Some Operations On Anti Fuzzy Graph

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Abstract *- Graphs are simple model of relation. It is a convenient way of representing information involving relationship between objects. In this chapter, we discussed the concept some operations such as null anti-fuzzy graph and the relationship between complete anti fuzzy graph and strong anti-fuzzy graphs are discussed. Suitable example is illustrated to demonstrate the null identity fuzzy graph.*

Keywords: - *Anti-fuzzy graph, strong anti-fuzzy graph, complete anti-fuzzy graph, null anti fuzzy graph.*

I. INTRODUCTION

Zadeh [12] introduced fuzzy sets in 1965 to represent/manipulate data and information possessing non-statistical uncertainties. It was, particularly,designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to manyproblems. But it was Rosenfeld [10] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. Rosenfeld has obtained the fuzzy analogues of several basic graph-theoretic concepts like bridges, paths,cycles, trees and connectedness and established some of their properties. There are several operations on $G1$ and $G2$ which result in agraph G whose set of points. These include the Cartesian product, the composition and the tensor product. Other operations of this form are developed in Harary and Wilcox [3] and they investigated some invariant properties of them. Operations on (crisp) graphs such as conjunction, disjunction, rejection and symmetric difference were extended to fuzzy graphs and a methodology is proposed to find the resulting fuzzy graphs of the same operations using adjacency matrices of $G1$ and $G2$ [7]. Bhutani [1] introduced the notion of weak isomorphism and isomorphism between fuzzy graphs. Nagoorgani and Malarvizhi [8] discussed the order, size and degree of the vertices of the isomorphic fuzzy graphs. Nagoorgani and Latha [6] introduced neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs and a comparative study between them had been analysed. In this chapter, we discussed the concept some operations such as null - identity anti fuzzy graph and the relationship between complete anti-fuzzy graph and strong anti-fuzzy graphs are discussed. We derived some theorem and example on them.

II. PRELIMINARIES

DEFINITION: 2.1

Let G be a simple fuzzy graph M of G is called anti fuzzy graph such that $M(x,y) \leq S(x)S((y), \forall$ (x, y) ∈ G An anti-fuzzy graph $AfG = (S, M)$ over the set V is called ***strong anti-fuzzy graph*** if $M(x,y) \leq S(x)S((y), \forall$ (x, y) ∈ G

DEFINITION: 2.2

An anti-fuzzy graph $G_A = (\sigma, \mu)$ is a strong anti-fuzzy graph if $\mu(u,v) = \sigma(u) \vee \sigma(v), \forall (u,v) \in \mu^*$, and is a complete anti fuzzy graph if $\mu(u,v) = \sigma(u) \vee \sigma(v)$, $\forall (u,v) \in \sigma^*$. Two nodes u and v are said to be neighbours if μ (u, v) > 0.

DEFINITION: 2.3

Let G be simple graph, the anti-fuzzy graph
$AFI \in AF, (V,E)$ is called ***identity anti-fuzzy graph*** such that denoted by I- if $AFI(e) = 1, \forall e \in E$.

DEFINITION: 2.4

An IFG $G:(V,E)$ is said to be a ***strong IFG*** if
$\mu_2(v_i, v_j) = \min\left(\mu_1(v_i), \mu_1(v_j)\right)$ and $v_2(v_i, v_j) = max\left(v_1(v_i), v_1(v_j)\right), \forall (v_i, v_j) \in E$.

III. OPERATIONS OF ANTI FUZZY GRAPH

DEFINITION 3.1

Consider an anti join $G_A^* = G_{A1}^* + G_{A2}^* = (v_1 \cup v_2, E_1 \cup E_2 \cup E')$ of anti fuzzy graphs where E' is the set of all edges joining the vertices of V_1 and V_2 where we assume that $V_1 \cap V_2 = \emptyset$. Then the anti join of anti fuzzy graphs G_{A1} and G_{A2} is an anti fuzzy graph $G_A = G_{A1} + G_{A2} : (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ is defined by

$$\sigma_1 + \sigma_2(u) = \sigma_1 \cup \sigma_2(u) \; if \; u \in V_1 \cup V_2$$

$$\mu_1 + \mu_2 \, (u,v) = \mu_1 \cup \mu_2(u,v) if (u,v) \in E_1 \cup E_2$$

$$\mu_1 + \mu_2 \, (u,v) = \max\{\sigma_1(u), \sigma_2(v)\} \; if \; (u,v) \in E'$$

DEFINITION: 3.2

Let G be a simple graph let $AfH = (S_i, M_i)$ and
$AfG = (S, M)$ is two anti-fuzzy graph over set v then AfH is called anti fuzzy sub-graph of anti-fuzzy graph such that

$$S_i(x) \leq S(x) M_i(x,y) \leq M(x,y) \forall (x,y) \in AfG$$

Now we define null-identity anti fuzzy graph.

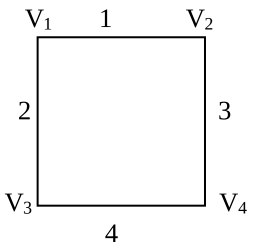
EXAMPLE: 3.1

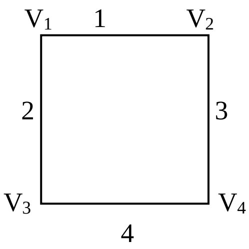
Let $= [v_1, v_2, v_3, v_4]$ and $E = [e_1, e_2, e_3, e_4]$. Here anti fuzzy graph AfG such that

$$M(x,y) = \begin{cases} 0 \; if \; v_i is \; not \; end \; of \; e_i \\ 1 \quad if \;\; v_i is \;\; end \; of \; e_i \end{cases}$$

$$V(\,x_i, x_j = 1, \forall(\,x_i, x_j) \in V)\; while\; V\;\;(\,x_i, x_j) = i,$$
$$i = 2,3,4,5, \forall(\,x_i\,, x_j) \notin \mathrm{V}$$

In the following graph G_1 and G_2





Of the graph G_1 we apply Anti fuzzy graph

$M(v_1, v_2) = 1\; and\; S(v_1)S(v_2) = 1$

$Then\; M(v_1, v_2) \leq\; S(v_1)S(v_2)$

$M(v_1, v_2) = 2$

$S(v_1) = 1\,, S(v_3) = 3$

$Then\; M(v_1, v_3) \leq\; S(v_1)S(v_3)$

$If\; M(v_3, v_4) = 3$

$S(v_3) = 3\,, S(v_4) = 4$

$M(v_3, v_4) \leq\; S(v_3)S(v_4)$

$If\; M(v_2, v_4) = 4$

$S(v_2) = 1\,, S(v_4) = 4$

$Then\; M(v_2, v_4) \leq\; S(v_2)S(v_4)$

Of the graph G_2 we apply Anti fuzzy graph

$M(v_1, v_2) = 2\; and\;\; S(v_1) = 2, S(v_2) = 3$

such that $S(v_1)\; S(v_2) = 6$

$Then\; M(v_1, v_2) \leq\; S(v_1)S(v_2)$

$If\;\; M(v_1, v_3) = 4$

$S(v_1) = 2\,, S(v_3) = 4$

$Then\; M(v_1, v_3) \leq\; S(v_1)S(v_3)$

$If\; M(v_3, v_4) = 5$

$S(v_3) = 4\,, S(v_4) = 5$

$Then\; M(v_3, v_4) \leq\; S(v_3)S(v_4)$

$If\; M(v_2, v_4) = 3$

$S(v_2) = 3\,, S(v_4) = 5$

$Then\; M(v_2, v_4) \leq\; S(v_2)S(v_4)$

THEOREM: 3.1

If $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ are complete anti fuzzy graphs, then $G_1 \oplus G_2$ is a strong anti-fuzzy graph.

PROOF:

Let $G_1 \oplus G_2 = G = (\sigma, \mu)$ where $\sigma = \sigma_1 \oplus \sigma_2$ and $\mu = \mu_1 \oplus \mu_2$ *and*

$G: (V, X)$ *where* $V = V_1 \times V_2$ *and*

$$X = \left\{\{(u_1, v_1), (u_1, v_2)/(u_1 \in v_{1,}(v_{1,}v_2) \in X_2\right\} \cup \left\{((u_{1,}v_1)(u_{2,}v_1)\right)$$

$$/v_1 \in v_2, (u_1, u_2) \in X_1\}$$

$$\cup \ \{((u_1, v_1), (u_2, v_2)), u_1, u_2 \in V_1, u_1 \neq u_2, v_1, v_2 \in V_1, v_1 \neq v_2$$

Either $\{(u_1, u_2) \in X_1 \, or (v_1, v_2) \in X_2\}$

CASE (i):

Let $e = ((u_1, v_1), (u_1, v_2)), \forall \, u_1 \in V_1, (v_1, v_2) \in X_2$

Then,

$$(\mu_1 \oplus \mu_2)((u_1, v_1), (u_1, v_2)) = \min\{\sigma_1(u_1), \mu_2(v_1, v_2)\}$$

$$= \sigma_1(u_1) \vee [\sigma_2(v_1) \vee \sigma_2(v_2)$$

Since G_2 is a complete anti fuzzy graph.

$$= [\sigma_1(u_1) \vee \sigma_2(v_1)] \vee [\sigma_1(u_1) \vee \sigma_2(v_2)]$$

$$= (\sigma_1 \oplus \sigma_2)(u_1, v_1) \vee (\sigma_1 \oplus \sigma_2)(u_1, v_2)$$

CASE (ii):

Let $e = ((u_1, v_1), (u_2, v_1)), \forall \, v_1 \in V_2, (u_1, u_2) \in X_1$

Then,

$$(\mu_1 \oplus \mu_2)((u_1, v_1), (u_2, v_1)) = \min\{\sigma_2(v_1), \mu_1(u_1, u_2)\}$$

$$= \sigma_2(v_1) \vee [\sigma_1(u_1) \vee \sigma_1(u_2)$$

Since G_1 is a complete anti fuzzy graph.

$$= [\sigma_1(u_1) \vee \sigma_2(v_1)] \vee [\sigma_1(u_2) \vee \sigma_2(v_1)]$$

$$= (\sigma_1 \oplus \sigma_2)(u_1, v_1) \vee (\sigma_1 \oplus \sigma_2)(u_2, v_1)$$

CASE (iii):

Let e $= ((u_1, v_1), (u_2, v_2)), \forall\, u_1, u_2 \in V_1, (v_1, v_2) \in V_2$

a) Suppose $(u_1, u_2) \notin X_1\ and (v_1, v_2) \in X_2$

Then,

$$(\mu_1 \oplus \mu_2)((u_1, v_1), (u_2, v_2)) = \min\{\sigma_1(u_1), \sigma_1(u_2)\mu_2(v_1, v_2)\}$$

$$= \sigma_1(u_1) \vee \sigma_1(u_2) \vee \{\sigma_2(v_1), \sigma_2(v_2)$$

Since G_2 is a complete anti fuzzy graph.

$$= [\sigma_1(u_1) \vee \sigma_2(v_1)] \vee [\sigma_1(u_2) \vee \sigma_2(v_2)]$$

$$= (\sigma_1 \oplus \sigma_2)(u_1, v_1) \vee (\sigma_1 \oplus \sigma_2)(u_2, v_2)$$

b) Suppose $(u_1, u_2) \in X_1\ and (v_1, v_2) \notin X_2$

Then,

$$(\mu_1 \oplus \mu_2)((u_1, v_1), (u_2, v_2)) = \min\{\sigma_2(v_1), \sigma_2(v_2), \mu_1(u_1, u_2)\}$$

$$= \sigma_2(v_1) \vee \sigma_2(v_2) \vee \{\sigma_1(u_1), \sigma_1(u_2)$$

Since G_1 is a complete anti fuzzy graph.

$$= [\sigma_1(u_1) \vee \sigma_2(v_1)] \vee [\sigma_1(u_2) \vee \sigma_2(v_2)]$$

$$= (\sigma_1 \oplus \sigma_2)(u_1, v_1) \vee (\sigma_1 \oplus \sigma_2)(u_2, v_2)$$

Thus in all cases, it is true that $G_1 \oplus G_2$ is a strong anti-fuzzy graph.

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