Some Expressions on Logarithmic Sum

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Abstract *— in this paper author try to establish three expression or formulas whose are purely based on logarithmic sum for proving these expressions author use very well known or core concepts of series and sequence and logarithm. Main objective is to calculate sum of logarithmic values for a specific type of sequence and in last author prove expression number first with the help of concept of mathematical induction.*

Keywords — *logarithmic sum; Mathematical induction; Base change of logarithm.*

I. INTRODUCTION

In many books such as [1] R.D Sharma problem number 2page no.21.19 seventh edition 2017 class-XI there are problems on sum of logarithm but most of the time it happen that there is a chapter based on logarithm there are 1 and 2 problems in it whose are based on sum of two or more than two logs such as [2] M .L .Khanna and J .N .Sharma problem number 37 page no.1240 150th edition in 2006 and some times it takes much time for solving summation problems, for a better idea there are three expression which helps a lot, are present in second section and their proofs are presented in third section.

II. EXPRESSIONS

Three expressions on logarithmic sum are as follows which we'll prove in part three of this paper:-

1.

$$\sum_{i=1}^{k}\log_b(bi) = k + \log_b(\lfloor k)$$

2.

$$\sum_{i=x}^{k}\log_b(bi) = (k-c) + \log_b \frac{1}{c!}(\lfloor k)$$

Here,c is equal to (b-1)

or

$$\sum_{i=x}^{k}\log_b(bi) = (k-(b-1)) + \log_b \frac{1}{(b-1)!}(\lfloor k)$$

3.

$$\sum_{i=1}^{n}\log_b(bi) + \sum_{i=2}^{n}\log_b(bi) + \dots\dots\dots + \sum_{i=n}^{n}\log_b(bi) = n^2 - \frac{n(n-1)}{2} + \log_b \frac{(n!)^n}{\prod_{n=1}^{n}(n-1)!}$$

These above expressions are applicable only when :

- The value of base which is b is greater than zero : b>0
- In multiplication base of log is always presented with any number i which is belongs to any positive number respectively
- And finally n and k are also greater then zero n>0

III. PROOF OF EXPRESSIONS

In this section we can show the proofs of above expressions
so,

Expression 1.

$$\sum_{i=1}^{k}\log_b(bi) = k + \log_b\left(\lfloor\!\underline{k}\right)$$

Where, b,k>0

Proof:-

So, from the left hand side LHS:-

$$\sum_{i=1}^{k}\log_b(bi)$$

$$\log_b(b)+\log_b(2b)+\log_b(3b)+......+\log_b(kb)$$

$$\log_b(b\cdot 2b\cdot 3b......\cdot kb)$$

$$\log_b\left(1\cdot 2\cdot 3....\cdot k\cdot b^k\right)$$

$$\log_b\left(\lfloor\!\underline{k}\cdot b^k\right)$$

$$\log_b(b)^k + \log_b\lfloor\!\underline{k}$$

$$k + \log_b\lfloor\!\underline{k}$$

By using it we can solve $\log_{10}(10)+\log_{10}(20)+\log_{10}(30)+..........+\log_{10}(180)$

With the aid of scientific calculator as presented on Google [3] https://www.google.com/search?q=scientific+calculator&rlz=1C1GIWA_enIN826IN826&oq=scientific+ca&aqs=chrome.1.69i57j0l7.12590j0j7&sourceid=chrome&ie=UTF-8 as most of us use calculator for solving it so, we take adding all but it can be easier if we add log of factorial 18 with base 10 which is 15.8 and 18 by which we get same result which is 33.8 also and by using it we can save our's precious time respectively.

But as we know that it is only possible when series starts from i=1 and it is not essential that's why for different start of I expression second is applicable

Expression 2.

$$\sum_{i=x}^{k}\log_b(bi) = (k-c) + \log_b\frac{1}{c!}\left(\lfloor\!\underline{k}\right)$$

Where c=(x-1) and b, k>0

Proof:

Let consider that series starts from 2 or i=2

so, for it expression number first becomes as :

If series starts from 3 or i=3 so for it:

$$\sum_{i=3}^{k} \log_b (bi)$$

$$\log_b (3b) + \ldots\ldots + \log_b (kb)$$

$$\log_b (3b \cdot 4b \ldots\ldots \cdot kb)$$

$$\log_b \left(\frac{1 \cdot 2 \cdot 3 \ldots \cdot k \cdot b^k}{1(b)2(b)} \right)$$

$$\log_b \left(\frac{\lfloor k \cdot b^{k-2}}{2!} \right)$$

$$\log_b \left(\frac{\lfloor k}{2} \cdot b^{k-2} \right)$$

$$\log_b (b)^{k-2} + \log_b \frac{\lfloor k}{2}$$

$$(k-2) + \log_b \frac{\lfloor k}{2}$$

By these above activities to consider i=2,3 we can write one another expression:

$$\sum_{i=x}^{k} \log_b (bi) = (k-c) + \log_b \frac{1}{c!} (\lfloor k)$$

Where c=(x-1) and b,k>0

Or

$$\sum_{i=x}^{k} \log_b (bi) = (k-(b-1)) + \log_b \frac{1}{(b-1)!} (\lfloor k)$$

Expression 3.

$$\sum_{i=1}^{n} \log_b(bi) + \sum_{i=2}^{n} \log_b(bi) + \ldots\ldots\ldots + \sum_{i=n}^{n} \log_b(bi) = n^2 - \frac{n(n-1)}{2} + \log_b \frac{(n!)^n}{\prod_{n=1}^{n}(n-1)!}$$

Where,b,n>0

Proof:

From LHS

$$\sum_{i=1}^{n} \log_b(bi) + \sum_{i=2}^{n} \log_b(bi) + \ldots\ldots\ldots + \sum_{i=n}^{n} \log_b(bi) =$$

$$n + \log_b n! + (n-1) + \log_b \frac{n!}{1!} + (n-2) + \log_b \frac{n!}{2!} + \cdots \ldots\ldots\ldots\ldots (n-(n-1)) + \log_b \frac{n!}{(n-1)!}$$

$$\sum_{i=1}^{n} \log_b(bi) + \sum_{i=2}^{n} \log_b(bi) + \ldots\ldots\ldots + \sum_{i=n}^{n} \log_b(bi) =$$

$$(n+n+\cdots\ldots\ldots n) - \left(1+2+\cdots\ldots(n-1)\right) + \log_b \frac{n!}{0!} + \log_b \frac{n!}{1!} + \log_b \frac{n!}{2!} + \cdots\ldots\ldots\ldots\ldots\ldots + +\log_b \frac{n!}{(n-1)!}$$

As n is added with n times then it becomes as n square and by the property of logarithm [4] Dr. SK Goyal Chapter 4 ,page no.317 Textbook of Algebra ISBN-978-93-13191-88-9

$$\sum_{i=1}^{n} \log_b(bi) + \sum_{i=2}^{n} \log_b(bi) + \dots \dots \dots + \sum_{i=n}^{n} \log_b(bi) =$$
$$n \cdot n - \frac{n(n-1)}{2} + \log_b\left(\frac{n!}{0!} \cdot \frac{n!}{1!} \cdot \frac{n!}{2!} \dots\dots\dots\dots\dots\dots \frac{n!}{(n-1)!}\right)$$

$$\sum_{i=1}^{n} \log_b(bi) + \sum_{i=2}^{n} \log_b(bi) + \dots \dots \dots + \sum_{i=n}^{n} \log_b(bi) =$$
$$n^2 - \frac{n(n-1)}{2} + \log_b \frac{(n!)^n}{\prod_{n=1}^{n}(n-1)!}$$

IV. PROOF OF EXPRESSION FROM MATHEMATICAL INDUCTION

EXPRESSION 1.

$$\sum_{i=1}^{k} \log_b(bi) = k + \log_b(\lfloor k)$$

Where, b,k>0

Step1.

For first term p(1) equation should be true

So,

K=1 on both side (LHS and RHS)

$$\sum_{i=1}^{k} \log_b(bi) = k + \log_b(\lfloor k)$$

$$\sum_{i=1}^{1} \log_b(bi) = 1 + \log_b \lfloor 1$$

$$\log_b b = 1 + \log_b 1$$

$$1 = 1$$

Hence LHS and RHS gives same value which is equal to 1 that is why they equal to each other which means is that equation is true for first term which is nothing but equal to p(1)

Step2.

Let consider any number which is equal to c for it equation is true or in other words it is also said that for p(c) equation is true so, put c on place of k then equation becomes as :-

$$\sum_{i=1}^{c} \log_b(bi) = c + \log_b(\lfloor c) - - - - - - - - - - - - (i)$$

Step3.

It should be true for next term : term after c which is also written as (c+1) so, for it equation becomes as:-

$$\sum_{i=1}^{c+1} \log_b(bi) = (c+1) + \log_b(\lfloor c+1)$$

Or, it is also written as:-

$$\sum_{i=1}^{c+1}\log_b(bi)=(c+1)+\log_b\left(\underline{\lfloor c+1}\right)$$

$$\sum_{i=1}^{c}\log_b(bi)+\log_b\left(b(c+1)\right)=(c+1)+\log_b\left(\underline{\lfloor c+1}\right)$$

From equation 1

$$c+\log_b\underline{\lfloor c}+\log_b(c+1)+\log_b b=(c+1)+\log_b\left(\underline{\lfloor c+1}\right)$$

$$(c+1)+\log_b\left[(c+1)\cdot\underline{\lfloor c}\right]=(c+1)+\log_b\left(\underline{\lfloor c+1}\right)$$

$$(c+1)+\log_b\left(\underline{\lfloor c+1}\right)=(c+1)+\log_b\left(\underline{\lfloor c+1}\right)$$

Hence,
From above equation on both side (on LHS and on RHS) the same result becomes. So it is proven that for next term P of c+1 it is true so now it is easily said that formula for summation of logarithm is true easily because the main motive of proof by mathematical induction is that Formula is true or not for a given series only [5] Chapter -4 Mathematical induction page no-86-96 NCERT Text book class-XI.

V. CONCLUSIONS

In this article author develop three Properties on sum of two or more than two logs which gives help during calculation and also save ours precious time but it is valid for a type of problem which is looking lime as:

$$\log_{10}(10)+\log_{10}(20)+\log_{10}(30)+\ldots\ldots\ldots\ldots+\log_{10}(180)$$

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