Harmonic Mean Labeling Of Zig-Zag Triangle graphs

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Abstract

A graph G with p vertices and q edges is called a harmonic mean graph if it is possible to label the vertixnodes x∈V with distinct labels f(x) from {1,2,…..q+1} in such a way that each edge e = uv is labeled with f(uv) = $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ (or) $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$then the edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper we prove that some families of graphs such as zig-zag triangle of

Z $(T_n)\odot K_1$,Z$(T_n)\odot\overline{K_2}$, Z$(T_n)\odot K_2$,are harmonic mean graphs.

Keywords:
Harmonic mean graph, zig-zag triangle of Z $(T_n)\odot K_1$, Z$(T_n)\odot\overline{K_2}$, Z$(T_n)\odot K_2$.

AMS subject classification :- 05078

Introduction

Let G=(V,E) be a (p,q) graph with p = |V(G)| vertices and q =|E(G)| edges, where V(G) and E(G) respectively denote the vertex set and edge set of the graph G. In this paper, we refer the graphs which are simple, finite and undirected. Harary's graph theory used for theoretic terminology and notations [3].

The concept of graph labeling was introduced by Rosa in 1967. A detailed survey of graph labeling is available in Gallian[2]. S.Somasundaram, R.Ponraj and S.S.Sandhya were introduced the concept of harmonic mean labeling of graphs. They investigated the existence of harmonic mean labeling of several family of graphs studied by several authors. We have proved Harmonic mean labeling of subdivision graphs such as $P_n\odot K_1$, $P_n\odot\overline{K_2}$, H-graph, crown, $C_n\odot K_1$,$C_n\odot\overline{K_2}$, quadrilateral snake, Triangular snake and also proved Harmonic mean labeling of some graphs such as Triple triangular snake T(T_n), Alternate Triple triangular snake A[T(T_n)],Triple quadrilateral snake T(Q_n), Alternate Triple quadrilateral snake A[T(Q_n)], Twig graph T(n), balloon triangular snake $T_n(C_m)$, and key graph Ky(m,n),[6,7].The following definitions are useful for the present investigation.

Notations:

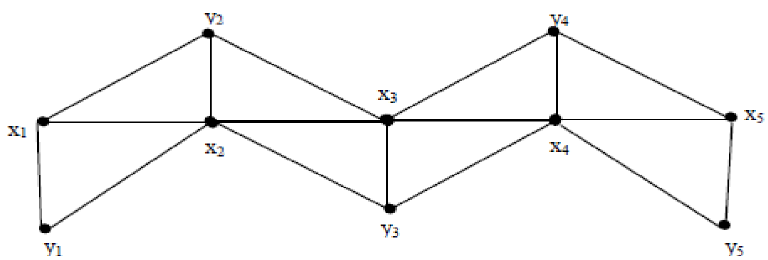
$\lfloor x \rfloor$ – Largest integer less than or equal to x.

Definition: 1.1 [8]

A Graph G = (V, E) with p points and q lines is called a Harmonic mean graph if it is possible to label the points $v \in V$ with distinct labels f(v) from {1,2,…,q+1} in such a way that when each line e = uv is labeled with f(uv) = $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ (or) $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the resulting edge labels are distinct. In this case f is called Harmonic mean labeling of G.

Definition:1.2 [4]

Let G be the graph obtained from the path P_n = $x_1, x_2, \ldots x_n$ adding a new vertex point $y_1, y_2, \ldots y_n$ and new edge line y_1x_2, y_nx_{n-1}; x_iy_i for $1 \le i \le n$, y_ix_{i-1}, y_ix_{i+1}, for $2 \le i \le n$-1.The resultant graph is called zig-zag triangle Z (T_n).



In this paper we prove that zig-zag triangle Z $(T_n) \odot K_1$, Z$(T_n) \odot \overline{K_2}$, Z$(T_n) \odot K_2$,are harmonic mean graphs.

II. Harmonic mean labeling of graphs

Theorem:2.1

The zig-zag triangle Z $(T_n) \odot K_1$ is a harmonic mean graph.

Proof:

Let $x_1, x_2, \ldots, x_n$ be the vertices of the path P_n and let G= Z (T_n) be the zig-zag triangle graph.

Let V (G) ={$x_i y_i$ /$1 \le i \le n$ } and

E (G) = {y_ix_{i-1}, y_ix_{i+1} / $2 \le i \le n$-1} ∪ {y_ix_i / $1 \le i \le n$} ∪ {y_1x_2,y_nx_{n-1} }.

Let u_i,v_i and s_i be the pendent vertices attached at zig-zag triangle Z (T_n).

Define a function f : V(G) $\rightarrow \{1,2,\ldots,q+1\}$ by

$f(x_1)$ = 4

$f(x_1)=$ 6i-3 for $3\leq i\leq n$ if i is odd

$f(x_i)$ = 6i-3 for $2\leq i\leq n$ if i is even .

$f(y_2)$ = 7

$f(y_i)$ = 6i -4 for $4\leq i\leq n$ if i is even.

$f(y_1)$ = 2

$f(y_i)$ = 6i -5 for $3\leq i\leq n$ if i is odd.

$f(u_1)$ = 5

$f(u_i)$ = 12i - 10 for $2\leq i\leq \left\lceil\frac{n}{2}\right\rceil$

$f(s_1)$ = 6

$f(s_i)$ = 12i - 7 for $2\leq i\leq \left\lfloor\frac{n}{2}\right\rfloor$

$f(v_1)$ = 1

$f(v_i)$ = 12i - 12 for $2\leq i\leq \left\lceil\frac{n}{2}\right\rceil$

$f(t_1)$ = 11

$f(t_i)$ = 12i - 2 for $2\leq i\leq \left\lceil\frac{n}{2}\right\rceil$

Then the resulting edge labels are distinct.

$f(x_1x_2)$ = 6

$f(x_{i+1}x_{i+2})$ = 6i +6 for $2\leq i\leq n$ -1 if i is even .

$f(x_{i+1}x_{i+2})$ = 6i +6 for $1\leq i\leq n$ -1 if i is odd .

$f(x_1u_1)$ = 4

$f\left(x_iu_{\frac{i+1}{2}}\right)$ = 6i -3 for $3\leq i\leq n$ if i is odd.

$f\left(s_{\frac{i}{2}}\,y_i\right)$ = 6i -5 for $2\leq i\leq n$-1 if i is even .

$f(x_iy_{i+1})$ = 6i -1 for $1\leq i\leq n$ -2 if i is odd .

$f(y_2x_3)$ = 9

$f(y_ix_{i+1})$ = 6i-1 for $4\leq i\leq n$ if i is even .

$f(y_ix_i)$ = 6i -4 for $2\leq i\leq n$ -1 if i is even.

$f(x_iy_i)$ = 6i -4 for $1\leq i\leq n$ if i is odd .

$f(x_2t_1)$ = 10

$f\left(x_i t_{\frac{i}{2}}\right)$ = 6i -3 for $4\leq i \leq n$ -1 if i is even .

$f(y_1 x_2)$ = 3

$f(y_i x_{i+1})$ = 6i -2 for $3\leq i \leq n$ -2 if i is odd .

$f(x_2 y_3)$ = 11

$f(x_i y_{i+1})$ = 6i -2 for $4\leq i \leq n$ -1 if i is even.

$f(v_1 y_1)$ = 1

$f\left(v_{\frac{i+2}{2}} y_{i+1}\right)$ = 6i +1 for $2\leq i \leq n$ if i is even.

Thus f provides a harmonic mean labeling of graph G.Hence G is a harmonic mean graph.

Example:2.1.1

A harmonic mean labeling of zig –zag triangle Z $(T_{11})\odot K_1$ is given in fig 2.1.1

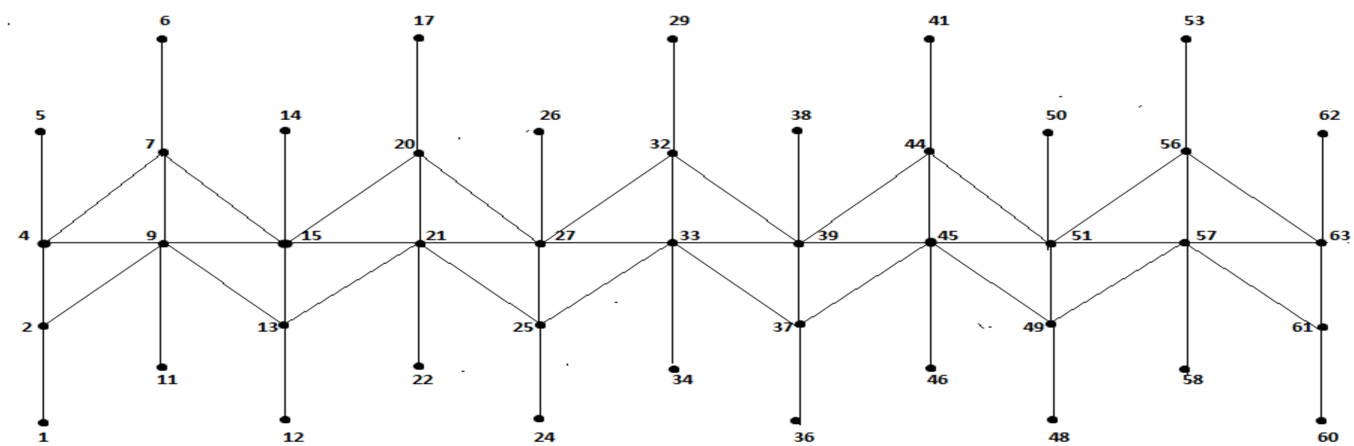


fig 2.1.1

Theorem:2.2

The zig-zag triangle Z(T_n)$\odot\overline{K_2}$is a harmonic mean graph.

Proof:

Let $x_1, x_2, \ldots, x_n$ be the vertices of the path P_n and let G= Z (T_n) be the zig-zag triangle graph.

Let V (G) ={$x_i y_i$ /1$\leq i \leq n$ } and

E (G) = {$y_i x_{i-1}$, $y_i x_{i+1}$ / 2$\leq i \leq n$-1} ∪ {$y_i x_i$ / 1$\leq i \leq n$} ∪ {$y_1 x_2$,$y_n x_{n-1}$ }.

Let u_i,v_i be the vertices of $\overline{K_2}$ which are joined to the vertex $x_i$of the path $p_n$if i is odd and let p_i,q_i be the vertices of $\overline{K_2}$ which are joined to the vertex $x_i$of the path $p_n$if i is even.

Let z_i,$t_i$be the vertices of $\overline{K_2}$ which are joined to the vertex $y_i$of the triangleif i is even and let r_i , $s_i$be the vertices of $\overline{K_2}$ which are joined to the vertex $y_i$of the triangle if i is odd.

Define a function f : V(G) $\rightarrow \{1,2,\ldots,q+1\}$ by

$f(x_1) = 5$

$f(x_i) = 8i-4$ for $3 \le i \le n$ if i is odd

$f(x_2) = 15$

$f(x_i) = 8i-5$ for $4 \le i \le n$ if i is even

$f(y_1) = 3$

$f(y_i) = 8i-3$ for $3 \le i \le n$ if i is odd

$f(y_2) = 13$

$f(y_i) = 8i-6$ for $4 \le i \le n$ if i is even

$f(u_1) = 4$

$f(u_2) = 19$

$f(u_i) = 16i-16$ for $3 \le i \le \left\lceil \frac{n}{2} \right\rceil$

$f(v_1) = 6$

$f(v_i) = 16i-15$ for $2 \le i \le \left\lceil \frac{n}{2} \right\rceil$

$f(z_1) = 9$

$f(z_i) = 16i-3$ for $2 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(t_1) = 10$

$f(t_i) = 16i-1$ for $2 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(p_1) = 7$

$f(p_i) = 16i-8$ for $2 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(q_1) = 8$

$f(q_i) = 16i-7$ for $2 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(r_1) = 2$

$f(r_2) = 11$

$f(r_i)$ = 16i-14 for $3 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$

$f(s_1)$ = 1

$f(s_2)$ = 12

$f(s_i)$ = 16i-13 for $3 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$

Then the resulting edge labels are distinct.

$f(x_1 x_2)$ = 7

$f(x_{i+1} x_{i+2})$ = 8i +7 for $2 \leq i \leq n$ -3 if i is even .

$f(x_2 x_3)$ = 17

$f(x_{i+1} x_{i+2})$ = 8i +7 for $3 \leq i \leq n$ -2 if i is odd .

$f(x_1 u_1)$ = 5

$f(x_3 u_2)$ = 20

$f\left(x_i u_{\frac{i+1}{2}}\right)$ = 8i -7 for $5 \leq i \leq n$ if i is odd .

$f(x_1 v_1)$ = 6

$f(x_3 v_2)$ = 19

$f\left(x_i v_{\frac{i+1}{2}}\right)$ = 8i -6 for $5 \leq i \leq n$ if i is odd .

$f(x_1 y_2)$ = 8

$f(x_i y_{i+1})$ = 8i -2 for $3 \leq i \leq n$ -2 if i is odd .

$f(y_2 x_2)$ = 13

$f(y_i x_i)$ = 8i -5 for $4 \leq i \leq n$ -1 if i is even.

$f(y_2 x_3)$ = 16

$f(y_i x_{i+1})$ = 8i -2 for $4 \leq i \leq n$ -1 if i is even.

$f(z_1 y_2)$ = 11

$f\left(z_{\frac{i}{2}} y_i\right)$ = 8i -4 for $4 \leq i \leq n$ -1 if i is even.

$f(t_1 y_2)$ = 12

$f\left(t_{\frac{i}{2}} y_i\right)$ = 8i -3 for $4 \leq i \leq n$ -1 if i is even.

$f(x_1 y_1)$ = 3

$f(x_i y_i)$ = 8i -3 for $3 \leq i \leq n$ if i is odd .

$f(y_1x_2)$ = 4

$f(y_ix_{i+1})$ = 8i for $3 \le i \le n-2$ if i is odd .

$f(x_2y_3)$ = 18

$f(x_iy_{i+1})$ = 8i for $4 \le i \le n-1$ if i is even.

$f\left(x_i p_{\frac{i}{2}}\right)$ = 8i -7 for $2 \le i \le n-1$ if i is even.

$f\left(x_i q_{\frac{i}{2}}\right)$ = 8i -6 for $2 \le i \le n-1$ if i is even.

$f(y_1r_1)$ = 2

$f(y_3r_2)$ = 14

$f\left(y_i r_{\frac{i+1}{2}}\right)$ = 8i-5 for $5 \le i \le n$ if i is odd .

$f(y_1s_1)$ = 1

$f(y_3s_2)$ = 15

$f\left(y_i s_{\frac{i+1}{2}}\right)$ = 8i-4 for $5 \le i \le n$ if i is odd

Thus f provides a harmonic mean labeling of graph G.Hence G is a harmonic mean graph.

Example:2.2.1

A harmonic mean labeling of zig –zag triangle Z(T_{11})$\odot\overline{K_2}$ is given in fig 2.2.1

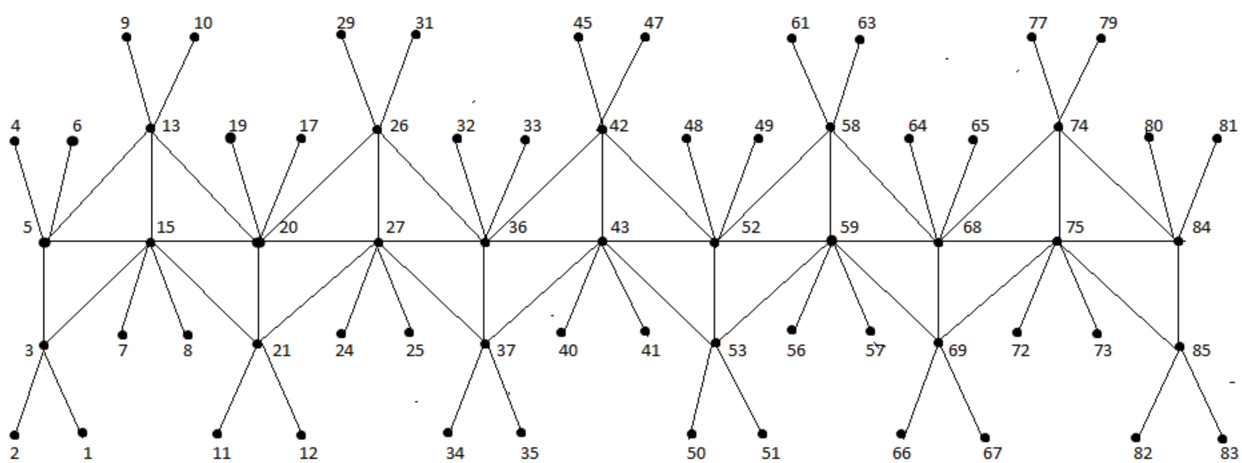


fig 2.2.1

Theorem:2.3

The zig-zag triangle Z(T_n)⊙ $K_2$is a harmonic mean graph.

Proof:

Let $x_1, x_2, \dots, x_n$ be the vertices of the path P_n and let G= Z (T_n) be the zig-zag triangle graph.

Let V (G) = { $x_i y_i$ /1≤ $i \le n$ } and

E (G) = $\{y_i x_{i-1}, y_i x_{i+1}$ / 2≤ $i \le n$-1} ∪ $\{y_i x_i$ / 1≤ $i \le n\}$ ∪ $\{y_1 x_2, y_n x_{n-1}\}$.

Let u_i,v_i be the vertices of K_2which are joined to the vertex $x_i$of the path $p_n$if i is odd and let p_i,q_i be the vertices of K_2which are joined to the vertex $x_i$of the path $p_n$if i is even.

Let z_i,$t_i$be the vertices of K_2which are joined to the vertex $y_i$of the triangleif i is even and let r_i , $s_i$be the vertices of K_2which are joined to the vertex $y_i$of the triangle if i is odd.

Define a function f : V(G) $\rightarrow \{1,2,\dots,q+1\}$ by

$f(x_1)$	=	6		
$f(x_2)$	=	15		
$f(x_i)$	=	10 i -3	for 3≤ $i \le n$	
$f(v_1)$	=	5		
$f(v_i)$	=	20 i-16	for 2≤ $i \le \left\lceil\frac{n}{2}\right\rceil$	
$f(w_1)$	=	10		
$f(w_i)$	=	20 i-14	for 2≤ $i \le \left\lceil\frac{n}{2}\right\rceil$	
$f(y_1)$	=	4		
$f(y_i)$	=	10i-7	for 3≤ $i \le n$	if i is odd
$f(y_2)$	=	20		
$f(y_i)$	=	10 i-9	for 4≤ $i \le n$ -1	if i is even
$f(u_1)$	=	13		
$f(u_i)$	=	20 i-10	for 2≤ $i \le \left\lfloor\frac{n}{2}\right\rfloor$	
$f(z_i)$	=	20i- 6	for 1≤ $i \le \left\lfloor\frac{n}{2}\right\rfloor$	
$f(p_1)$	=	11		
$f(p_i)$	=	20 i-5	for 2≤ $i \le \left\lfloor\frac{n}{2}\right\rfloor$	
$f(q_1)$	=	12		

$f(q_i) = 20i-4$ for $2 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$

$f(r_1) = 2$

$f(r_i) = 20i-21$ for $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$

$f(s_1) = 1$

$f(s_i) = 20i-18$ for $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$

Then the resulting edge labels are distinct.

$f(x_1x_2) = 9$

$f(x_{i+1}x_{i+2}) = 10i+11$ for $2 \leq i \leq n-3$ if i is even .

$f(x_2x_3) = 19$

$f(x_{i+1}x_{i+2}) = 10i+12$ for $3 \leq i \leq n-2$ if i is odd .

$f(x_1v_1) = 5$

$f\left(x_iv_{\frac{i+1}{2}}\right) = 10i-4$ for $3 \leq i \leq n$ if i is odd .

$f(x_1w_1) = 8$

$f\left(x_iw_{\frac{i+1}{2}}\right) = 10i-3$ for $3 \leq i \leq n$ if i is odd .

$f(v_1w_1) = 7$

$f(v_iw_i) = 20i-15$ for $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$

$f(x_1y_2) = 10$

$f(x_iy_{i+1}) = 10i-2$ for $3 \leq i \leq n-2$ if i is odd.

$f(y_2x_2) = 17$

$f(y_ix_i) = 10i-6$ for $4 \leq i \leq n-1$ if i is even.

$f(y_2x_3) = 23$

$f(y_ix_{i+1}) = 10i-2$ for $4 \leq i \leq n-1$ if i is even.

$f(u_1y_2) = 15$

$f\left(u_{\frac{i}{2}}y_i\right) = 10i-10$ for $4 \leq i \leq n$ if i is even.

$f(z_1y_2) = 16$

$f\left(z_{\frac{i}{2}}y_i\right) = 10i-7$ for $4 \leq i \leq n$ if i is even.

$f(u_1z_1) = 14$

$f(u_i z_i)$ = 20i-8 for $2 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$

$f(x_1 y_1)$ = 4

$f(x_i y_i)$ = 10i -6 for $3 \leq i \leq n$ if i is odd.

$f(y_1 x_2)$ = 6

$f(y_i x_{i+1})$ = 10i -1 for $3 \leq i \leq n$ -2 if i is odd.

$f(x_2 y_3)$ = 18

$f(x_i y_{i+1})$ = 10i-1 for $4 \leq i \leq n$ -1 if i is even.

$f(x_2 p_1)$ = 12

$f\left(x_2 p_{\frac{i}{2}}\right)$ = 10i -4 for $4 \leq i \leq n$ -1 if i is even .

$f(x_2 q_1)$ = 13

$f\left(x_i q_{\frac{i}{2}}\right)$ = 10i -3 for $4 \leq i \leq n$ -1 if i is even

$f(p_1 q_1)$ = 11

$f(p_i q_i)$ = 20i -5 for $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$

$f(y_1 r_1)$ = 3

$f\left(y_i r_{\frac{i+1}{2}}\right)$ = 10i-9 for $3 \leq i \leq n$ if i is odd .

$f(y_1 s_1)$ = 1

$f(y_3 s_2)$ = 22

$f\left(y_i s_{\frac{i+1}{2}}\right)$ = 10i -7 for $5 \leq i \leq n$ if i is odd .

$f(r_1 s_1)$ = 2

$f(r_i s_i)$ = 20i -20 for $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$

Thus f provides a harmonic mean labeling of graph G.Hence G is a harmonic mean graph.

Example:2.3.1

A harmonic mean labeling of zig –zag triangle Z(T_{11})$\odot$ K_2 is given in fig 2.4.1

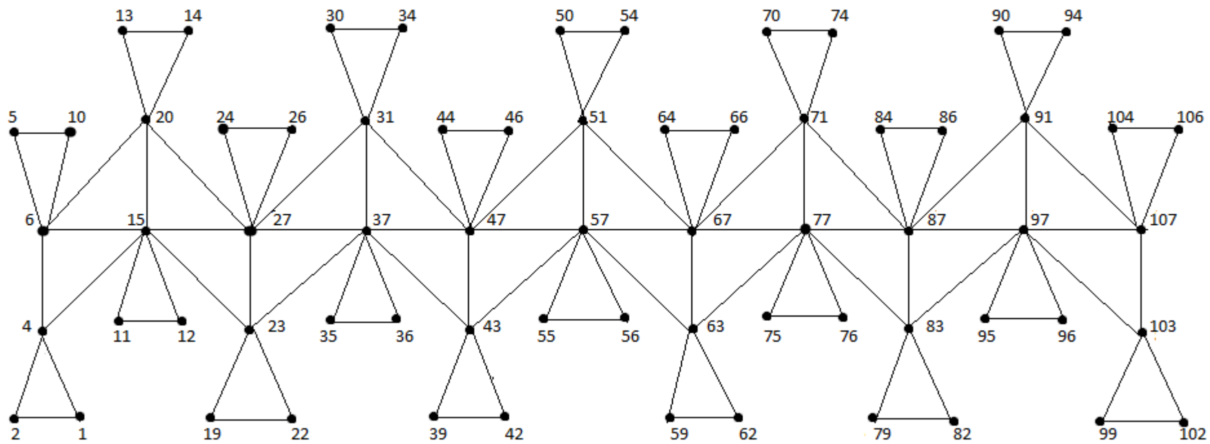


fig 2.3.1

Conclusion:

We have presented a few new results on Harmonic mean labeling of certain classes of graphs like the zig-zag triangle Z $(T_n) \odot K_1$, Z$(T_n) \odot \overline{K_2}$, Z$(T_n) \odot K_2$. Analogous work can be carried out for other families and in the context of different types of graph labeling technique.

References

[1] Gallian, "A Dynamic Survey Of Graph Labeling", The Electronic Journal Of Combinatories. (2012).

[2] F. Harary , "Graph Theory", NarosaPupulising House Reading, New Delhi. (1988).

[3] A.Maheswari,K.Thanalakshmi, "Vertex Equitable Labeling Of Union Of Cycle Snake Transformed Trees", Internation Journal Of Advance Research In Science And Engineering.Vol.No.6,Issue No.02,February 2017.

[4] S.Meena, M.Renugha,M.Sivasakthi, "Cardial Labeling For Different Type Of Shell Graphs" International Journal Of Scientific and Engineering Research, Vol.No.6,Issue 9,Sep 2015.

[5] S.Meena, M.Sivasakthi, "Harmonic Mean Labeling Subdivision Graphs" International Journal Of Research And Analytical Reviews,Volume 6, Issue 1, Jan-March2019,E-ISSN 2348-1269, P-ISSN 2349-5138.

[6] S.Meena, M.Sivasakthi, "Some Results On Harmonic Mean Graphs" International Journal Of Research And Analytical Reviews,Volume 6, Issue 2, June 2019,E-ISSN 2348-1269, P-ISSN 2349-5138.

[7] S.S. Sandhya, S.Somasundaram and R.Ponraj ,"Some Results On Harmonic Mean graphs "Internation Journal Of Comtemporary Mathematical Sciences vol.7 No.4, PP-197-208. (2012)

[8] S.S. Sandhya, S.Somasundaram and R.Ponraj , " Harmonic Mean Labeling of Some Cycle Related Graphs"Internation Journal Of Math.Analysis,vol.6,2012, No.40, PP-1997-2005.

[9] S.S. Sandhiya, S.Somasundaram and R.Ponraj , "Some More Result On Harmonic Mean Graphs", Journal Of Mathematics Research,vol.4,Feb(2012),No.1 Pg: 21– 29.

[10] S.S.Sandhya, C.Jayasekaran and C.David raj , "Some New Families Of Harmonic Mean Graphs", International Journal Of Mathematical Research ,5(1) (2013) Pg 223-232.

[11] S.S.Sandhya, C.David raj, and C.Jayasekaran, "Some New Results On Harmonic Mean Graphs",International Journal Of Mathematical Archive-4(5), (2013) Pg 240-245.