# Minkowski-4 Mean Labeling of Graphs 

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$$
\begin{aligned}
& \text { Abstract - Let } G=(V, E) \text { be an undirected graph with } p \text { vertices and } q \text { edges. Define a function } \\
& f: V(G) \rightarrow\{1,2,3, \ldots, q+1\} \text { is called Minkowski-4 Mean Labeling of a graph } G \text { if we could able to } \\
& \text { label the vertices } x \in V \text { with distinct elements from } 1,2, \ldots, q+1 \text { such that it induces an edge labeling } \\
& f^{*}: E(G) \rightarrow\{1,2,3, \ldots, q+1\} \text { defined as, } \\
& \qquad \boldsymbol{f}^{*}(\boldsymbol{e}=\boldsymbol{u v})=\left\lfloor\left(\frac{f(\boldsymbol{u})^{4}+\boldsymbol{f}(\boldsymbol{v})^{4}}{2}\right)^{\frac{1}{4}}\right\rfloor
\end{aligned}
$$

is distinct for all edges $e=u v \in E$. (i,e.) It indicates that the distinct vertex labeling induces a distinct edge labeling on the graph. The graph which admits Minkowski-4 Mean Labeling is called a Minkowski-4 Mean Graph. In this paper, we have investigated the Minkowski-4 Mean Labeling of some standard graphs like Path, Comb, Caterpillar, $P_{n} \odot K_{1,2}$, etc.

Keywords-Minkowski-4 Mean Labeling, Minkowski-4 Mean Graph, Path, Comb, Caterpillar, $P_{n} \odot K_{1,2}$.

## I. Introduction

The graph $G$ we used here are simple, finite and undirected graphs. $V(G)$ and $E(G)$ denotes the vertex set and edge set of a graph $G$. For graph theoretic terminology, we refer to Harary.F [3], Douglas B. West [1] and Gallian.J.A [2]. The concept of Mean Labeling of graphs was introduced by Somasundaram.S and Ponraj.R [4] in 2003. Sandhya.S.S, Somasundaram.S and Anusa.S [6] introduced the concept of Root Square Mean Labeling of graphs in 2014. On the same lines we define and study Minkowski-4 Mean Labeling of graphs.

## II. BASIC DEFINITIONS

The following definitions are needed for the present study.

## A. Definition

A walk in which all the vertices are distinct is called a Path. A Path is denoted by $P_{n}$. The Path $P_{n}$ has $n$ vertices and $n-1$ edges.

## B. Definition

The graph attained by joining a single pendent edge to each vertex of a Path is called Comb. It has $2 n$ vertices and $2 n-1$ edges.

## C. Definition

A tree which yields a Path when its pendant vertices are removed is called a Caterpillar. It has $3 n$ vertices and $3 n-1$ edges.

## D. Definition

The $\boldsymbol{P}_{\boldsymbol{n}} \odot \boldsymbol{K}_{\mathbf{1 , 2}}$ is a graph attained by attaching the complete bipartite graph $\boldsymbol{K}_{\mathbf{1 , 2}}$ to each vertex of the path $P_{n}$. It has $3 n$ vertices and $3 n-1$ edges.

## III.MAIN RESULTS

## Theorem: 1

For every $n$, Path $P_{n}$ is a Minkowski-4 Mean graph .
Proof:
Let $P_{n}$ be a path $u_{1}, u_{2}, u_{3}, \ldots, u_{n}$ of length $n$. The gragh $G$ has $n$ vertices and $n-1$ edges.

Define a function $f: V(G) \rightarrow\{1,2,3, \ldots, q+1\}$ by

$$
f\left(u_{i}\right)=i \quad, 1 \leq i \leq n
$$

Then the induced edge labels are,

$$
f^{*}\left(u_{i} u_{i+1}\right)=i \quad, 1 \leq i \leq n-1
$$

Then the edge labels are distinct.
Therefore, $P_{n}$ is a Minkowski-4 Mean graph.


Figure 1: $P_{8}$

## Theorem: 2

For every $n, \operatorname{Comb} P_{n} \odot K_{1}$ is a Minkowski-4 Mean graph .

## Proof:

Let $P_{n} \odot \mathrm{~K}_{1}$ be a comb attained from a path $P_{n}=u_{1}, u_{2}, u_{3}, \ldots, u_{n}$ by joining a vertex $u_{i}$ to a pendent vertex $v_{i}(1 \leq i \leq n)$. The gragh $G$ has $2 n$ vertices and $2 n-1$ edges.
Define a function $f: V(G) \rightarrow\{1,2,3, \ldots, q+1\}$ by

$$
\begin{aligned}
& f\left(u_{i}\right)=\left\{\begin{array}{cl}
2 i & , \text { when } i=1 \\
2 i-1 & , 2 \leq i \leq n
\end{array}\right. \\
& f\left(v_{i}\right)=\left\{\begin{array}{cl}
i & , \text { when } i=1 \\
2 i & , 2 \leq i \leq n
\end{array}\right.
\end{aligned}
$$

Then the induced edge labels are,

$$
\begin{array}{ll}
f^{*}\left(u_{i} u_{i+1}\right)=2 i & , 1 \leq i \leq n-1 \\
f^{*}\left(u_{i} v_{i}\right)=2 i-1 & , 1 \leq i \leq n
\end{array}
$$

Then the edge labels are distinct.
Therefore, $P_{n} \odot \mathrm{~K}_{1}$ is a Minkowski-4 Mean graph.


Figure 2: $P_{5} \odot \mathrm{~K}_{1}$

Theorem: 3
Let $G$ be a graph attained by attaching a pendant edges to both sides of each vertex of a path $P_{n}$. Then $G$ is
a Minkowski-4 Mean graph .

## Proof:

Consider a graph $G$ which is attained by attaching a pendant edges to both sides of each vertex of a path $\mathrm{P}_{\mathrm{n}}$. Let $P_{n}$ be a path $v_{1}, v_{2}, v_{3}, \ldots, v_{n}$. Let $u_{i}$ and $w_{i}$ be the pendant vertices adjacent to $v_{i}(1 \leq i \leq n)$. The gragh $G$ has $3 n$ vertices and $3 n-1$ edges.
Define a function $f: V(G) \rightarrow\{1,2,3, \ldots, q+1\}$ by

$$
\begin{aligned}
& f\left(u_{i}\right)= \begin{cases}3 i-2 & , \text { if } i \text { is odd } \\
3 i-1 & , \text { if } i \text { is even }\end{cases} \\
& f\left(v_{i}\right)= \begin{cases}3 i-1 & , \text { if } i \text { is odd } \\
3 i-2 & , \text { if } i \text { is even }\end{cases} \\
& f\left(w_{i}\right)=\{3 i
\end{aligned}
$$

Then the induced edge labels are,

$$
\begin{array}{ll}
f^{*}\left(v_{i} v_{i+1}\right)=3 i & , 1 \leq i \leq n-1 \\
f^{*}\left(v_{i} u_{i}\right)=3 i-2 & , 1 \leq i \leq n \\
f^{*}\left(v_{i} w_{i}\right)=3 i-1 & , 1 \leq i \leq n
\end{array}
$$

Then the edge labels are distinct.
Therefore, $G$ is a Minkowski-4 Mean graph.


Figure 3: Caterpillar

## Theorem: 4

For every $n, P_{n} \odot K_{1,2}$ is a Minkowski -4 Mean graph .

## Proof:

Let $G$ be a graph attained by attaching each vertex of $P_{n}$ to the central vertex of the complete bipartite graph $K_{1,2}$. Let $P_{n}$ be a path $u_{1}, u_{2}, u_{3}, \ldots, u_{n}$ and $v_{i}, w_{i}$ be the vertices of $K_{1,2}$, which are attached to the vertex $u_{i}$ of $P_{n}$. The gragh $G$ has $3 n$ vertices and $3 n-1$ edges.
Define a function $f: V(G) \rightarrow\{1,2,3, \ldots, q+1\}$ by

$$
\begin{aligned}
& f\left(u_{i}\right)= \begin{cases}i & , \text { when } i=1 \\
i+2 & , \text { when } i=2 \\
3 i-1 & , 3 \leq i \leq n\end{cases} \\
& f\left(u_{i}\right)= \begin{cases}2 i & , \text { when } i=1 \\
i+3 & , \text { when } i=2 \\
3 i-2 & , 3 \leq i \leq n\end{cases} \\
& f\left(w_{i}\right)=\{3 i
\end{aligned}
$$

Then the induced edge labels are,

$$
\begin{array}{ll}
f^{*}\left(u_{i} u_{i+1}\right)=3 i & , 1 \leq i \leq n-1 \\
f^{*}\left(u_{i} v_{i}\right)=3 i-2 & , 1 \leq i \leq n \\
f^{*}\left(u_{i} w_{i}\right)=3 i-1 & , 1 \leq i \leq n
\end{array}
$$

Then the edge labels are distinct.
Therefore, $P_{n} \odot \mathrm{~K}_{1,2}$ is a Minkowski-4 Mean graph.


Figure 4: $P_{4} \odot \mathrm{~K}_{1,2}$

## Theorem: 5

Let $G$ be a graph attained by attaching $K_{1}$ at each pendant vertex of a comb. Then $G$ admits a Minkowski-4
Mean graph.
Proof:
Let $G$ be a graph attained by attaching $K_{1}$ at each pendant vertex of a comb. The gragh $G$ has $3 n$ vertices and $3 n-1$ edges.
Define a function $f: V(G) \rightarrow\{1,2,3, \ldots, q+1\}$ by

$$
\begin{aligned}
& f\left(u_{i}\right)= \begin{cases}i & , \text { when } i=1 \\
i+2 & , \text { when } i=2 \\
3 i-1 & , 3 \leq i \leq n\end{cases} \\
& f\left(u_{i}\right)= \begin{cases}2 i & , \text { when } i=1 \\
i+3 & , \text { when } i=2 \\
3 i-2 & , 3 \leq i \leq n\end{cases} \\
& f\left(w_{i}\right)=\{3 i
\end{aligned}
$$

Then the induced edge labels are,

$$
\begin{array}{ll}
f^{*}\left(u_{i} u_{i+1}\right)=3 i & , 1 \leq i \leq n-1 \\
f^{*}\left(u_{i} v_{i}\right)=3 i-1 & , 1 \leq i \leq n
\end{array}
$$

$$
f^{*}\left(v_{i} w_{i}\right)=3 i-2 \quad, 1 \leq i \leq n
$$

Then the edge labels are distinct.
Therefore, $G$ is a Minkowski-4 Mean graph.


Figure 5: $G$

## IV.CONCLUSIONS

In this paper, we have introduced the notion of Minkowski-4 Mean Labeling and studied for some standard graphs. Illustrative examples are provided to support our investigation.

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