An EPQ Model for Non-Instantaneous Weibully Decaying Items with Ramp Type Demand and Partially Backlogged Shortages

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Abstract: *In this paper, an EPQ model is developed for deteriorating items with general ramp-type demand rate and time-varying holding cost; where the "Time-varying holding cost" means that the holding cost is a linear function of time,. Shortages are allowed to occur and partially backlogged. Sensitivity analysis is conducted with respect to model parameters and to make this model more realistic to the current scenario, it is also explained with the help of numerical example.*

Keywords: *Ramp Type Demand, Weibull distribution, Deterioration, Partially Backlogged*

I. INTRODUCTION

Many of the researchers have developed inventory model by assuming the demand of the items to be constant, linearly increasing or decreasing, exponentially increasing or decreasing with time, stock-dependent etc. But such type of demand patterns do not satisfy the demand of newly launched items, cosmetics, garments, automobile etc. for which demand raises at the time of their launches into the market and after sometime, it becomes constant. Such kind of demand is known as Ramp type demand. It depicts a demand which increases up to a certain time, after which, it stabilizes and becomes constant.

The first inventory model with ramp type demand rate was developed by Hill in 1995 and later on it was extended by Mandal and Pal (1998) by assuming ramp type demand for deteriorating item with shortages. Wu (2001) presented an EOQ inventory model which depleted not only by demand but also by Weibull distribution deterioration, in which the demand rate is assumed with a ramp type function of time. In the model, shortages were allowed to occur and partially backlogged, the backlogging rate was variable and was dependent on waiting time for the next replenishment. Moreover, Jain and Kumar (2010) developed an inventory model with ramp-type demand and three-parameter Weibull distribution deterioration rate.

Manna and Chaudhari (2006) also solved an order level inventory system for deteriorating items with demand rate as a ramp type function of time. The finite production rate was proportional to the demand rate and deterioration rate was time proportional. The unit production cost was inversely proportional to the demand rate. Skouri et al. (2009) studied inventory model with general ramp-type demand, time dependent Weibull deterioration and partial backlogging under different replenishment policies. An EPQ model for deteriorating items with ramp type demand was developed by Manna and Chiang (2010) by assuming that the finite production rate is proportional to the time-dependent demand rate and the unit production cost was inversely proportional to the production rate.

Garg and Bansal (2014) has developed an inventory model for non-instantaneous decaying items with ramp type demand and partial backlogging shortages. Chandra (2017) discussed an inventory model with ramp -type demand with time varying holding cost. Recently, Gothi, Pandya & Parmar (2018) discussed an EOQ model with Ramp type demand for three parameter Weibully distributed deterioration. In the model shortages are allowed and completely backlogged. Apart from the aforesaid studies, the work of Yadav et al. (2012), Bhojak and Gothi (2015), Singh and Sharma (2013), Chatterji and Gothi (2015) and many more have given significant contributions in the domain of inventory modelling.

In our present study, considering some of the assumptions and notations of Garg and Bansal (2014), we have revised the model for deteriorating items by considering two parameter Weibull distribution as deterioration rate. Shortages are allowed to occur and partially backlogged. Inventory holding cost is time dependent. At the end numerical examples are proposed to demonstrate our developed model and sensitivity analysis with respect to system parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

Following **assumptions** are used to develop the model.

1. Demand rate is following ramp type demand pattern:
   $$D(t) = \begin{cases} ae^{bt} \,;\, t < \mu \\ ae^{b\mu} \,;\, t \geq \mu \end{cases}$$
   Where a and b are positive constants, $a > b$ and $0 \leq b \leq 1$.
2. The production rate is dependent on demand:
   $P(t) = \lambda D(t)$
   Where λ is constant, $\lambda > 1$
3. Deterioration rate is non-instantaneous and it is
   $\theta(t) = \alpha\beta t^{\beta-1} \,;\, \mu \leq t \leq t_2$
   where α is a scale parameter $(0 < \alpha \ll 1)$ and β is a shape parameter $(\beta > 0)$.
4. The system operates for a prescribed period of a planning horizon.
5. Shortages are allowed and are partially backlogged.
6. There is no repair or replacement of deteriorating items during the period under consideration.
7. Single item inventory system is considered over a set period of time.
8. Holding cost is a linear function of Time.
   $C_h = h + rt, (h, r > 0)$
9. Production cost, shortage cost, lost sale cost and set up cost are known and constants.
10. Lead time is zero.

Following **notations** are used in developed model.

1.	$I(t)$	:	Inventory level at any time t.
2.	t_1	:	Time at which production stops.
3.	t_2	:	Decision variable representing the time at which inventory level drops at zero (0) level.
4.	t_3	:	Time at which maximum shortages occurs.
5.	t_4	:	The replenishment cycle.
6.	μ	:	The time at which deterioration starts.
7.	Q	:	The economic production quantity.
8.	A	:	The setup cost.
9.	C_h	:	Inventory holding cost per unit per unit time, where h>0.
10.	C_p	:	The production cost per unit item.
11.	C_s	:	The shortage cost per unit item.
12.	C_l	:	The lost sales cost per unit.
13.	TC	:	Total cost per unit time.
14.	$e^{-\delta t}$	:	The backlogging rate $(0 < \delta < 1)$.
15.	S_1	:	Inventory level at time $t = \mu$.

III. MATHEMATICAL FORMULATION

For ease of understanding, time period $[0, t_4]$ of derived inventory model can be divided in to five sub time periods i.e. $[0, \mu]$, $[\mu, t_1]$, $[t_1, t_2]$, $[t_2, t_3]$ & $[t_3, t_4]$ where zero (0) is the initial time and t_4 is the replenishment time. During the time period $[0, t_1]$ inventory level raises due to production and μ is the time at which deterioration starts. Combined effect of deterioration and demand is also observed during the time period $[\mu, t_1]$ where demand becomes constant after the time $t = \mu$. As the maximum stock level of inventory reaches, the production stops at the time $t = t_1$. During the time period $[t_1, t_2]$, inventory decreases due to the mutual effect of demand and deterioration. At time $t = t_2$ inventory becomes zero and thereafter shortages starts and continue during the time period $[t_2, t_3]$. Maximum shortages occurs at the time $t = t_3$ and at the same time production starts and continue up to the time at which backlogged is completed i.e. $t = t_4$. During the time period $[t_3, t_4]$ shortages are partially backlogged and backlogging rate is an exponential decreasing function of time. The same cycle is repeated for the further time period T.

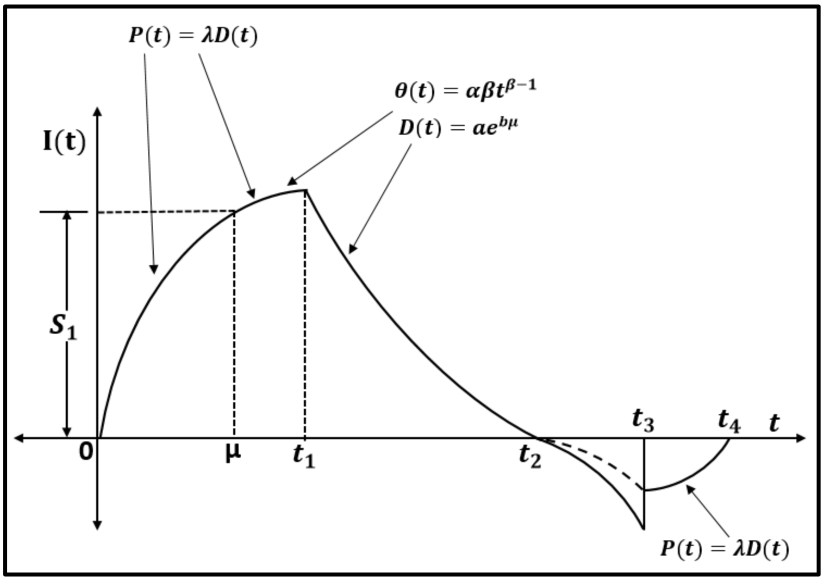

Figure: 1 Graphical Presentation of Developed Model

The differential equations which describe the instantaneous state of inventory at time t over the period $[0, t_4]$ are given by

$$I'(t) = (\lambda - 1)ae^{bt} \qquad 0 \le t \le \mu \qquad (1)$$

$$I'(t) = (\lambda - 1)ae^{b\mu} - \alpha\beta t^{\beta-1} I(t) \qquad \mu \le t \le t_1 \qquad (2)$$

$$I'(t) = -ae^{b\mu} - \alpha\beta t^{\beta-1} I(t) \qquad t_1 \le t \le t_2 \qquad (3)$$

$$I'(t) = -e^{-\delta(t_3 - t)} ae^{b\mu} \qquad t_2 \le t \le t_3 \qquad (4)$$

$$I'(t) = (\lambda - 1)ae^{b\mu} \qquad t_3 \le t \le t_4 \qquad (5)$$

By using the boundary conditions $I(0) = 0, I(\mu) = S_1, I(t_2) = 0$ *and* $I(t_4) = 0$, we get the solutions of equation (1) to (5) as under:

$$I(t) = (\lambda - 1)\frac{a}{b}(e^{bt} - 1) \qquad 0 \le t \le \mu \qquad (6)$$

$$I(t) = (\lambda - 1)ae^{b\mu}\left[t - \frac{\alpha\beta}{\beta+1}t^{\beta+1} - \mu\left(1 + \frac{\alpha\mu^\beta}{\beta+1}\right)\right] + S_1(1 + \alpha\mu^\beta - \alpha t^\beta) \qquad \mu \le t \le t_1 \qquad (7)$$

$$I(t) = K(1 - \alpha t^\beta) - ae^{b\mu}\left(t - \frac{\alpha\beta}{\beta+1}t^{\beta+1}\right) \qquad t_1 \le t \le t_2 \qquad (8)$$

$$\text{where } K = ae^{b\mu}\left(t_2 + \frac{\alpha t_2^{\beta+1}}{\beta+1}\right)$$

$$I(t) = \frac{ae^{b\mu}}{\delta}\left[e^{-\delta(t_3 - t_2)} - e^{-\delta(t_3 - t)}\right] \qquad t_2 \le t \le t_3 \qquad (9)$$

$$I(t) = -(\lambda - 1)ae^{b\mu}(t_4 - t) \qquad t_3 \le t \le t_4 \qquad (10)$$

By using $I(\mu) = S_1$ in equation (6), we get

$$S_1 = (\lambda - 1)\frac{a}{b}(e^{bt} - 1) \qquad (11)$$

Since inventory level is continuous at time $t = t_3$, from the equation (9) and (10), we get

$$\frac{ae^{b\mu}}{\delta}\left[e^{-\delta(t_3 - t_2)} - 1\right] = -(\lambda - 1)ae^{b\mu}(t_4 - t_3) \qquad (12)$$

$$\Rightarrow t_4 = t(e^{-\delta(t_3 - t_2)} - 1) \qquad (13)$$

Thus, t_4 can be expressed as a function of t_2 and t_3 (i.e. $t_4 = f(t_2, t_3)$).
The economic production quantity over the time period $[0, t_1]$ and $[t_3, t_4]$ can be determined as,

$$Q = \int_0^{\mu} P(t)dt + \int_{\mu}^{t_1} P(t)dt + \int_{t_3}^{t_4} P(t)dt$$

$$Q = \int_0^{\mu} \lambda a e^{bt}dt + \int_{\mu}^{t_1} \lambda a e^{b\mu}dt + \int_{t_3}^{t_4} \lambda a e^{b\mu}dt$$

$$Q = \frac{\lambda a}{b}\left(e^{b\mu} - 1\right) + \lambda a e^{b\mu}[(t_1 - \mu) + (t_4 - t_3)] \tag{14}$$

IV. COST COMPONENTS

The total cost per replenishment cycle consists of the following cost components.

1) Setup Cost

The set up cost over the time period $[0, t_4]$ is given by,

$$CR = A \tag{15}$$

2) Holding Cost

The holding cost over the time period $[0, t_2]$ is given by,

$$IHC = \int_0^{\mu} (h + rt)I(t)dt + \int_{\mu}^{t_1} (h + rt)I(t)dt + \int_{t_1}^{t_2} (h + rt)I(t)dt \tag{16}$$

$$= (\lambda - 1)\frac{a}{b}\left[h\left\{\frac{1}{b}\left(e^{b\mu} - 1\right) - \mu\right\} + r\left\{\mu\left(\frac{1}{b}e^{b\mu} - \mu\right) - \left(\frac{1}{b^2}e^{b\mu} - \frac{\mu^2}{2}\right) + \frac{1}{b^2}\right\}\right]$$

$$h\left[(\lambda - 1)ae^{b\mu}\left[\frac{1}{2}(t_1^2 - \mu^2) - \frac{\alpha\beta}{(\beta + 1)(\beta + 2)}\left(t_1^{\beta+2} - \mu^{\beta+2}\right) - \mu\left(1 + \frac{\alpha\mu^{\beta}}{\beta + 1}\right)(t_1 - \mu)\right]\right.$$
$$\left. + S_1\left[\left(1 + \alpha\mu^{\beta}\right)(t_1 - \mu) - \frac{\alpha}{\beta + 1}\left(t_1^{\beta+1} - \mu^{\beta+1}\right)\right]\right]$$

$$+r\left[(\lambda - 1)ae^{b\mu}\left[\frac{1}{3}(t_1^3 - \mu^3) - \frac{\alpha\beta}{(\beta + 1)(\beta + 3)}\left(t_1^{\beta+3} - \mu^{\beta+3}\right) - \mu\left(1 + \frac{\alpha\mu^{\beta}}{\beta + 1}\right)\frac{1}{2}(t_1^2 - \mu^2)\right]\right.$$
$$\left. + S_1\left[\left(1 + \alpha\mu^{\beta}\right)\frac{1}{2}(t_1^2 - \mu^2) - \frac{\alpha}{\beta + 2}\left(t_1^{\beta+2} - \mu^{\beta+2}\right)\right]\right]$$

$$+h\left[k_3\left((t_2 - t_1) - \frac{\alpha}{\beta + 1}\left(t_2^{\beta+1} - t_1^{\beta+1}\right)\right) - ae^{b\mu}\left(\frac{1}{2}(t_2^2 - t_1^2) - \frac{\alpha\beta}{(\beta + 1)(\beta + 2)}\left(t_2^{\beta+2} - t_1^{\beta+2}\right)\right)\right]$$
$$+r\left[k_3\left(\frac{1}{2}(t_2^2 - t_1^2) - \frac{\alpha}{\beta + 2}\left(t_2^{\beta+2} - t_1^{\beta+2}\right)\right) - ae^{b\mu}\left(\frac{1}{3}(t_2^3 - t_1^3) - \frac{\alpha\beta}{(\beta + 1)(\beta + 2)}\left(t_2^{\beta+3} - t_1^{\beta+3}\right)\right)\right]$$

3) Production Cost

The production cost over the period $[0, t_1]$ and $[t_3, t_4]$ is given by,

$$PC = C_pQ \tag{17}$$

$$PC = C_p\left[\frac{\lambda a}{b}\left(e^{b\mu} - 1\right) + \lambda a e^{b\mu}[(t_1 - \mu) + (t_4 - t_3)]\right]$$

4) Shortage Cost

The shortage cost over the period $[t_2, t_4]$ is given by,

$$SC = C_s\left[\int_{t_2}^{t_3} -I(t)dt + \int_{t_3}^{t_4} -I(t)dt\right] \tag{18}$$

$$= C_s\left[-\frac{ae^{b\mu}}{\delta}\left\{e^{-\delta(t_3-t_2)}(t_3-t_2)-\frac{1}{\delta}\left(1-e^{-\delta(t_3-t_2)}\right)\right\}+(\lambda-1)ae^{b\mu}\frac{(t_3-t_4)^2}{2}\right]$$

5) Lost Sale Cost

The lost sale cost over the period $[t_2, t_3]$ is given by,

$$LSC = C_l\int_{t_2}^{t_3}(1-e^{-\delta(t_3-t)})ae^{b\mu}dt \tag{19}$$

$$= C_l ae^{b\mu}\left[(t_3-t_2)-\frac{1}{\delta}\left(1-e^{-\delta(t_3-t_2)}\right)\right]$$

6) Deterioration Cost

The deterioration cost over the period $[\mu, t_2]$ is given by

$$CD = C_d\int_{\mu}^{t_2}\alpha\beta t^{\beta-1}I(t)\,dt \tag{20}$$

$$= C_d\alpha\beta\left[\begin{array}{c}a(\lambda-1)e^{b\mu}\left((S_1-\mu)\frac{1}{\beta}(t_1^{\beta}-\mu^{\beta})+\frac{1}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1})\right)\\ +K\frac{1}{\beta}(t_2^{\beta}-t_1^{\beta})+ae^{b\mu}\frac{1}{\beta+1}(t_2^{\beta+1}-t_1^{\beta+1})\end{array}\right]$$

Hence, the total cost per unit time is given by

$$TC = CR + IHC + PC + SC + LSC + DC \tag{21}$$

Our objective is to determine optimum values t_1^* *and* t_2^* of t_1 and t_2 respectively so that TC is minimum. The values t_1^* *and* t_2^*, for which the TC is minimum, are the solutions of equations $\frac{\partial TC(t_1,t_2)}{\partial t_1} = 0$ and $\frac{\partial TC(t_1,t_2)}{\partial t_2} = 0$ satisfying the conditions

$$\left(\frac{\partial^2 TC\,(t_1,t_2)}{\partial t_1^2}\right) > 0, \left(\frac{\partial^2 TC\,(t_1,t_2)}{\partial t_2^2}\right) > 0 \text{ and } \left(\frac{\partial^2 TC\,(t_1,t_2)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC\,(t_1,t_2)}{\partial t_2^2}\right) - \left(\frac{\partial^2 TC\,(t_1,t_2)}{\partial t_1\,\partial t_2}\right)^2 > 0$$

The optimal solution of the equation (21) can be obtained by using appropriate mathematical software.

V. NUMERICAL EXAMPLE

To illustrate the proposed model, an inventory system with the following hypothetical values is considered. By taking, $A = 50, a = 1.5, b = 0.2, \lambda = 1.8, \mu = 0.1, \alpha = 0.0005, \beta = 2, h = 3.5, r = 0.05, C_p = 3.2, C_s = 4, C_l = 3.8, C_d = 2.1, \delta = 0.05, t_3 = 3.5$ (with appropriate units). The optimal values $t_1^* = 0.9556514239$*units*, $t_2^* = 3.336793693$*units* and the optimal average total cost $TC = 77.968430$*units*.

VI. SENSITIVITY ANALYSIS

To study the impact of variation (i.e. an increase and decrease variation of 10% and 20%) in an independent variable's value on a particular dependent variables, below sensitivity analysis is carried out. Here, we've tried to capture the sensitivity of TC per unit time for every change in the value of parameters $A, a, b, \lambda, \mu, \alpha, \beta, h, r, C_p, C_s, C_l, C_d, \delta$ *and* t_3. The results are presented in the Table-1 and the last column shows the % change in TC as compared to the original value, for each of the parameter.

Table-1 Partial Sensitivity Analysis

Parameter	% change	t_1	t_2	TC	%change in TC
A	-20	0.95565	3.33679	67.96843	-12.82571
	-10	0.95565	3.33679	72.96843	-6.41285
	10	0.95565	3.33679	82.96843	6.41285
	20	0.95565	3.33679	87.96843	12.82570
a	-20	0.95566	3.33680	72.37472	-7.17433
	-10	0.95566	3.33679	75.17157	-3.58717
	10	0.95565	3.33679	80.76529	3.58717
	20	0.95564	3.33679	83.56215	7.17434
b	-20	0.95555	3.33676	77.85930	-0.13997
	-10	0.95560	3.33678	77.91381	-0.07006
	10	0.95570	3.33681	78.02316	0.07019
	20	0.95576	3.33682	78.07800	0.14053
	-20	1.70715	3.73611	73.72004	-5.44886

Parameter	% change	t_1	t_2	TC	%change in TC
λ	-10	1.27997	3.52082	76.51081	-1.86950
	10	0.70506	3.18171	78.72137	0.96570
	20	0.50770	3.05111	79.06569	1.40732
μ	-20	0.95546	3.33674	77.86131	-0.13738
	-10	0.95555	3.33676	77.91494	-0.06860
	10	0.95576	3.33683	78.02177	0.06841
	20	0.95589	3.33686	78.07497	0.13665
α	-20	0.95522	3.33726	77.95990	-0.01094
	-10	0.95544	3.33703	77.96417	-0.00547
	10	0.95586	3.33656	77.97269	0.00546
	20	0.95608	3.33633	77.97694	0.01092
β	-20	0.95519	3.33797	77.95153	-0.02167
	-10	0.95542	3.33746	77.95907	-0.01200
	10	0.95588	3.33591	77.98003	0.01488
	20	0.95610	3.33474	77.99442	0.03334
h	-20	0.78777	3.43860	74.25990	-4.75645
	-10	0.88407	3.38717	76.18890	-2.28237
	10	1.00959	3.28751	79.62968	2.13067
	20	1.05053	3.23934	81.19381	4.13678
r	-20	0.95552	3.34091	77.88870	-0.10226
	-10	0.95558	3.33885	77.92860	-0.05108
	10	0.95572	3.33474	78.00817	0.05097
	20	0.95578	3.33269	78.04783	0.10184
C_p	-20	1.09538	3.26168	75.72200	-2.88120
	-10	1.02559	3.29921	76.89649	-1.37484
	10	0.88556	3.37442	78.93756	1.24297
	20	0.81531	3.41210	79.80360	2.35373
C_s	-20	0.93643	3.30280	77.92444	-0.05642
	-10	0.94695	3.32140	77.94850	-0.02556
	10	0.96298	3.34975	77.98520	0.02151
	20	0.96923	3.36080	77.99952	0.03987
C_l	-20	0.95531	3.33619	77.96765	-0.00100
	-10	0.95548	3.33649	77.96804	-0.00050
	10	0.95582	3.33709	77.96881	0.00049
	20	0.95599	3.33739	77.96920	0.00099
C_d	-20	0.95565	3.33697	77.96507	-0.00431
	-10	0.95565	3.33688	77.96675	-0.00216
	10	0.95565	3.33671	77.97011	0.00215
	20	0.95565	3.33662	77.97179	0.00431
δ	-20	0.95612	3.33761	77.96937	0.00120
	-10	0.95588	3.33721	77.96890	0.00060
	10	0.95542	3.33638	77.96796	-0.00061
	20	0.95518	3.33596	77.96748	-0.00122
t_3	-20	0.62385	2.74962	69.25524	-11.17528
	-10	0.78989	3.04354	73.48027	-5.75638
	10	1.12112	3.62936	82.72093	6.09541
	20	1.28632	3.92125	87.73895	12.53138

VII. GRAPHICAL PRESENTATION

Graphical presentation of the above sensitivity analysis is shown in Figure-2 and Figure-3.

Figures: Graphical Presentation of Sensitivity Analysis

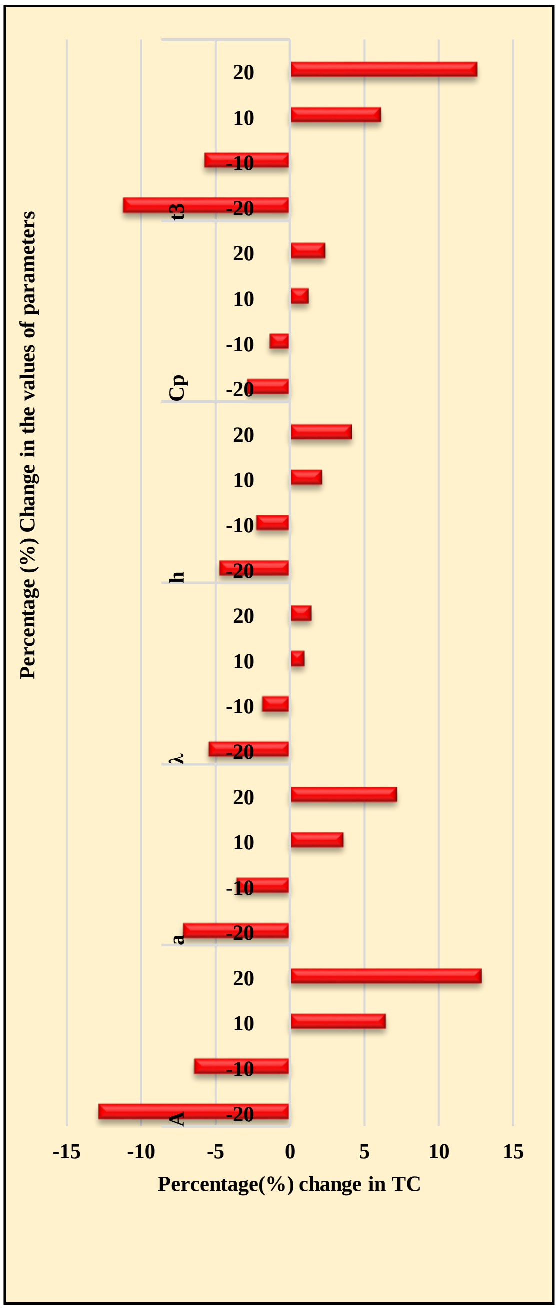

Figure-2

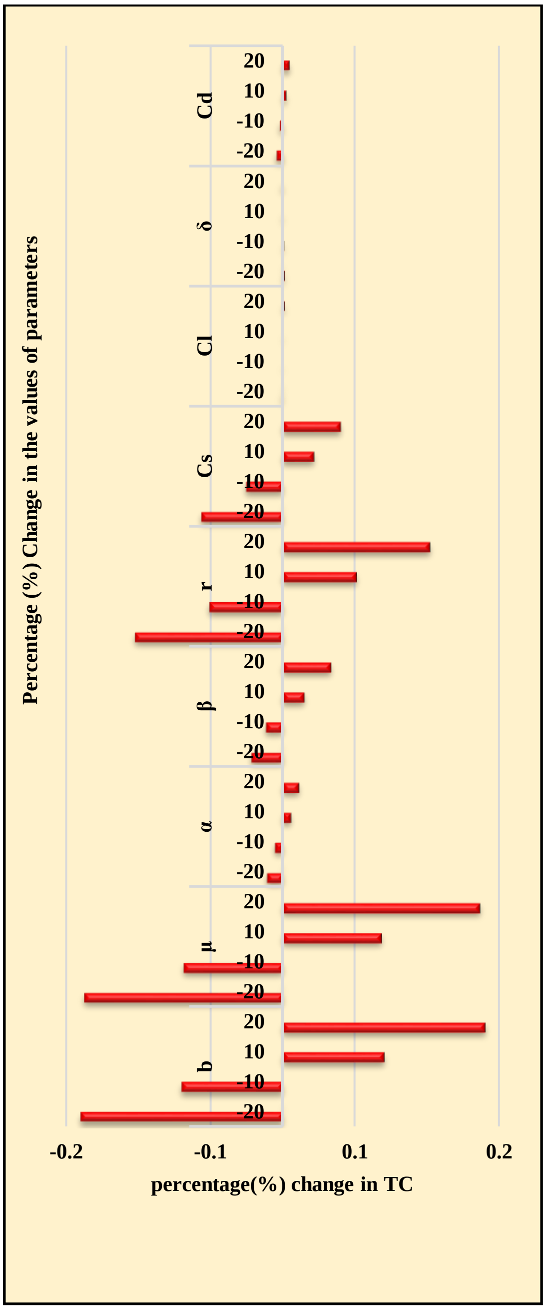

Figure-3

VIII. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with ramp type demand. Two parameter Weibull distribution is considered as deterioration rate while shortages are partially backlogged and backlogging rate is an exponential decreasing function of time. As the changes of the time value of money and in the price index, holding cost may not remain constant over time. It is assumed that the holding cost is linearly increasing function of time. A numerical assessment of the theoretical model has been done to illustrate the theory.

From partial sensitivity analysis (table-1), it is established that the total cost TC per time unit is highly sensitive to the changes in the values of the parameters $A, a \text{ and } t_3$; moderately sensitive to the changes in the values of the parameters $\lambda, h, C_p, b, \mu \text{ and } r$; less sensitive to the changes in the values of the parameters $\alpha, \beta, C_s, C_l, \delta \text{ and } C_d$.

Here, it should be noted that the negative percentage values shown in the above graph shows the changes in the parameter values are in the negative direction.

Of course, the paper provides an interesting topic for the further study of such kind of important inventory models, and at the same time, the following two problems can be considered in future research, either.
(1) One can consider one more case like $t_1 < \mu < t_2$.
(2) How about the inventory model starting with shortages?

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