Minkowski-4 Mean Labeling of Graphs

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Abstract — *Let $G = (V, E)$ be an undirected graph with p vertices and q edges. Define a function $f: V(G) \to \{1,2,3, \dots, q+1\}$ is called* ***Minkowski-4 Mean Labeling*** *of a graph G if we could able to label the vertices $x \in V$ with distinct elements from $1,2, \dots, q+1$ such that it induces an edge labeling $f^*: E(G) \to \{1,2,3, \dots, q+1\}$ defined as,*

$$\boldsymbol{f^*(e = uv)} = \left\lfloor \left(\frac{f(u)^4 + f(v)^4}{2} \right)^{\frac{1}{4}} \right\rfloor,$$

is distinct for all edges $e = uv \in E$. (i,e.) It indicates that the distinct vertex labeling induces a distinct edge labeling on the graph. The graph which admits Minkowski-4 Mean Labeling is called a ***Minkowski-4 Mean Graph.*** *In this paper, we have investigated the Minkowski-4 Mean Labeling of some standard graphs like Path, Comb, Caterpillar, $P_n \odot K_{1,2}$, etc.*

Keywords— *Minkowski-4 Mean Labeling, Minkowski-4 Mean Graph, Path, Comb, Caterpillar, $P_n \odot K_{1,2}$.*

I. INTRODUCTION

The graph G we used here are simple, finite and undirected graphs. $V(G)$ and $E(G)$ denotes the vertex set and edge set of a graph G. For graph theoretic terminology, we refer to Harary.F [3], Douglas B. West [1] and Gallian.J.A [2]. The concept of Mean Labeling of graphs was introduced by Somasundaram.S and Ponraj.R [4] in 2003. Sandhya.S.S, Somasundaram.S and Anusa.S [6] introduced the concept of Root Square Mean Labeling of graphs in 2014. On the same lines we define and study **Minkowski-4 Mean Labeling of graphs**.

II. BASIC DEFINITIONS

The following definitions are needed for the present study.

A. Definition

A walk in which all the vertices are distinct is called a **Path.** A Path is denoted by P_n . The Path P_n has n vertices and $n-1$ edges.

B. Definition

The graph attained by joining a single pendent edge to each vertex of a Path is called **Comb.** It has $2n$ vertices and $2n-1$ edges.

C. Definition

A tree which yields a Path when its pendant vertices are removed is called a **Caterpillar.** It has 3n vertices and $3\text{n} - 1$ edges.

D. Definition

The $\boldsymbol{P_n \odot K_{1,2}}$ is a graph attained by attaching the complete bipartite graph $\boldsymbol{K_{1,2}}$ to each vertex of the path P_n . It has $3n$ vertices and $3n-1$ edges.

III.MAIN RESULTS

Theorem: 1

For every n, Path P_n is a $Minkowski-4\ Mean\ graph$.

Proof:

Let P_n be a path $u_1, u_2, u_3, \dots, u_n$ of length n. The gragh G has n vertices and $n-1$ edges.

Define a function $f: V(G) \rightarrow \{1,2,3, \dots, q+1\}$ by

$$f(u_i) = i \qquad ,1 \leq i \leq n$$

Then the induced edge labels are,

$$f^*(u_i u_{i+1}) = i \qquad ,1 \leq i \leq n-1$$

Then the edge labels are distinct.
Therefore, P_n is a Minkowski-4 Mean graph.

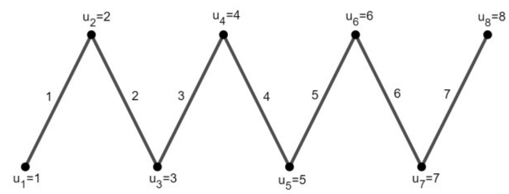

Figure 1: P_8

Theorem: 2

For every n, Comb $P_n \odot K_1$ is a $Minkowski - 4\ Mean\ graph$.

Proof:

Let $P_n \odot K_1$ be a comb attained from a path $P_n = u_1, u_2, u_3, \dots, u_n$ by joining a vertex u_i to a pendent vertex $v_i (1 \leq i \leq n)$ .The gragh G has $2n$ vertices and $2n-1$ edges.
Define a function $f: V(G) \rightarrow \{1,2,3, \dots, q+1\}$ by

$$f(u_i) = \begin{cases} 2i & ,when\ i = 1 \\ 2i-1 & ,2 \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} i & ,when\ i = 1 \\ 2i & ,2 \leq i \leq n \end{cases}$$

Then the induced edge labels are,

$$f^*(u_i u_{i+1}) = 2i \qquad ,1 \leq i \leq n-1$$

$$f^*(u_i v_i) = 2i-1 \qquad ,1 \leq i \leq n$$

Then the edge labels are distinct.
Therefore, $P_n \odot K_1$ is a Minkowski-4 Mean graph.

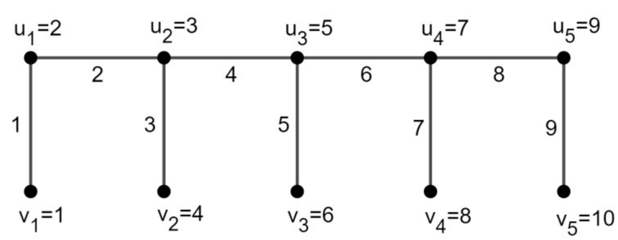

Figure 2: $P_5 \odot K_1$

Theorem: 3

Let G be a graph attained by attaching a pendant edges to both sides of each vertex of a path P_n. Then G is a $Minkowski - 4\ Mean\ graph$.

Proof:
Consider a graph G which is attained by attaching a pendant edges to both sides of each vertex of a path $\mathrm{P_n}$. Let P_n be a path $v_1, v_2, v_3, \dots, v_n$. Let u_i and w_i be the pendant vertices adjacent to $v_i (1 \le i \le n)$. The gragh G has $3n$ vertices and $3n - 1$ edges.
Define a function $f: V(G) \to \{1,2,3, \dots, q+1\}$ by

$$f(u_i) = \begin{cases} 3i - 2 & , if\ i\ is\ odd \\ 3i - 1 & , if\ i\ is\ even \end{cases}$$

$$f(v_i) = \begin{cases} 3i - 1 & , if\ i\ is\ odd \\ 3i - 2 & , if\ i\ is\ even \end{cases}$$

$$f(w_i) = \{3i \qquad ,1 \le i \le n$$

Then the induced edge labels are,

$$f^*(v_i v_{i+1}) = 3i \qquad ,1 \le i \le n - 1$$

$$f^*(v_i u_i) = 3i - 2 \qquad ,1 \le i \le n$$

$$f^*(v_i w_i) = 3i - 1 \qquad ,1 \le i \le n$$

Then the edge labels are distinct.
Therefore, G is a Minkowski-4 Mean graph.

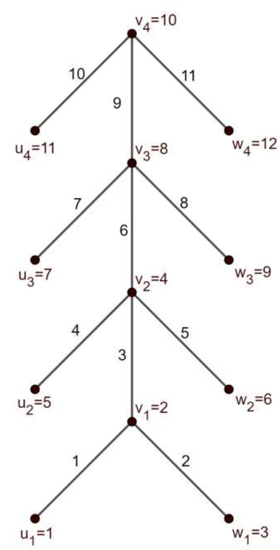

Figure 3: Caterpillar

Theorem: 4

For every n, $P_n \odot K_{1,2}$ is a $Minkowski - 4\ Mean\ graph$.

Proof:
Let G be a graph attained by attaching each vertex of P_n to the central vertex of the complete bipartite graph $K_{1,2}$. Let P_n be a path $u_1, u_2, u_3, \dots, u_n$ and v_i, w_i be the vertices of $K_{1,2}$, which are attached to the vertex u_i of P_n.The gragh G has $3n$ vertices and $3n - 1$ edges.
Define a function $f: V(G) \to \{1,2,3, \dots, q+1\}$ by

$$f(u_i) = \begin{cases} i & ,when\ i = 1 \\ i+2 & ,when\ i = 2 \\ 3i-1 & ,3 \le i \le n \end{cases}$$

$$f(u_i) = \begin{cases} 2\,i & ,when\ i = 1 \\ i+3 & ,when\ i = 2 \\ 3i-2 & ,3 \le i \le n \end{cases}$$

$$f(w_i) = \{3i \qquad ,1 \le i \le n$$

Then the induced edge labels are,

$$f^*(u_i u_{i+1}) = 3i \qquad ,1 \le i \le n-1$$

$$f^*(u_i v_i) = 3i-2 \qquad ,1 \le i \le n$$

$$f^*(u_i w_i) = 3i-1 \qquad ,1 \le i \le n$$

Then the edge labels are distinct.
Therefore, $P_n \odot K_{1,2}$ is a Minkowski-4 Mean graph.

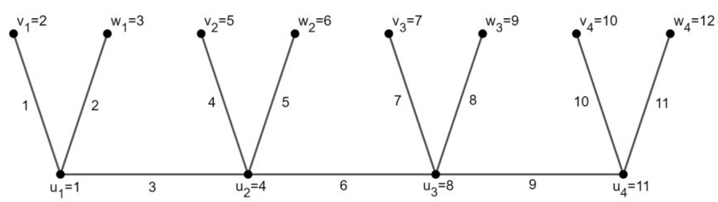

Figure 4: $P_4 \odot K_{1,2}$

Theorem: 5

Let G be a graph attained by attaching K_1 at each pendant vertex of a comb. Then G admits a Minkowski-4 Mean graph.

Proof:

Let G be a graph attained by attaching K_1 at each pendant vertex of a comb. The gragh G has $3n$ vertices and $3n-1$ edges.
Define a function $f: V(G) \to \{1,2,3, \dots, q+1\}$ by

$$f(u_i) = \begin{cases} i & ,when\ i = 1 \\ i+2 & ,when\ i = 2 \\ 3i-1 & ,3 \le i \le n \end{cases}$$

$$f(u_i) = \begin{cases} 2\,i & ,when\ i = 1 \\ i+3 & ,when\ i = 2 \\ 3i-2 & ,3 \le i \le n \end{cases}$$

$$f(w_i) = \{3i \qquad ,1 \le i \le n$$

Then the induced edge labels are,

$$f^*(u_i u_{i+1}) = 3i \qquad ,1 \le i \le n-1$$

$$f^*(u_i v_i) = 3i-1 \qquad ,1 \le i \le n$$

$$f^*(v_i w_i) = 3i - 2 \qquad ,1 \le i \le n$$

Then the edge labels are distinct.
Therefore, G is a Minkowski-4 Mean graph.

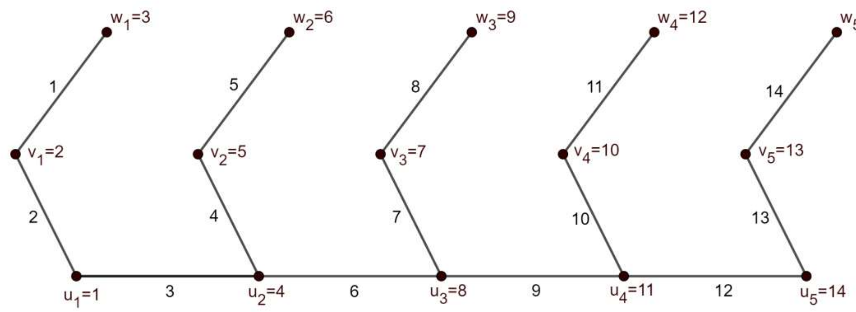

Figure 5: G

IV. Conclusions

In this paper, we have introduced the notion of Minkowski-4 Mean Labeling and studied for some standard graphs. Illustrative examples are provided to support our investigation.

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