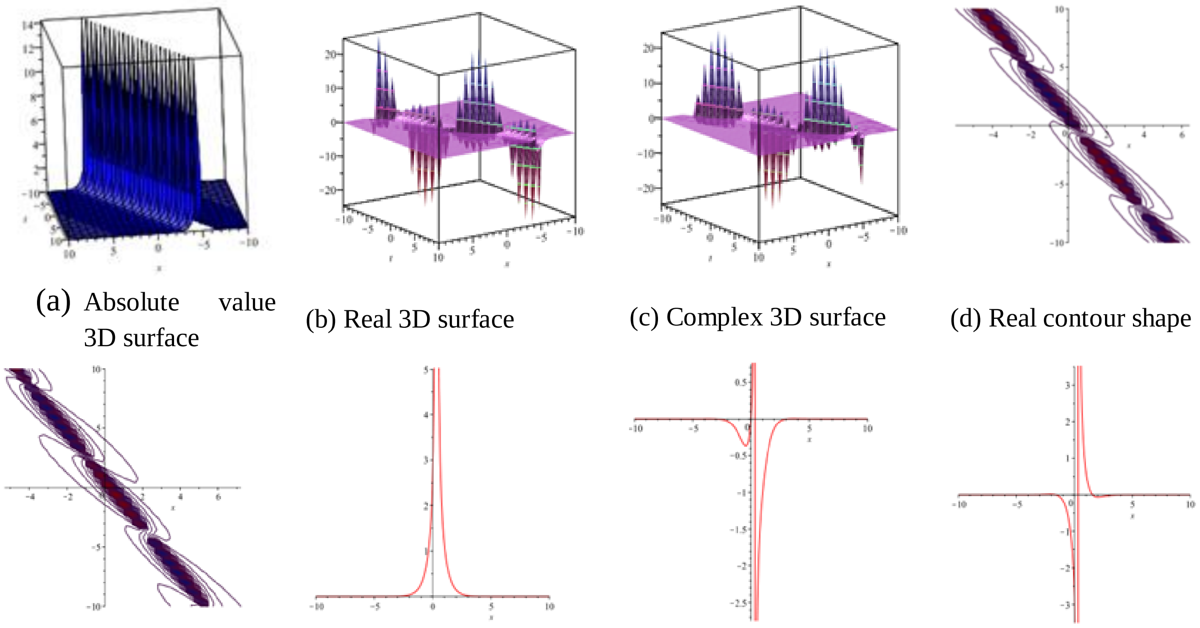
$$\Phi(\xi) = \frac{A_0 + A_1\Psi + A_2\Psi^2}{B_0 + B_1\Psi} \tag{11}$$

By equation (11) and equation (10) and then equating each coefficients of β^i to zeros, we get:



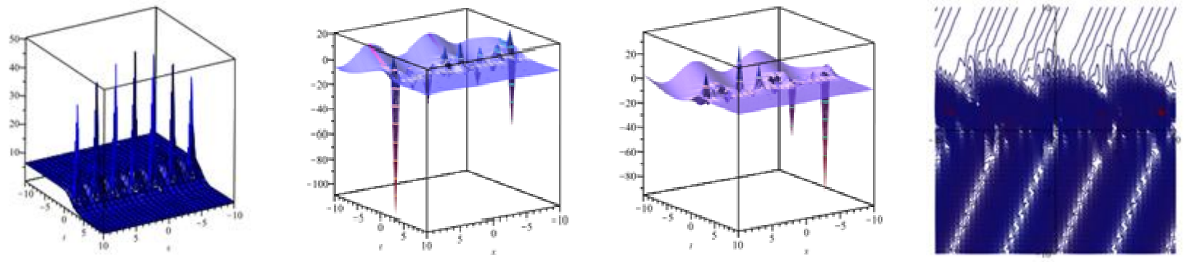
(a) Absulute value 3D surface
(b) Real 3D surface
(c) Complex 3D surface
(d) Real contour shape
(e) Complex contour shape
(f) Absulute value 2D shape
(g) Real 2D shape
(h) Complex 2D shape

Figure 1: Graphical representation of the solution in $W_1(x,t)$ and its projection at $t = 0.01$ for the unknown parameters $a = 0.35$ and $b_2 = 0.5$ within the interval $-10 \le x, t \le 10$.

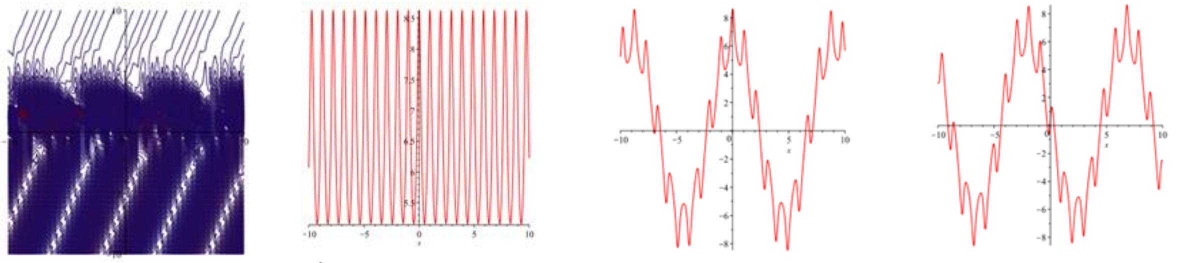


(a) Absolute value 3D surface
(b) Real 3D surface
(c) Complex 3D surface
(d) Real contour shape
(e) Complex contour shape
(f) Absolute value 2D shape
(g) Real 2D shape
(h) Complex 2D shape

Figure 2: Graphical representation of the solution in $W_3(x,t)$ and its projection at $t = 0.01$ for the unknown parameters $a = 0.35$ and $b_2 = 0.5$ within the interval $-10 \le x, t \le 10$.

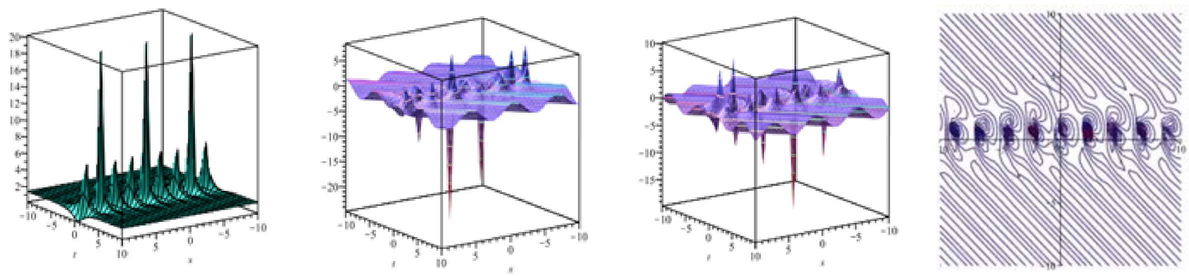


(a) Absolute value 3D surface (b) Real 3D surface c) Complex 3D surface (d) Real contour shape

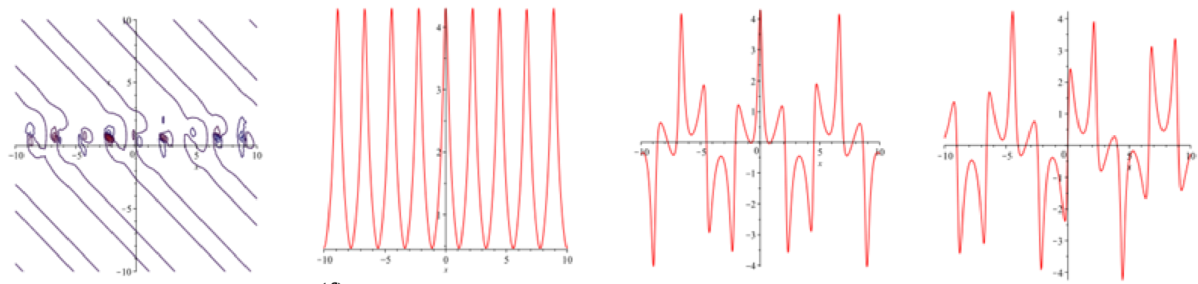


(e) Complex contour shape (f) Absolute value 2D shape (g) Real 2D shape (h) Complex 2D shape

Figure 3: Graphical representation of the solution in $W_5(x,t)$ and its projection at $t = 0.01$ for the unknown parameters $a = 0.35$ and $b_2 = 0.5$ within the interval $-10 \le x,t \le 10$.



(a) Absolute value 3D surface (b) Real 3D surface c) Complex 3D surface d) Real contour shape



(e) Complex contour shape (f) Absolute value 2D shape (g) Real 2D shape (h) Complex 2D shape

Figure 4: Graphical representation of the solution in $W_7(x,t)$ and its projection at $t = 0.01$ for the unknown parameters $a = 0.35$ and $b_2 = 0.5$ within the interval $-10 \le x,t \le 10$.

Phase one:

$$p=\sqrt{-2r^2-4p_2-2s}\,,\; p_3=\sqrt{\frac{r^2+2p_2+s}{2p_1}}\,,\; A_0=0,\; A_1=p_3B_1,$$

$$A_2=-\frac{B_1(r^2+2p_2+s)}{p_1p_3},\; B_0=0,$$

where, q, p_0, p_2 and B_1 are constants. Using the values of phase one, equation (10) and (11), we have

$$W_1(x,t)=\left\{\frac{p_3B_1\left(\frac{1}{1+pe^{px+qt}}\right)-\frac{B_1(r^2+2p_2+s)}{p_1p_3}\left(\frac{1}{1+pe^{px+qt}}\right)^2}{B_1\left(\frac{1}{1+pe^{px+qt}}\right)}\right\}e^{i(rx+st)}$$

Phase two:

$$p=-\sqrt{-2r^2-4p_2-2s}\,,\; p_4=-\sqrt{\frac{r^2+2p_2+s}{2p_1}}\,,\; A_0=0,\; A_1=p_4B_1,$$

$$A_2=-\frac{B_1(r^2+2p_2+s)}{p_1p_4},\; B_0=0,$$

Where q, p_0, p_2 and B_1 are constants. Similarly, we get

$$W_2(x,t)=\left\{\frac{p_4B_1\left(\frac{1}{1+pe^{px+qt}}\right)-\frac{B_1(r^2+2p_2+s)}{p_1p_4}\left(\frac{1}{1+pe^{px+qt}}\right)^2}{B_1\left(\frac{1}{1+pe^{px+qt}}\right)}\right\}e^{i(rx+st)}$$

Phase three:

$$p=\sqrt{r^2-4p_2+s}\,,\; p_5=\sqrt{\frac{-4r^2-8p_2-4s}{p_1}}\,,\; A_0=0,\; A_1=p_5B_0,$$

$$A_2=-p_5B_0,\; B_1=-2B_0,$$

where, q, p_0, p_2 and B_1 are constants. Similarly, we get

$$W_3(x,t)=\left\{\frac{p_5B_0\left(\frac{1}{1+pe^{px+qt}}\right)-p_5B_0\left(\frac{1}{1+pe^{px+qt}}\right)^2}{B_0-2B_0\left(\frac{1}{1+pe^{px+qt}}\right)}\right\}e^{i(rx+st)}$$

Phase four:

$$p=-\sqrt{r^2+2p_2+s}\,,\; p_6=-\sqrt{\frac{-4r^2-8p_2-4s}{p_1}}\,,\; A_0=0,,$$

$$A_1=p_6B_0,\;\; A_2=-p_6B_0,\; B_1=-2B_0,$$

where, q, p_0, p_2, B_0 and B_1 are constants. Similarly, we get

$$W_4(x,t)=\left\{\frac{p_6B_1\left(\frac{1}{1+pe^{px+qt}}\right)-p_6B_0\left(\frac{1}{1+pe^{px+qt}}\right)^2}{B_0-2B_0\left(\frac{1}{1+pe^{px+qt}}\right)}\right\}e^{i(rx+st)}$$

Phase five:

$$p=\sqrt{\frac{-r^2-2p_2-s}{2}},\ p_7=\sqrt{\frac{r^2+2p_2+s}{2p_1}},\ A_0=\frac{B_1(r^2+2p_2+s)}{4p_1p_7},$$

$$A_1=-\frac{B_1(r^2+2p_2+s)}{2p_1p_7},\ A_2=p_7B_1,\ B_1=-0.5B_0,$$

where, q, p_0, p_2, B_0 and B_1 are constants. Similarly, we get

$$W_5(x,t)=\left\{\frac{\frac{B_1(r^2+2p_2+s)}{4p_1p_7}-\frac{B_1(r^2+2p_2+s)}{2p_1p_7}\left(\frac{1}{1+pe^{px+qt}}\right)+p_7B_1\left(\frac{1}{1+pe^{px+qt}}\right)^2}{B_0-0.5B_0\left(\frac{1}{1+pe^{px+qt}}\right)}\right\}e^{i(rx+st)}$$

Phase six:

$$p=-\sqrt{\frac{-r^2-2p_2-s}{2}},\ p_8=-\sqrt{\frac{r^2+2p_2+s}{2p_1}},\ A_0=\frac{B_1(r^2+2p_2+s)}{4p_1p_8},$$

$$A_1=-\frac{B_1(r^2+2p_2+s)}{2p_1p_8},\ A_2=p_8B_1,\ B_1=-0.5B_0,$$

where, q, p_0, p_2, B_0 and B_1 are constants. Similarly, we get

$$W_6(x,t)=\left\{\frac{\frac{B_1(r^2+2p_2+s)}{4p_1p_8}-\frac{B_1(r^2+2p_2+s)}{2p_1p_8}\left(\frac{1}{1+pe^{px+qt}}\right)+p_8B_1\left(\frac{1}{1+pe^{px+qt}}\right)^2}{B_0-0.5B_0\left(\frac{1}{1+pe^{px+qt}}\right)}\right\}e^{i(rx+st)}$$

Phase seven:

$$p=\sqrt{-2r^2-4p_2-2s},\ p_9=\sqrt{\frac{2r^2+4p_2+2s}{p_1}},\ A_0=-\frac{B_0(r^2+2p_2+s)}{p_1p_9},$$

$$A_1=\frac{2r^2B_0-r^2B_1+4p_2B_0-2p_2B_1+2sB_0-sB_1)}{p_1p_9},\ A_2=p_9B_1,$$

where, p_1, r, s, p_2, B_0 and B_1 are constants. Similarly, we get

$$W_7(x,t)=\left\{\frac{-\dfrac{B_0(r^2+2p_2+s)}{p_1p_9}+\dfrac{2B_0r^2-r^2B_1+4p_2B_0-2p_2B_1+2sB_0-sB_1)}{2p_1p_9}\left(\dfrac{1}{1+pe^{px+qt}}\right)+p_9B_1\left(\dfrac{1}{1+pe^{px+qt}}\right)^2}{B_0+B_1\left(\dfrac{1}{1+pe^{px+qt}}\right)}\right\}e^{i(rx+st)}$$

Phase eight:

$$p==\sqrt{-2r^2-4p_2-2s}\,,\ p_{10}=-\sqrt{\frac{2r^2+4p_2+2s}{p_1}}\,,\ A_0=-\frac{B_0(r^2+2p_2+s)}{p_1p_{10}},$$

$$A_1=\frac{2r^2B_0-r^2B_1+4p_2B_0-2p_2B_1+2sB_0-sB_1)}{p_1p_{10}},\ \ A_2=p_{10}B_1\,,$$

where, p_1, r, s, p_2, B_0 and B_1 are constants. Similarly, we get

$$W_8(x,t)=\left\{\frac{-\dfrac{B_0(r^2+2p_2+s)}{p_1p_{10}}+\dfrac{2B_0r^2-r^2B_1+4p_2B_0-2p_2B_1+2sB_0-sB_1)}{p_1p_{10}}\left(\dfrac{1}{1+pe^{px+qt}}\right)+p_{10}B_1\left(\dfrac{1}{1+pe^{px+qt}}\right)^2}{B_0+B_1\left(\dfrac{1}{1+pe^{px+qt}}\right)}\right\}e^{i(rx+st)}$$

IV. CONCLUSIONS

In this paper, we have extracted different types of solitons which are shown in the Figures (1- 4) of the equation (1) and the equation through the generalized Kudryshov method. The study model is a critical nonlinear evolution equations that arises in the vast areas of applied sciences, such as nonlinear optics, plasma physics, quantum mechanics. The studied technique is straightforward, brief, outspoken, and sincere to execute as well as it is pretty proficient for generating new different types of solitons of distinct nonlinear evolution equations.

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