Convective Heat And Mass Transfer Flow of A Micropolar Fluid In A Rectangular Duct With Heat Generating Sources

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Abstract:

The convective heat and Mass transfer through a porous medium in a Rectangular enclosure with Darcy model. The transport equations of liner momentum, angular momentum and energy are solved by employing Galerkine finite element analysis with linear triangular elements. The computation is carried out for different values of Rayleigh number – Ra micro polar parameter – R, spin gradient parameter - λ, Eckert number Ec and heat source parameter - α. The rate of heat transfer and couple stress on the side wall is evaluated for different variation of the governing parameters.

Keywords*: Heat and Mass Transfer, Micro polar Fluid, Rectangular Duct, Heat Generating Sources*

INTRODUCTION:

As high power electronic packaging and component density keep increasing substantially with the fast growth of electronic technology, effective cooling of electronic equipment has become exceptionally necessary. Therefore, the natural convection in an enclosure has become increasingly important in engineering applications in recent years. Through studies of the thermal behavior of the fluid in a partitioned enclosure is helpful to understand the more complex processes of natural convection in practical applications number of studies, numerical and experimental, concerned with the natural convection in an enclosure with or without a divider were conducted in past years.

Several relevant analytical and experimental studies have been reported during the past decades. Excellent reviews have been given by Ostrach [12] and Catton [3]. The critical Rayleigh numbers for natural convection in rectangular boxes heated from below and cooled from above have been obtained theoretically by Davis [7] and Cotton [4]. Samuels and Churchill [15] presented the stability of fluids in rectangular region heated from below and obtained the critical Rayleigh numbers with finite differences approximation. Ozoe *et al.* [13, 14] determined experimentally and numerically the natural convection in an inclined long channel with a rectangular cross-section, and found the effects of inclination angle and aspect ratio on the circulation and rate of heat transfer. Wilson and Rydin [16] discussed bifurcation phenomenon in a rectangular cavity, as calculated by a nodal integral method. They obtained critical aspect ratios and Rayleigh numbers and found good agreement with the results of Cliffe and Winter [5]. Numerous studies have been presented for a different arrangement of boundary conditions [14-15]. Several analytical studies of natural heat transfer in a rectangular porous cavity have been carried out recently [1, 2]. The theory of micro polar fluids developed by Erigen [8, 9 and 10] has been a popular filed of research in recent years. In this theory, the local effects arising from the microstructure and the intrinsic motions of the fluid elements are taken into account. It has been expected to describe properly the non-Newtonian behaviour of certain fluids, such as liquid crystals, ferroliquids, colloidal fluid, and liquids with polymer additives. Recently, Jena and Bhattacharya [11] studied the effect of microstructure on the thermal convection in a rectangular box heated from below with Galerkin's method, and obtained critical Rayleigh numbers for various material parameters, Cehn and Hsu [6] considered the effect of mesh size on the thermal convection in an enclosed cavity. Wang *et al.* [18] presented the study of the natural convection of micro polar fluids in an inclined rectangular enclosure; HSU *et al.* [18] investigated the thermal convection of micro polar fluids in a lid-driven cavity. They reported that the material parameters such as voters' viscosity and sin gradient viscosity strongly influence the flow structure and heat transfer. Cha-Kaungchen *et al.* [6] have investigated numerically the steady laminar natural convection flow of a micro polar fluid in a rectangular cavity. The angular momentum and energy are solved with the aid of the cubic spline collocation method. Parametric studies of the effects of microstructure on the fluid flow and heat transfer in the enclosure have been performed.

Wang *et al.* (17) have investigated natural convection flow of a micro polar fluid in a partially divided rectangular enclosure. The enclosure is partially divided protruding flore of the enclosure. The effect of the highest on the location of the divider is investigated. Also the effects of material parameters or micro polar fluids or the thermal characters, are also studied for different Rayleigh numbers. Tsan – Hsu *et al.* (6) have investigated the effects, the characteristic parameters of micro polar fluids on mixed convection in a cavity. The equations are solved with the help of the cubic spin collocation method.

In this chapter we make an investigation of the convective heat transfer through a porous medium in a Rectangular enclosure with Darcy model. The transport equations of liner momentum, angular momentum and energy are solved by employing Galerkine finite element analysis with linear triangular elements. The computation is carried out for different values of Rayleigh number – Ra micro polar parameter – R, spin gradient parameter - λ, Eckert number Ec and heat source parameter - α. The rate of heat transfer and couple stress on the side wall is evaluated for different variation of the governing parameters.

FORMULATION OF THE PROBLEM:

We consider the mixed convective heat and mass transfer flow of a viscous, incompressible, micro polar fluid in a saturated porous medium confined in the rectangular duct whose base length is a and height b. The heat flux on the base and top walls is maintained constant. The Cartesian coordinate system 0(x, y) is chosen with origin on the central axis of the duct and its base parallel to X-axis, we assume that

(i) The convective fluid and the porous medium are everywhere in local thermodynamic equilibrium.
(ii) There is no phase change of the fluid in the medium.
(iii) The properties of the fluid and of the porous medium are homogeneous and isotropic.
(iv) The porous medium is assumed to be closely packed so that Darcy's momentum law is adequate in the porous medium.
(v) The Boussinesq approximation is applicable,

Under the assumption the governing equations are given by

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \tag{4.1}$$

$$\frac{-\partial p}{\partial x}-\frac{(\mu+k)}{k_1}u+k\frac{\partial \omega}{\partial y}=0 \tag{4.2}$$

$$\frac{-\partial p}{\partial x}-\frac{(\mu+k)}{k_1}v+k\frac{\partial \omega}{\partial y}-\rho\bar{g}=0 \tag{4.3}$$

$$\rho_\sigma C_P\left[u'\frac{\partial T'}{\partial x}v'\frac{\partial T'}{\partial y'}\right]=k_f\left[\frac{\partial^2 T'}{\partial x'^2}+\frac{\partial^2 T'}{\partial y'^2}\right]-Q(T-T_0)+(\frac{\mu}{k_1})(u^2+v^2) \tag{4.4}$$

$$\rho_j\left[u'\frac{\partial \omega}{\partial x'}+v'\frac{\partial \omega}{\partial y'}\right]=\gamma\left[\frac{\partial^2 \omega}{\partial x'^2}+\frac{\partial^2 \omega}{\partial y'^2}\right]+K\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}-2\omega\right) \tag{4.5}$$

$$\rho_\sigma C_P\left[u'\frac{\partial C}{\partial x}v'\frac{\partial C}{\partial y'}\right]=D_m\left[\frac{\partial^2 C}{\partial x'^2}+\frac{\partial^2 C}{\partial y'^2}\right] \tag{4.6}$$

$$\rho'=\rho_0\{1-\beta_0(T-T_0)-\beta^\bullet(C-C_0)\}$$

$$T_0 = \frac{T_h + T_c}{2}, \qquad\qquad C_0 = \frac{C_h + C_c}{2} \tag{4.7}$$

Where u′ and v′ are Darcy velocities along 0(x, y) direction. T′, N, p′ and g′ are the temperature, micro rotation, pressure and acceleration due to gravity, T_c and T_h are the temperature on the cold and warm side walls respectively. ρ′, μ, v and β are the density, coefficients of viscosity, kinematic viscosity and thermal expansion of the fluid, k_1 is the permeability of the porous medium, γ, k are the micro polar and material constant pressure , Q is the strength of the heat source.

The boundary conditions are

u′ = v′ = 0 on the boundary of the duct

T′ = T_h , C=C_h on the side wall to the right (x = 1)

T′ = T_c , C=C_c on the side wall to the right (x = 0) (4.8)

$\frac{\partial T'}{\partial y} = 0, \frac{\partial C}{\partial y} = 0$ on the top (y = 0) and bottom

u = v = 0 walls (y = 0) which are insulated,

$$\left.\begin{aligned} &\omega = \frac{-1}{2}\frac{\partial u}{\partial x} \quad \text{On } y = 0 \ \&\ 1 \\ &\omega = \frac{-1}{2}\frac{\partial v}{\partial x} \quad \text{On } x = 0 \ \&\ 1 \end{aligned}\right\} \tag{4.9}$$

Eliminating the pressure p from equations (4.2) and (4.3) and using (4.6) we get

$$\left. \begin{aligned} 0 = \frac{-(\mu + k)}{k_1}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + k\left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x^2}\right) + \rho\beta g(\frac{\partial}{\partial x}(T - T_0) + \\ + \rho\beta^{\bullet} g\frac{\partial}{\partial x}(C - C_0) \end{aligned} \right| \tag{4.10}$$

On introducing the stream functionψ, as

$$u = -\frac{\partial \Psi}{\partial y}, \qquad\qquad v = \frac{\partial \Psi}{\partial y}$$

The equations (4.10) and (4.5) reduce to

$$\frac{(\mu + k)}{k_1}\nabla^2\Psi + k\nabla^2\omega + (\beta_0 g\frac{\partial}{\partial x}(\mathrm{T} - \mathrm{T}_0) + \beta^{\bullet} g\frac{\partial}{\partial x}(\mathrm{C} - \mathrm{C}_0)) \tag{4.11}$$

$$\rho_0 C_P\left(\frac{\partial \Psi}{\partial x}\frac{\partial T}{\partial y} - \frac{\partial \Psi}{\partial y}\frac{\partial T}{\partial y}\right) = k_f\nabla^2 T - Q(T - T_0) + (\frac{\mu}{k_1})((\frac{\partial \psi}{\partial x})^2 + (\frac{\partial \psi}{\partial y})^2) \tag{4.12}$$

$$\rho_j\left(\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial\psi}{\partial x}\right)=\gamma\nabla^2\omega+K\left(\nabla^2\psi-2\omega\right) \tag{4.13}$$

$$\left(\frac{\partial\Psi}{\partial x}\frac{\partial C}{\partial y}-\frac{\partial\Psi}{\partial y}\frac{\partial C}{\partial y}\right)=D_m\nabla^2 C \tag{4.14}$$

Now introducing the following non-dimensional variables

$$x'=ax: \qquad y'=by: \qquad h=\frac{b}{a}, \quad \Psi'=\frac{\Psi}{v}$$

$$u'=\left(\frac{v}{a}\right)u: \qquad v'=\left(\frac{v}{a}\right)v: \qquad p'=\left(\frac{v^2\rho}{a^2}\right)p$$

$$T'=T_c+\theta(T_h-T_c): \qquad \omega^1=\left(\frac{\frac{\omega}{v}}{a^{22}}\right);$$

$$C'=C_c+C\,(C_h-C_c): \tag{4.15}$$

the equations (4.11) – (4.13) in the non-dimensional form are

$$\nabla\ \psi=-RD^{-1}\nabla^2\omega-GD^{-1}\left(\frac{\partial\theta}{\partial x}+N\frac{\partial C}{\partial x}\right) \tag{4.16}$$

$$P\left[\frac{\partial\Psi}{\partial x}\frac{\partial\theta}{\partial y}-\frac{\partial\Psi}{\partial y}\frac{\partial\theta}{\partial y}\right]=\left[\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right]-\alpha\theta+Ec(((\frac{\partial\psi}{\partial x})^2+(\frac{\partial\psi}{\partial y})^2) \tag{4.17}$$

$$\nabla^2\omega+2\left(\frac{R}{\lambda}\right)\omega=\left(\frac{R}{\lambda}\right)\nabla^2\Psi \tag{4.18}$$

$$Sc\left[\frac{\partial\Psi}{\partial x}\frac{\partial C}{\partial y}-\frac{\partial\Psi}{\partial y}\frac{\partial C}{\partial y}\right]=\left[\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right] \tag{4.19}$$

Where

$G=\dfrac{g\beta(T_h-T_c)a^3}{v^2}$ (Grashof number)

$P=\mu C_p/k_f$ (Prandtl number)

$D^{-1}=\dfrac{a^2}{k_1}$ (Darcy parameter)

$$R = \frac{k}{\mu} \qquad \text{(Micropolar parameter)}$$

$$\lambda = \frac{\gamma}{va^2} \qquad \text{(Micropolar material constant)}$$

$$Sc = \frac{\nu}{D_m} \qquad \text{(Schmidt number)}$$

$$N = \frac{\beta^{\bullet}(C_h - C_c)}{\beta(T_h - T_c)} \qquad \text{(Buoyancy ratio)}$$

$$\alpha = \frac{Qa^2}{k_f C_p} \qquad \text{(Heat source parameter)}$$

$$Ec = \frac{a^4}{\mu k k_f \Delta T} \qquad \text{(Eckert number)}$$

And the corresponding boundary conditions are

$\psi_y = \psi_x = 0$ on the boundary (4.17a)

$$\left.\begin{array}{llll} \theta = 1 & , C=1 & \text{on} & x = 1 \\ \theta = 0 & , C=0 & \text{on} & x = 0 \end{array}\right\} \qquad (4.17b)$$

$$\left.\omega = \frac{1}{2}\frac{\partial^2 \Psi}{\partial x \partial y} \qquad \text{On} \qquad y = 0 \ \& \ 1 \right\}$$

$$\omega = -\frac{1}{2}\frac{\partial^2 \Psi}{\partial y \partial x} \qquad \text{On} \qquad x = 0 \ \& \ 1 \qquad (4.17c)$$

METHOD OF SOLUTION:

Finite Element Analysis:

The region is divided into a finite number of three noded triangular elements, in each of which the element equation is derived using Galerkin weighted residual method. In each element f_i the approximate solution for an unknown f in the variation formulation is expressed as a linear combination of shape function $\left(N_k^i\right)$k = 1, 2, 3, which are linear polynomials sin x and y. This approximate solution of the unknown f coincides with actual values of each node of the element. The variation formulation results in 3x3 matrix equation (stiffness matrix) for the unknown local nodal values of the given element. The stiffness matrices are assembled in terms of global nodal values using inter element continuity and boundary conditions resulting in global matrix equation.

In each case there are r distinct global nodes in the finite element domain and f_p = (p =1, 2...r) is the global nodal values of any unknown f defined over the domain

Then $f = \sum_{\propto=1}^{s} \sum_{p=1}^{r} f_p \phi'_p$ (4.18)

Where the first summation denotes summation over s elements and the second one represents summation over the independent global nodes and

$\phi_p^i = N_N^i$, if p is one of the local nodes say k of the element e_i = 0, f'_p s are determined from the global matrix equation. Based on these lines we now make a finite element analysis of the given problem governed by (4.14) – (4.16) subject to the conditions (4.17a) – (4.17c).

Let ψ^i, θ^i and N^i be the approximate valuesψ, θ and N in a element e_i

$$\Psi^i = N_1^i \Psi_1^i + N_2^i \Psi_2^i + N_3^i \Psi_3^i \quad (4.18a)$$

$$\theta^i = N_1^i \theta_1^i + N_2^i \theta_2^i + N_3^i \theta_3^i \quad (4.18b)$$

$$C^i = N_1^i C_1^i + N_2^i C_2^i + N_3^i C_3^i \quad (4.18c)$$

$$\omega^i = N_1^i \omega_1^i + N_2^i \omega_2^i + N_3^i \omega_3^i \quad (4.18d)$$

Substituting the approximate value ψ^i, θ^i, C^i and ω^i for ψ, θ, C and ω respectively in (4.14)

$$E_1^i = \left(\frac{\partial^2 \theta^i}{\partial x^2}\right) + \frac{\partial^2 \theta^i}{\partial y^2} - P\left[\frac{\partial \Psi^i}{\partial y}\frac{\partial \theta^i}{\partial x} - \frac{\partial \Psi^i}{\partial x}\frac{\partial \theta^i}{\partial y}\right] - \alpha\theta^i + + Ec\left(\left(\frac{\partial \psi^i}{\partial x}\right)^2 + \left(\frac{\partial \psi^i}{\partial y}\right)^2\right) \quad (4.19)$$

$$E_2^i = \frac{\partial^2 \omega^i}{\partial x^2} + \frac{\partial^2 \omega^i}{\partial y^2} + 2R\omega^i - \frac{R}{\lambda}\left(\frac{\partial^2 \Psi^i}{\partial x^2} + \frac{\partial^2 \Psi^i}{\partial y^2}\right) \quad (4.20)$$

Under Galaerkin method this is made orthogonal over the domain e_i to the respective shape functions (weight function)

Where $\int_{e_i} E_1^i N_k^i d\Omega = 0$

$$\int_{e_i} E_2^i N_k^i d\Omega = 0 \quad (4.21)$$

$$\Rightarrow \int_{e_i} N_k^u \left(\frac{\partial^2 \theta^i}{\partial x^2}\right) + \frac{\partial^2 \theta^i}{\partial y^2} + P\left[\frac{\partial \Psi^i}{\partial y}\frac{\partial \theta^i}{\partial x} - \frac{\partial \Psi^i}{\partial x}\frac{\partial \theta^i}{\partial y}\right] - \alpha\theta^i + Ec\left(\left(\frac{\partial \psi^i}{\partial x}\right)^2 + \left(\frac{\partial \psi^i}{\partial y}\right)^2\right) d\Omega = 0$$

$$\int_{e_i} N_k^i \left[\frac{\partial^2 \omega^i}{\partial x^2} + \frac{\partial^2 \omega^i}{\partial y^2}\right] + 2R\omega^i - \frac{R}{\lambda}\left(\frac{\partial^2 \Psi^i}{\partial x^2} + \frac{\partial^2 \Psi^i}{\partial y^2}\right) d\Omega = 0 \quad (4.22)$$

Using Green's theorem we reduce the surface integral (4.21) and (4.22) without affecting ψ terms and obtain

$$\int_{e_i} N_k^i \left\{ \frac{\partial N_k^i}{\partial x}\frac{\partial \theta^i}{\partial x} + \frac{\partial N_k^i}{\partial y}\frac{\partial \theta^i}{\partial y} - PN_k\left[\frac{\partial \Psi^i}{\partial y}\frac{\partial \theta^i}{\partial x} - \frac{\partial \Psi^i}{\partial x}\frac{\partial \theta^i}{\partial y}\right] - \alpha\theta^i + Ec((\frac{\partial \psi^i}{\partial x})^2 + (\frac{\partial \psi^i}{\partial y})^2) \right\} d\Omega$$

$$= \int_{\Gamma_i} N_k^i \left(\frac{\partial \theta^i}{\partial x} nx + \frac{\partial \theta^i}{\partial x} ny \right) d\Gamma_i \qquad (4.23)$$

$$\int_{e_i e_i} N_k^i \left\{ \frac{\partial N_k^i}{\partial x}\frac{\partial \omega^i}{\partial x} + \frac{\partial N_k^i}{\partial y}\frac{\partial \omega^i}{\partial y} \right\} + 2R\omega^i - \frac{R}{\lambda}\left(\frac{\partial N_k^i}{\partial x}\frac{\partial \Psi^i}{\partial x} + \frac{\partial N_k^i}{\partial y}\frac{\partial \Psi^i}{\partial y} \right)$$

$$= \int_{e_i} N_k^i \left[\left(\frac{\partial \omega^i}{\partial x} - \frac{R}{\lambda}\frac{\partial \Psi^i}{\partial x} \right) nx + \left(\frac{\partial \omega^i}{\partial y} - \frac{R}{\lambda}\frac{\partial \Psi^i}{\partial y} \right) ny + \right] d\Gamma_i \qquad (4.24)$$

Where Γ_1 is the boundary of e_i, substituting L.H.S of (4.18a) – (4.18d) for ψ^i, θ^i, C^i and ω^i in (4.23) and (4.24) we get

$$\sum_{i=1}^{3} \int_{e_i} \frac{\partial N_k^i}{\partial x}\frac{\partial N_L^i}{\partial x} - \frac{\partial N_L^i}{\partial y}\frac{\partial N_k^i}{\partial y}$$

$$-P\sum_{i=1}^{3}\Psi_m^i \int_{e_i} \left[\frac{\partial N_m^i}{\partial y}\frac{\partial N_L^i}{\partial x} - \frac{\partial N_m^i}{\partial x}\frac{\partial N_L^i}{\partial y} \right] - \alpha\sum_{i=1}^{3}\Psi_m^i \int_{e_i} N_k^i + Ec\sum_{i=1}^{3}\Psi_m^i \int_{e_i} ((\frac{\partial \psi^I}{\partial x})^2 + (\frac{\partial \psi^I}{\partial y})^2) d\Omega$$

$$= \int_{\Gamma_i} N_k^i \left[\frac{\partial \theta^i}{\partial x} nx + \frac{\partial \theta^i}{\partial y} ny \right] d\Gamma_i = W_k^i$$

(l, m, k = (1, 2, 3)) (4.25)

$$\sum_{i=1}^{3} \int_{e_i} \frac{\partial N_k^i}{\partial x}\frac{\partial N_L^i}{\partial x} - \frac{\partial N_L^i}{\partial y}\frac{\partial N_k^i}{\partial y}$$

$$-Sc\sum_{i=1}^{3}\Psi_m^i \int_{e_i} \left[\frac{\partial N_m^i}{\partial y}\frac{\partial N_L^i}{\partial x} - \frac{\partial N_m^i}{\partial x}\frac{\partial N_L^i}{\partial y} \right]$$

$$= \int_{\Gamma_i} N_k^i \left[(\frac{\partial \theta^i}{\partial x} nx + \frac{\partial \theta^i}{\partial y} ny \right] d\Gamma_i = W_k^i$$

(l, m, k = (1, 2, 3))

$$\sum_{1}\int_{e_i} N^i\left(\frac{\partial N_k^i}{\partial x}\frac{\partial N_L^i}{\partial y}+\frac{\partial N_L^i}{\partial y}\frac{\partial N_k^i}{\partial y}\right)$$

$$+\frac{2R}{\lambda}\Sigma\omega^i\int N_k^i N_L^i d\Omega-\frac{R}{\lambda}\Sigma\Psi^i\int\left(\frac{\partial N_k^i}{\partial x}\frac{\partial N_J^i}{\partial x}+\frac{\partial N_k^i}{\partial y}\frac{\partial N_J^i}{\partial y}\right)d\Omega \tag{4.26}$$

$$=\int_{\Gamma_i} N_k^i\left[\left(\frac{\partial\omega^i}{\partial x}-\frac{R}{\lambda}\frac{\partial\Psi^2}{\partial x}\right)nx+\left(\frac{\partial\omega^i}{\partial y}-\frac{R}{\lambda}\frac{R\partial\Psi^i}{\partial y}\right)ny\right]d\Gamma_i=Q_i^N$$

Where $Q_k^i=Q_{k1}^i+Q_{k2}^i+Q_{k3}^i, Q_k^i{}'s$ being the values of Q_k^i on the sides s = (1, 2, 3) of the element e_i. The sign of Q_k^i 's depends on the direction of the outward normal with reference to the element.

Choosing different N_k^i 's as weight functions and following the same procedure we obtain matrix equations for three unknowns (Q_p^i)

$$(Q_p^i)(Q_p^i)=(Q_k^i) \tag{4.27}$$

Where (Q_{pk}^i) 3 x 3 matrixes are, $(Q_p^i),(Q^i)$ are column matrices.

Repeating the above process with each of s elements, we obtain sets of such matrix equations. Introducing the global coordinates and global values for (Q_p^i) and making use of inter element continuity and boundary conditions relevant to the problem the above stiffness matrices are assembled to obtain a global matrix equation. This global matrix is rxr square matrix if there are r distinct global nodes in the domain of flow considered.

Similarly substituting ψ^i θ^i ,ω^i, and ϕ^i in (4.11) and defining the error and following the Galerkin method we obtain using Green's Theorem, (4.25) reduces to

$$\int_{\Omega}\left[\frac{\partial N_k^i}{\partial x}\frac{\partial\Psi^i}{\partial x}+\frac{\partial N_k^i}{\partial y}\frac{\partial\Psi^i}{\partial y}+GD^{-1}\frac{\partial N_k^i}{\partial x}+\frac{RD^{-1}}{\lambda}.\left(\frac{\partial N_k^i}{\partial x}\frac{\partial N^i}{\partial x}+\frac{\partial N_k^i}{\partial y}\frac{\partial N^i}{\partial y}\right)\right]d\Omega$$

$$=\int_{\Gamma}\left(\frac{\partial\Psi^i}{\partial x}nx+\frac{\partial\Psi^i}{\partial y}ny\right)d\Gamma+RD^{-1}\int_{\Gamma}N_k^i\left(\frac{\partial\omega^i}{\partial x}ny+\frac{\partial\omega^i}{\partial y}ny\right)d\Gamma \tag{4.28}$$

$$+GD^{-1}\int_{\Gamma}N_k^i nx d\Gamma$$

In obtaining (4.26) the Green's Theorem is applied with reference to derivatives of Ψ without affecting θ terms.

Using (4.18-4.18d) in (4.26) we have

$$\sum_m \Psi_m^i \left\{ \int_\Omega \left(\frac{\partial N_k^i}{\partial x}\frac{\partial N_m^i}{\partial x} + \frac{\partial N_m^i}{\partial y}\frac{\partial N_k^i}{\partial y} \right) d\Omega + GD^{-1} (\sum_L \theta_L^i \int_\Omega N_k^i \frac{\partial N_L^i}{\partial x} d\Omega \right\}$$

$$= \int N_k^i \left[\left(\frac{\partial \Psi^i}{\partial x} + \frac{\partial N^i}{\partial x} \right) nx + \left(\frac{\partial \Psi^i}{\partial y} + \frac{\partial N^i}{\partial y} \right) ny \right] d\Gamma + \int N_k^i \theta^i d\Omega_i = \Gamma ki \qquad (4.29)$$

In the problem under consideration, for computational purpose, we choose uniform mesh of 10 triangular elements. The domain has vertices whose global coordinates are (0, 0), (1, 0) and (1, c) in the non-dimensional form. Let e_1, e_2…..e_{10} be the ten elements and let θ_1, θ_2,….θ_{10} be the global values of θ, C_1, C_2,….C_{10} be the global values of C,ω_1, ω_2, ω_3,……., ω_{10}, be the global values of ω and ψ_1, ψ_2 ….ψ_{10} are the global values of ψ at the global nodes of the domain.

RESULTS AND DISCUSSION:

In this analysis we investigate the effect of dissipation and heat sources on convective heat and mass transfer flow of a micro polar fluid through a porous medium in a rectangular duct. The non-linear coupled equations governing the flow, heat and mass transfer have been solved by using Galerikin finite element analysis with three nodded triangular elements and Prandtl number Pr is taken as 0.71.

The non-dimensional temperature 'θ' is exhibited in figs for different values of G, D^{-1}, Sc, N, α, Ec, R & λ at different levels.

Figs. (1-4) represent θ with G, D^{-1}. It is found that an increase in G enhances the temperature at $y = \frac{2h}{3}$ level, and $x = \frac{2}{3}$ levels and reduces at $y = \frac{h}{3}$ and x=1/3 levels. With respect to Darcy parameter D^{-1}, we find the lesser the permeability of the porous medium larger the temperature at $y = \frac{2h}{3}$, and $x = \frac{2}{3}$ levels and smaller at $y = \frac{h}{3}$ and x=1/3 levels.

Figs. (5-8) represent 'θ' with α at different levels. The variation of θ with heat source parameter 'α' shows that an increase in the strength of the heat generating source enhances the temperature at y=2h/3, y=h/3 and x=1/3 levels and reduces it at higher vertical level x=2/3 while an increase in the heat absorbing source enhances the temperature at all levels y=2h/3,x=1/3 and x=2/3 and reduces at y=h/3 level .

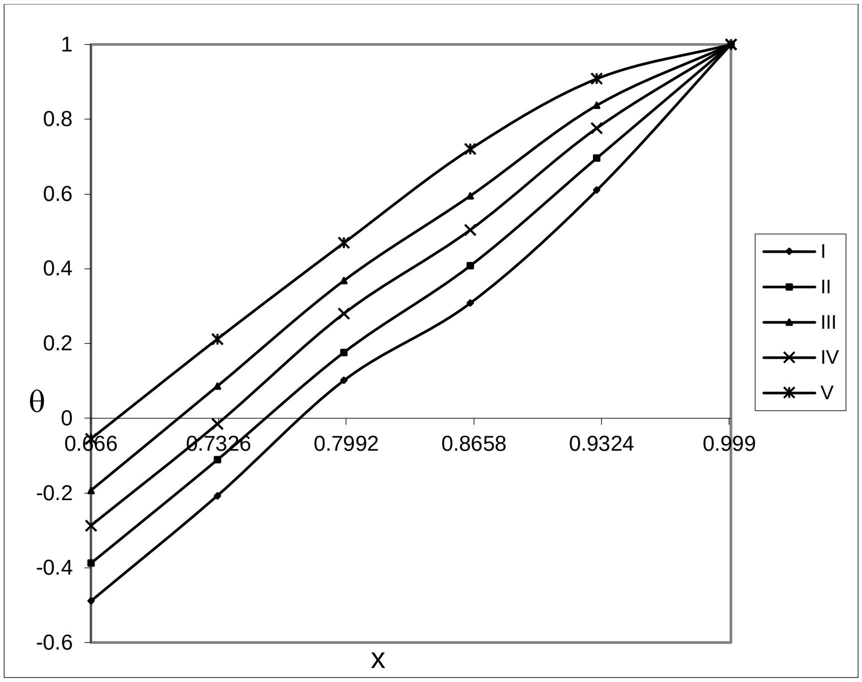

Fig. 1: Variation of θ with G, D^{-1} at y=2h/3 level

	I	II	III	IV	V
G	**10^2**	**$3x10^2$**	**$5x10^2$**	**10^2**	**10^2**
D^{-1}	**2**	**2**	**2**	**5**	**10**

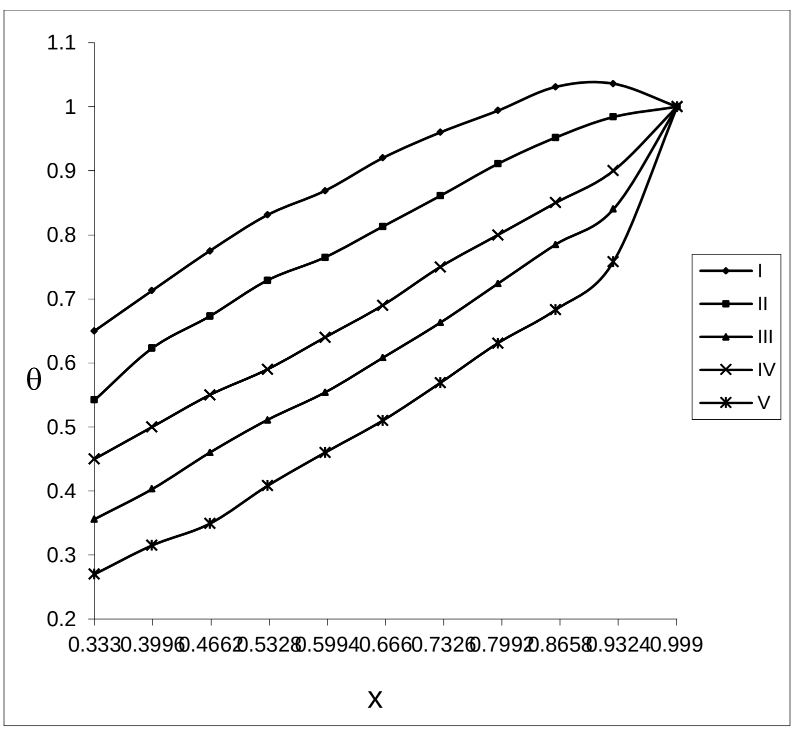

Fig. 2: Variation of θ with G, D^{-1} at y=h/3 level

I II III IV V

G 10^2 $3x10^2$ $5x10^2$ 10^2 10^2

D^{-1} 2 2 2 5 10

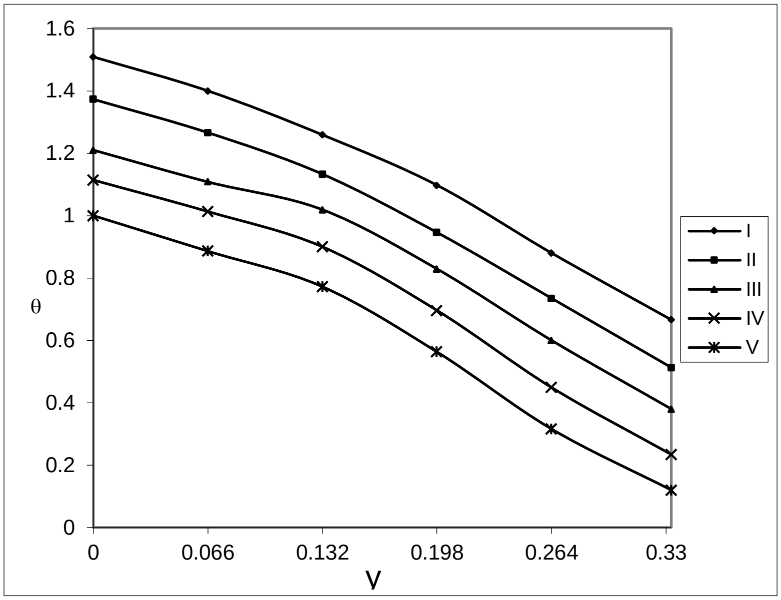

Fig. 3: Variation of θ with G, D^{-1} at x=1/3 level

I II III IV V

G 10^2 $3x10^2$ $5x10^2$ 10^2 10^2

D^{-1} 2 2 2 5 10

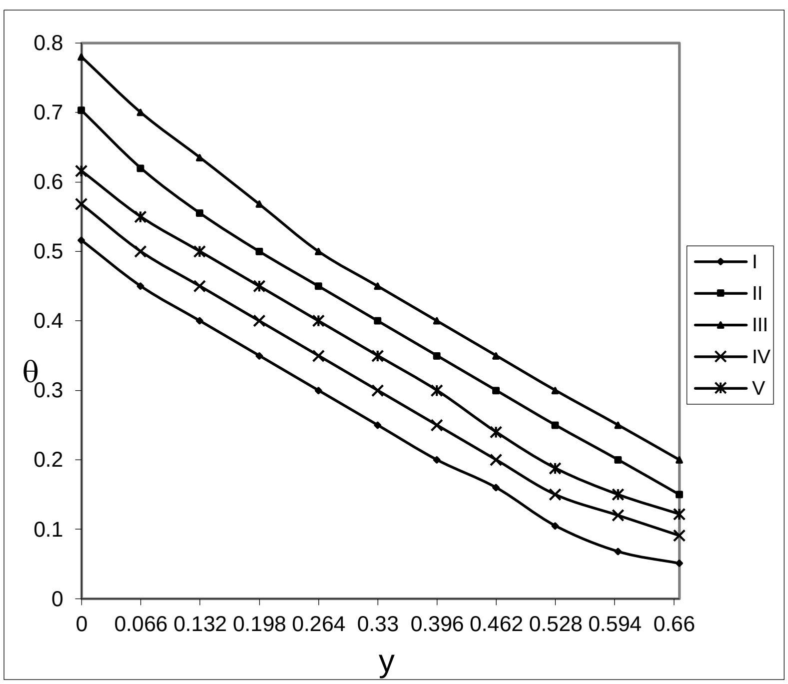

Fig. 4: Variation of θ with G& D^{-1} at x=2/3 level

I II III IV V

G	**10^2**	**$3x10^2$**	**$5x10^2$**	**10^2**	**10^2**
D^{-1}	**2**	**2**	**2**	**5**	**10**

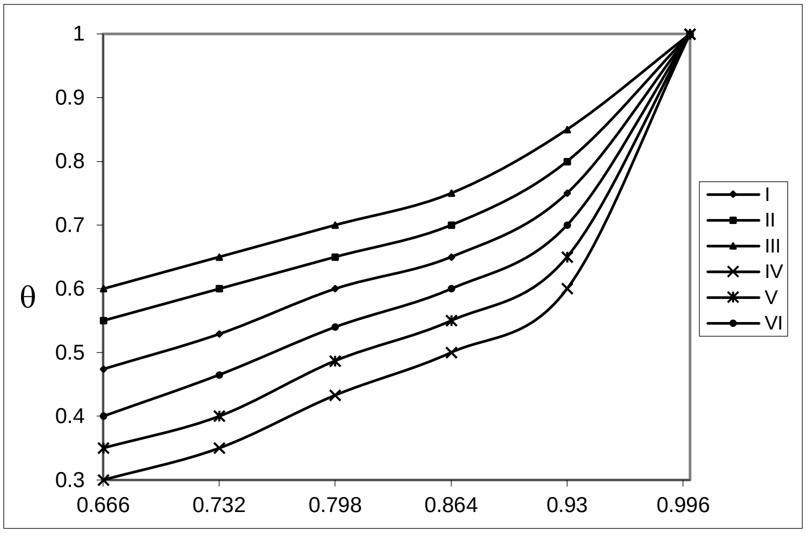

Fig. 5: Variation of θ with α at y=h/3 level

	I	II	III	IV	V	VI
α	**2**	**4**	**6**	**-2**	**-4**	**-6**

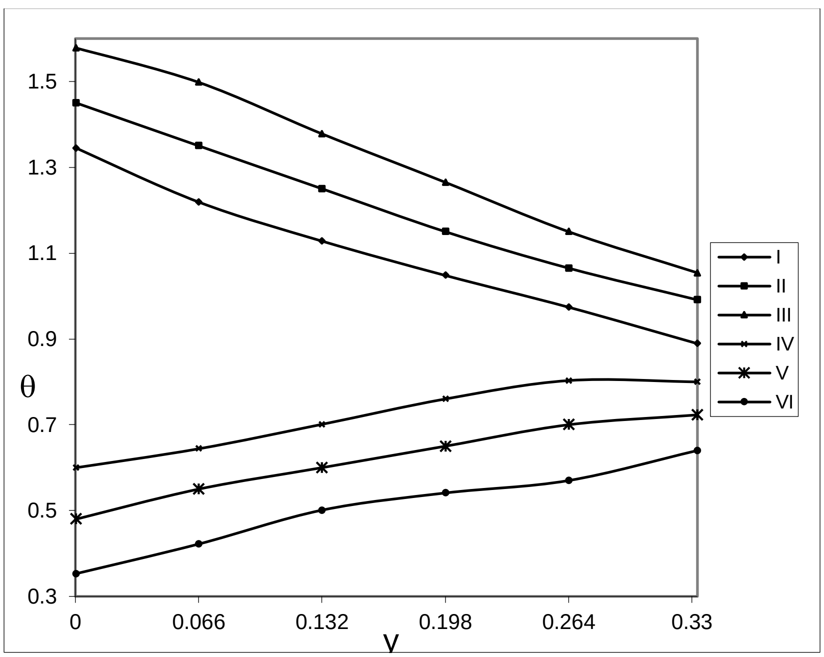

Fig. 6: Variation of θ with α at y=2h/3 level

	I	II	III	IV	V	VI
α	**2**	**4**	**6**	**-2**	**-4**	**-6**

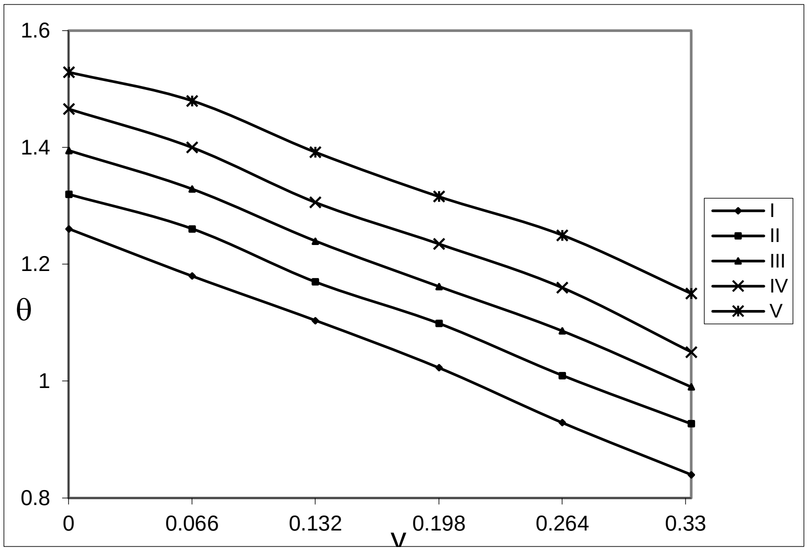

Fig. 7: Variation of θ with α at x=2/3 level

	I	II	III	IV	V
α	2	4	6	-2	-4

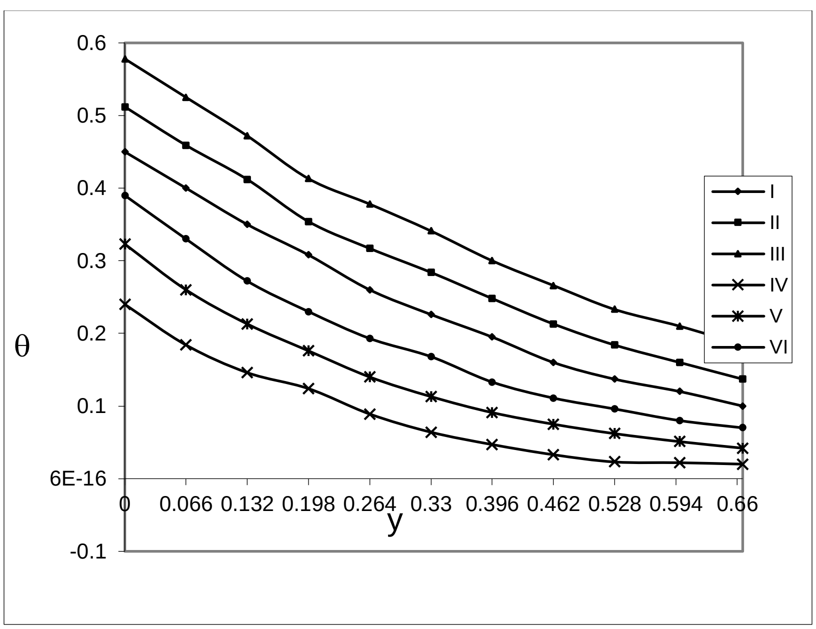

Fig.8: Variation of θ with α at x=1/3 level

	I	II	III	IV	V
α	2	4	6	-2	-4

The rate of mass transfer (Sherwood number (Sh)) on the side x = 1, is shown in Tables 4-6 for different parameter values. It is found that the rate of mass transfer on the lower and upper quadrants reduces with increase in

G and D^{-1}, and an increase $G \leq 2$ and $D^{-1} \leq 10$ leads to depreciation in |Sh| on the middle quadrant while for higher $G \geq 3$ or $D^{-1} \geq 15$, we notice an enhancement in |Sh|. The variation of Sh with micro polar parameter R shows that |Sh| enhances on the lower and middle and reduces on the upper quadrant for $R \leq 0.2$ and for higher $R \geq 0.3$, |Sh| reduces on the lower and upper quadrants and enhances on the middle quadrant. With reference to spin gradient parameter λ we find that |Sh| enhances on the lower and upper quadrants and reduces on the middle quadrant (Table 1).

The variation of Sh with buoyancy ratio 'N' shows that when molecular buoyancy force dominates over the thermal buoyancy force, the rate of mass transfer reduces on the lower and middle quadrants and enhances on the upper quadrant irrespective of the directions of buoyancy forces. With respect to Sc, we find that the rate of mass transfer enhances on the lower and middle quadrants and reduces on the middle with $Sc \leq 0.6$ and reduces for higher Sc = 1.3. An increase in $Sc \geq 2.01$ reduces |Sh| on all quadrants (Table 2). The variation of Sh with heat source parameter α shows that |Sh| on the lower and upper quadrant enhances with $\alpha > 0$, and an increases in $|\alpha|$ enhances |Sh| on the lower and middle quadrants and reduces on the upper quadrant. With respect to Ec, we find that higher the dissipative heat larger |Sh| on the lower and upper quadrants and reduces on the middle quadrant (Table 3).

Table – 1

Sherwood number (Sh) at x =1

	I	II	III	IV	V	VI	VII	VIII	IX	X
Sh_1	2.92756	2.957184	2.452456	2.844332	2.571332	2.838972	2.627792	2.87516	2.923492	2.926676
Sh_2	-0.73256	-0.86508	-0.95868	0.004192	0.521504	0.015348	0.207428	-0.78691	-0.72024	-0.71494
Sh_3	2.419748	2.349976	-0.83748	2.3850904	2.3096868	2.3794512	2.3165112	2.423884	2.425212	2.4311
R	0.1	0.2	0.3	0.1	0.1	0.1	0.1	0.1	0.1	0.1
G	10^2	10^2	10^2	$2x10^2$	$3x10^2$	10^2	10^2	10^2	10^2	10^2
D^{-1}	5	5	5	5	5	10	15	5	5	5
λ	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.5	1.5	5

Table – 2

Sherwood number (Sh) at x =1

	I	II	III	IV	V	VI	VII
Sh_1	2.92756	2.87138	2.1291572	1.285128	3.169424	3.24714	2.658196
Sh_2	-0.73256	-0.29384	-3.5408	-0.284008	-3.51908	-1.977964	0.298544
Sh_3	2.419748	2.438928	-0.127112	2.71452	0.452388	2.60730944	2.2528428
N	1	2	-0.5	-0.8	1	1	1
Sc	1.3	1.3	1.3	1.3	0.24	0.6	2.01

Table – 3

Sherwood number (Sh) at x =1

	I	II	III	IV	V	VI	VII	VIII
Sh_1	2.92756	3.107556	3.208556	3.102016	3.157228	3.202524	2.93652	2.93752
Sh_2	-0.73256	-0.43879	-0.248376	-0.81701	-0.88788	-0.93442	-0.72354	-0.71364
Sh_3	2.419748	2.50834	2.6761488	2.561924	2.561108	2.555248	2.42092	2.43192
α	2	4	6	-2	-4	-6	2	2
Ec	0.03	0.03	0.03	0.03	0.03	0.03	0.05	0.07

CONCLUSIONS

- Lesser the molecular diffusivity larger the temperature at all levels except at $x=\frac{2}{3}$ level. Lesser the molecular diffusivity smaller the concentration at both the horizontal levels and larger at both the vertical levels and micro rotation at all levels.
- The molecular buoyancy force dominates over the thermal buoyancy force, the temperature enhances at all levels except at $x=\frac{1}{3}$ level, the concentration reduces at both the horizontal levels and vertical level x=2/3, enhances at the vertical level x=1/3 and the micro-rotation enhances at all levels except at $y=\frac{h}{3}$ level.
- An increase in the strength of the heat generating source enhances the temperature at all levels and reduces it at higher vertical level x=2/3, the concentration enhances with increasing strength of the heat generating source at all levels and the micro rotation reduces at $y=\frac{h}{3}$, $x=\frac{1}{3}$ and $x=\frac{2}{3}$ levels except at $y=\frac{2h}{3}$ level.
- Higher the dissipative heat larger the temperature at $y=\frac{h}{3}$ and $x=\frac{1}{3}$ levels and smaller at $y=\frac{2h}{3}$ and $x=\frac{2}{3}$ levels, Higher the dissipative heat smaller the concentration at both the horizontal levels and larger at the vertical levels and higher the dissipative heat larger the micro- rotation at all levels.
- An increase R enhances the Nusselt number on all the three quadrants, |Sh| reduces on the lower and upper quadrants and enhances on the middle quadrant and an increase in R enhances C_w on the middle quadrant and enhances on the upper quadrant.
- |Nu| reduces with λ on all three quadrants, |Sh| enhances on the lower and upper quadrants and reduces on the middle quadrant and C_w reduces on the middle quadrant and enhances on the upper quadrant.
- Higher the dissipation heat larger |Nu| on all the three quadrants, higher the dissipative heat larger |Sh| on the lower and upper quadrants and reduces on the middle quadrant and also higher the dissipation heat larger the C_w on the middle and upper quadrants.
- An increase in Sc reduces |Sh| on all quadrants and C_w reduces on the middle quadrant and enhances on the upper quadrant with increase in Sc.

REFERENCES

[1] Born P.J., Chow, L.C. and Tien C.L.: Int. J. Heat Transfer. 20, 919 (1977).
[2] Bejan A. and Tiew C.L.: Trans, ASME, J. Heat Transfer 100, 191 (1978).
[3] Catton I.: Natural convection in enclosure, Proc. 6th Int. Heat Transfer conf. Toranto, Canada. Vol. 6. Pp. 13-31 (7-11 August 1978).
[4] Catton I.: Int. J. Heat Mass Transfer 15, 665 (1972).
[5] Cliffe K.A. and Winter K.H.: Numerical methods for predicating preferred and anomalous flows in Benard convection. Proc. 6th Int. conf-numerical methods in thermal problems (1989).
[6] Cha O-Kaung Chen and Tsan – Huihso: Natural convection of micropolar fluids in a Rectangular Enclosure: Int. J. Eng. Sci. Vol. 34. 4. pp. 407-415 (1966).
[7] Davis S.H.: J. Fluid Mech. 30. 465 (1967).
[8] Eringen A.C.: Int. J. Eng. Sci., 2, 205 (1964).
[9] Eringen A.C.: J. Math Mech. 16, 1 (1977).
[10] Eringen A.C.: J. Math Anal. Appl. 38, 480 (1972).
[11] Haajizadeh M. and Tien C.L.: Trans ASME, J. Heat Transfer 105, 803 (1978).
[12] Jena S.K. and Bhattacharya S.P.: Int. J. Eng. Sci., 24, 69 (1986).
[13] Ostrach S.: Adv. Heat Transfer, 8, 161 (1972).
[14] Ozoe H., Yamamtoto K., Sayama H. and Churchill S.W.: Int. J. Heat Mass Transfer 17, 1209 (1972).
[15] Ozoe H. and Sayama H.: Int. J. Heat Mass Transfer, 18, 1425 (1975).
[16] Samuels M.R. and Churchill S.W.: Alche. J. 10. 110 (1967).
[17] Walker K.L. and Homsy G.M.: J. Fluid Mech. 87. 449 (1978).
[18] Wilson G.L. and Rydin R.A.: Int. J. Numer. Mech. Fluids 10, 35 (1990).
[19] Wang M., Tsuji T. and Nagana: Mixed convection with flow reversal in the thermal entrance region of horizontal and vertical pipes. Int. J. Heat Mass Transfer, 37 (1994) 2305-2319.
[20] Wang M. and Tsan – Huishu: Int. J. Heat and Mass Transfer, 43 (2000) 1563-1572.