# Minkowski-4 Mean Labeling of Graphs

M.Kaaviya Shree<sup>#1</sup>, K.Sharmilaa<sup>\*2</sup>

II M.Sc Mathematics<sup>#</sup>, Assistant Professor<sup>\*</sup>, Department of Mathematics(PG)<sup>#\*</sup>, PSGR Krishnammal College for Women<sup>#\*</sup>, Coimbatore-641004, Tamilnadu, India.

Abstract — Let G = (V, E) be an undirected graph with p vertices and q edges. Define a function  $f:V(G) \rightarrow \{1,2,3,...,q+1\}$  is called **Minkowski-4 Mean Labeling** of a graph G if we could able to label the vertices  $x \in V$  with distinct elements from 1,2,...,q+1 such that it induces an edge labeling  $f^*: E(G) \rightarrow \{1,2,3,...,q+1\}$  defined as,

$$f^*(\boldsymbol{e} = \boldsymbol{u}\boldsymbol{v}) = \left\lfloor \left(\frac{f(\boldsymbol{u})^4 + f(\boldsymbol{v})^4}{2}\right)^{\frac{1}{4}} \right\rfloor,$$

is distinct for all edges  $e = uv \in E$ . (i,e.) It indicates that the distinct vertex labeling induces a distinct edge labeling on the graph. The graph which admits Minkowski-4 Mean Labeling is called a **Minkowski-4 Mean Graph.** In this paper, we have investigated the Minkowski-4 Mean Labeling of some standard graphs like Path, Comb, Caterpillar,  $P_n \odot K_{1,2}$ , etc.

**Keywords**— Minkowski-4 Mean Labeling, Minkowski-4 Mean Graph, Path, Comb, Caterpillar,  $P_n \odot K_{1,2}$ .

## I. INTRODUCTION

The graph G we used here are simple, finite and undirected graphs. V(G) and E(G) denotes the vertex set and edge set of a graph G. For graph theoretic terminology, we refer to Harary.F [3], Douglas B. West [1] and Gallian.J.A [2]. The concept of Mean Labeling of graphs was introduced by Somasundaram.S and Ponraj.R [4] in 2003. Sandhya.S.S, Somasundaram.S and Anusa.S [6] introduced the concept of Root Square Mean Labeling of graphs in 2014. On the same lines we define and study **Minkowski-4 Mean Labeling of graphs**.

#### **II. BASIC DEFINITIONS**

The following definitions are needed for the present study.

### A. Definition

A walk in which all the vertices are distinct is called a **Path.** A Path is denoted by  $P_n$ . The Path  $P_n$  has n vertices and n - 1 edges.

## B. Definition

The graph attained by joining a single pendent edge to each vertex of a Path is called **Comb.** It has 2n vertices and 2n - 1 edges.

#### C. Definition

A tree which yields a Path when its pendant vertices are removed is called a **Caterpillar.** It has 3n vertices and 3n - 1 edges.

#### D. Definition

The  $P_n \odot K_{1,2}$  is a graph attained by attaching the complete bipartite graph  $K_{1,2}$  to each vertex of the path  $P_n$ . It has 3n vertices and 3n - 1 edges.

#### **III.MAIN RESULTS**

#### Theorem: 1

For every *n*, Path  $P_n$  is a *Minkowski* - 4 *Mean graph*. **Proof:** Let  $P_n$  be a path  $u_1, u_2, u_3, ..., u_n$  of length *n*. The graph *G* has *n* vertices and n - 1 edges. Define a function  $f: V(G) \rightarrow \{1, 2, 3, \dots, q+1\}$  by

$$f(u_i) = i \qquad , 1 \le i \le n$$

Then the induced edge labels are,

$$f^*(u_i u_{i+1}) = i$$
 ,  $1 \le i \le n-1$ 

Then the edge labels are distinct.

Therefore,  $P_n$  is a Minkowski-4 Mean graph.



#### Theorem: 2

For every n, Comb  $P_n \odot K_1$  is a Minkowski - 4 Mean graph.

#### **Proof:**

Let  $P_n \odot K_1$  be a comb attained from a path  $P_n = u_1, u_2, u_3, ..., u_n$  by joining a vertex  $u_i$  to a pendent vertex  $v_i(1 \le i \le n)$ . The graph G has 2n vertices and 2n - 1 edges. Define a function  $f: V(G) \to \{1, 2, 3, ..., q + 1\}$  by

$$f(u_i) = \begin{cases} 2i & , when i = 1\\ 2i - 1 & , 2 \le i \le n \end{cases}$$
$$f(v_i) = \begin{cases} i & , when i = 1\\ 2i & , 2 \le i \le n \end{cases}$$

Then the induced edge labels are,

$$f^*(u_i u_{i+1}) = 2i$$
 ,  $1 \le i \le n-1$   
 $f^*(u_i v_i) = 2i - 1$  ,  $1 \le i \le n$ 

Then the edge labels are distinct.

Therefore,  $P_n \bigcirc K_1$  is a Minkowski-4 Mean graph.



#### Theorem: 3

Let G be a graph attained by attaching a pendant edges to both sides of each vertex of a path  $P_n$ . Then G is a *Minkowski* – 4 *Mean graph*.

#### Proof:

Consider a graph G which is attained by attaching a pendant edges to both sides of each vertex of a path  $P_n$ . Let  $P_n$  be a path  $v_1, v_2, v_3, ..., v_n$ . Let  $u_i$  and  $w_i$  be the pendant vertices adjacent to  $v_i (1 \le i \le n)$ . The gragh G has 3n vertices and 3n - 1 edges.

Define a function  $f: V(G) \rightarrow \{1, 2, 3, \dots, q+1\}$  by

$$f(u_i) = \begin{cases} 3i-2 & , if \ i \ is \ odd \\ 3i-1 & , if \ i \ is \ even \end{cases}$$
$$f(v_i) = \begin{cases} 3i-1 & , if \ i \ is \ odd \\ 3i-2 & , if \ i \ is \ even \end{cases}$$

$$f(w_i) = \{3i \qquad , 1 \le i \le n$$

Then the induced edge labels are,

$f^*(v_i v_{i+1}) = 3i$	$,1 \leq i \leq n-1$
$f^*(v_i u_i) = 3i - 2$	$,1 \leq i \leq n$
$f^*(v_i w_i) = 3i - 1$	$, 1 \leq i \leq n$

Then the edge labels are distinct.

Therefore, G is a Minkowski-4 Mean graph.



Figure 3: Caterpillar

## Theorem: 4

For every  $n, P_n \odot K_{1,2}$  is a *Minkowski* - 4 *Mean graph*.

#### Proof:

Let G be a graph attained by attaching each vertex of  $P_n$  to the central vertex of the complete bipartite graph  $K_{1,2}$ . Let  $P_n$  be a path  $u_1, u_2, u_3, ..., u_n$  and  $v_i, w_i$  be the vertices of  $K_{1,2}$ , which are attached to the vertex  $u_i$  of  $P_n$ . The graph G has 3n vertices and 3n - 1 edges. Define a function  $f: V(G) \rightarrow \{1, 2, 3, ..., q + 1\}$  by

$$f(u_i) = \begin{cases} i & ,when \ i = 1 \\ i+2 & ,when \ i = 2 \\ 3i-1 & ,3 \le i \le n \end{cases}$$

$$f(u_i) = \begin{cases} 2i & ,when \ i = 1 \\ i+3 & ,when \ i = 2 \\ 3i-2 & ,3 \le i \le n \end{cases}$$

$$f(w_i) = \{3i & ,1 \le i \le n \end{cases}$$

Then the induced edge labels are,

$$f^{*}(u_{i}u_{i+1}) = 3i , 1 \le i \le n - 1$$
  
$$f^{*}(u_{i}v_{i}) = 3i - 2 , 1 \le i \le n$$
  
$$f^{*}(u_{i}w_{i}) = 3i - 1 , 1 \le i \le n$$

Then the edge labels are distinct.

Therefore,  $P_n \odot K_{1,2}$  is a Minkowski-4 Mean graph.



## Theorem: 5

Let G be a graph attained by attaching  $K_1$  at each pendant vertex of a comb. Then G admits a Minkowski-4

Mean graph.

## Proof:

Let G be a graph attained by attaching  $K_1$  at each pendant vertex of a comb. The graph G has 3n vertices and 3n - 1 edges.

Define a function  $f: V(G) \rightarrow \{1, 2, 3, \dots, q+1\}$  by

$$f(u_i) = \begin{cases} i & , when \ i = 1 \\ i + 2 & , when \ i = 2 \\ 3i - 1 & , 3 \le i \le n \end{cases}$$

$$f(u_i) = \begin{cases} 2i & , when \ i = 1 \\ i + 3 & , when \ i = 2 \\ 3i - 2 & , 3 \le i \le n \end{cases}$$

$$f(w_i) = \{3i & , 1 \le i \le n \end{cases}$$

Then the induced edge labels are,

$$f^*(u_i u_{i+1}) = 3i$$
,  $1 \le i \le n-1$   
 $f^*(u_i v_i) = 3i - 1$ ,  $1 \le i \le n$ 

$$f^*(v_i w_i) = 3i - 2$$
 ,  $1 \le i \le n$ 

Then the edge labels are distinct.

Therefore, G is a Minkowski-4 Mean graph.



## **IV. CONCLUSIONS**

In this paper, we have introduced the notion of Minkowski-4 Mean Labeling and studied for some standard graphs. Illustrative examples are provided to support our investigation.

## REFERENCES

- Douglas B.West, Introduction to Graph Theory, Second Edition, PHI Learning Private Limited (2009). [1]
- [2] Gallian.J.A, A Dynamic Survey of Graph Labeling, The Electronic Journal of combinatorics(2013).
- Harary.F, *Graph Theory*, Narosa publishing House, New Delhi. Ponraj.R and Somasundaram.S, "Mean Labeling of graphs," in National Academy of Science Letters, vol.26, pp.210-213, 2003. [3] [4]
- Sandhya.S.S, Somasundaram.S and Anusa.S, "Root Square Mean Labeling of Graphs," International Journal of Contemporary [5] Mathematical Sciences, Vol.9, 2014, no. 667-676.
- Sandhya.S.S, Somasundaram.S and Anusa.S, "Some More Results on Root Square Mean Labeling of Graphs," Journal of [6] Mathematics Research, Vol.7, No.1;2015.
- [7] Sandhya.S.S, Somasundaram.S and Anusa.S, "Root Square Mean Labeling of Some New Disconnected Graphs,"International Journal of Mathematics Trends and Technology, volume 15, number 2,2014.page no:85-92.