Analysis And Observation of Conventional Method of Partial Fraction

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Abstract:- *Various partial fraction decomposition technique is very useful in many areas including mathematics and engineering. In this Review paper, We study a various approach to compute the partial fraction decompositions of rational functions, Non Repeated Linear Factor, Repeated Linear Factor, Irreducible Linear Factor, Synthetic Division Factor, and study the outcome of its trials by different Literature Review .We observed an elementary and efficient method to find the partial fraction decomposition of a rational function when the denominator is a product of two ,three linear factors. Conventional method is based on a recursive computation of the $\hbar$-adic polynomial in commutative algebra which is a generalization of the Taylor polynomial. In this paper we only study, observed, Analysis and Compare the method with non conventional method of partial fraction.*

Keywords:- *The partial fraction decompositions of rational functions , Non Repeated Linear Factor, Repeated Linear Factor, Irreducible Linear Factor, Synthetic Division Factor.*

Introduction

Various method of partial fraction decomposition is a classic topic with applications in calculus, differential equations, control theory, and other fields of mathematics and engineering. Theoretically, it is well-known that every rational function has unique partial fraction decomposition, Non Repeated Linear Factor, Repeated Linear Factor, Irreducible Linear Factor, Synthetic Division Factor as it is an easy exercise in abstract algebra. However, actually decomposing a rational function into partial fractions is computationally intensive. From the aspect of computation, there has been previous study of method, developments in this topic for general rational functions. In this article, we Analysis Conventional method for the different cases, for example when the denominator is given as a product of two highly powered linear factors some method are applicable.

Only for related to h-Adic Polynomials and Partial Fraction Decomposition Let F be R or C and R = F[x] be a polynomial ring with the coefficients in F. We also assume the rational function to be proper (i.e., the degree of denominator is greater than the degree of numerator) with the denominator factored completely over F.

For the problem of non conventional decomposing a rational function into partial fractions is often encountered in the study of calculus, differential equations, discrete mathematics and control theory, etc. here are two common methods for computing the unknown partial fraction coefficients. One method is to use the method of undetermined coefficients, where the unknowns can be found by solving a system of linear equations. However, the drawback is that the calculations involved could be quite tedious. different process were to apply the Heaviside's cover-up technique, which uses simple substitutions to determine the unknown coefficients of the partial fractions with single poles, and successive differentiations to handle those with multiple poles. Now a before year an improved Heaviside approach for finding the partial fraction decompositions (PFD) of proper rational functions [6] .which involves substitutions and polynomial divisions only to determine the unknown partial fraction coefficients. Compared with the other techniques described in[8-9-10-11].

Literature Review

1. Yiu-Kwong Man present[1] a new approach to partial fraction decomposition of rational functions with irreducible quadratic factors in the denominators. It improves the Heaviside's cover-up technique to handle this type of problem via polynomial divisions and substitutions only, with no need to solve for the complex roots of the irreducible quadratic polynomial involved, to use differentiation or to solve a system of linear equations. Some examples of its applications in engineering mathematics are included. Yiu-Kwong Man introduced [1]a simple approach for computing the partial fraction decompositions of rational functions with irreducible quadratic factors in the denominators. By using this method, there is no need to solve for the complex roots of the quadratic polynomials involved, to use differentiation or to solve a system of linear equations. Due to its simplicity and useful applications in applied and engineering mathematics, this method can introduced to higher school or undergraduate students, as an alternative to those classic techniques mentioned in most mathematics textbooks.

2. Allen Leung Yiu-Kwong Man , "Teaching a New Method of Partial Fraction Decomposition to Senior Secondary Students: Results and Analysis from a Pilot Study"[2] introduce a new approach to compute the partial fraction decompositions of rational functions and describe the results of its trials at three secondary schools in Hong Kong. The data were collected via quizzes, questionnaire and interviews. In general, according to the responses from the teachers and students concerned, this new approach has potential to be introduced at the senior secondary level, as an alternative to the method of undetermined coefficients described in common secondary mathematics textbooks. Some remarks on the related pedagogical issues are included.
Allen Leung Yiu-Kwong Man observe that the Improved Heaviside approach (Method II) has the potential to be introduced to the students who are studying mathematics at the senior secondary level. In general, the students' performances in using Method II could be comparable to (or even statistically significantly better in School B) that of Method I in the calculation domain, the coefficients domain and the mastery domain in each school. In fact, over 85% of students responded in the questionnaire that Method II could be introduced at the secondary level and over 70% said that it could be introduced at F.6/F.7 or the lower forms. Also, all the three participating teachers and considerable number of students perceived that Method II was more interesting to them . However, there are also drawbacks from the pedagogical perspectives. According to the responses from the questionnaires and interviews, many students responded that Method II was more difficult to understand than Method I. From what we observed in the trial lessons, the teachers concerned had spent too little time (though Teacher B was a bit better) on explaining why Method II works, but rather put more effort on illustrating how it works by examples. Thus, many of the students perceived that Method I was easier to understand than Method II. Most students preferred to learn Method I before Method II because the former one was more simple and straightforward to them. It indicated that the understanding of the concepts behind Method II could significantly affect the perception of its interestingness, as well as the performance in applying it to solve problems,like The improved Heaviside approach.

For example:-
Input: a rational function F(x)= a(x) / b(x) , where a(x) and b(x) are polynomials such that dega(x) < degb(x) , b(x)= $(x-\alpha_1)^{n_1}$ $(x-\alpha_2)^{n_2}$ …..$(x-\alpha_s)^{n_s}$, $\alpha_1, \alpha_2, \ldots, \alpha_s$ are constants and $n_1, n_2, \ldots n_s$ are positive integers.

Output: The PFD of F(x), namely $f(x) = \sum_{i=1}^{s}\left(\frac{a_{i,1}}{(x-\alpha_i)} + \frac{a_{i,2}}{(x-\alpha_i)^2} + \ldots\ldots + \frac{a_{i,n_i}}{(x-\alpha_i)^{n_i}}\right)$ where $a_{i,j}$ are constants.
Procedure: The unknown coefficients i j a , can be found by the following steps.

S1: $a_{i,n_i} = \left.\frac{a(x)}{b(x)}.(x-\alpha_i)^{n_i}\right|_{x=\alpha_i}$ (Note: This is the so-called *cover-up* technique);

S2: $a_{i,n_i-j} = \left.\left[\frac{a(x)}{b(x)} - \sum_{k=0}^{j-1}\frac{a_{i,n_i-k}}{(x-\alpha_i)^{n_i-k}}\right](x-\alpha_i)^{n_i-j}\right|_{x=\alpha_i}$;

where $1 \le i \le s$ and $1 \le j \le n_i - 1$.

Thus, the cover-up technique is applied to compute a_{i,n_i} first. Then, the known partial fractions are subtracted from $F(x)$ and simplified to become a new function[1]. Next, we can apply the same technique to handle the new function(s) obtained again and again, until all the unknown coefficients a_{i,n_i-j} are found.

3. Youngsoo Kim, Byunghoon Lee "Partial Fraction Decomposition by Repeated Synthetic Division" [3]

present an efficient and elementary method to find the partial fraction decomposition of a rational function when the denominator is a product of two highly powered linear factors.[3] The synthetic division requires n multiplications and n additions where n is the degree of the polynomial the evaluation of functions through synthetic division, the cost is the same. This method is not the best algorithm in terms of asymptotic speed as the algorithm in [4] is performed in O(nlog2n) steps. However, this method is still interesting because it uses only one technique (synthetic division) in the whole process and hand calculation is straightforward.

4. Kwang Hyun Kim and Xin Zhang "h-Adic Polynomials and Partial Fraction Decomposition of Proper Rational Functions over R or C" [5] present a new and simple method on the partial fraction decomposition of proper rational functions which have completely factored denominators over R or C. The method is based on a recursive computation of the h-adic polynomial in commutative algebra which is a generalization of the Taylor polynomial. Since its computation requires only simple algebraic operations, it does not require a computer algebra system to be programmed.
Kwang Hyun Kim and Xin Zhang "h-Adic Polynomials and Partial Fraction Decomposition of Proper Rational Functions over R or C" [5] paper shows that the partial fraction of a proper rational function with denominator factored completely over R or C can be found by computing suitable h-adic polynomials with $\deg(h) \leqslant$ n. We also present recursive formulas (Propositions 6, 9, 11, and 12) to compute h-adic polynomials with $\deg(h) \leqslant$ n. Since this algorithm only requires simple arithmetic operations, it can be implemented easily without a complex computer algebra system.

Conclusion

In our study of Partial Fraction, we will have occasion to need to know some other study and other authors opinion . Yiu-Kwong Man says that ,there is no need to solve for the complex roots of the quadratic polynomials involved, to use differentiation or to solve a system of linear equations. Allen Leung Yiu-Kwong Man ,, Method I was easier to understand than Method II. It indicated that the understanding of the concepts behind Method II could significantly affect the perception of its interestingness, as well as the performance in applying it to solve problems. Youngsoo Kim, Byunghoon Lee,use this method is still interesting because it uses only one technique (synthetic division) in the whole process and hand calculation is straightforward. Kwang Hyun Kim and Xin Zhang paper shows that the partial fraction of a proper rational function with denominator factored completely over R or C can be found by computing suitable h-adic polynomials with $\deg(h) \leqslant$ n.

We have seen how to solve partial fraction in several special cases. In first case we introduced a simple approach for computing the partial fraction decomposition of rational functions with Irreducible quadratic factors in the denominators. Due to its simplicity and useful application in applied mathematics and engineering mathematics. This method can introduce to higher school or undergraduate student.

We will discover that new method of solve the partial fraction non repeated linear factor.(two or three linear factor).

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