Some More Operations On Multi Vague Sets

N. Ramakrishna

Department of Mathematics,
Mrs. A.V.N. College, Visakhapatnam-530 001,AP, India.

Abstract

The notion of Multi Vague set is introduced by Ramakrishna.N and the operations namely union, intersection, complement, addition, multiplication on Multi vague sets are discussed . In this paper we introduced some new operations like cross product, average operator on Multi Vaguesets and established various properties on Multi Vague sets. At the same time the behavior of modal operators over these operations are rigorously studied.

Keywords: *Vague set, Multi Fuzzy set, Multi Vagueset .*

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I. Introduction

Gau and Buehrer [4] in 1993 introduced and studied vague sets and Krassimir T,Atanassov [1] in 1986 studied Instutionistic Fuzzy sets. In fact both the notions are the same, and we prefer to use the terminology vague sets. The theory of vague sets started with the aim of interpreting the real life problems in a better way than the fuzzy sets do. In fact a vague set A on a set X is a pair (t_A, f_A) where t_A is called true membership function and f_A is called a false membership (non-membership) function are fuzzy sets on X with $t_A(x) \leq 1- f_A(x)$ for all x in X. This theory of vague sets are generalizations of study of fuzzy sets because fuzzy sets t_A is identified with pair $(t_A, 1 - t_A)$. ≤ −

In the theory of vague sets studied so far the membership function and non- membership function assume the values in [0, 1] of real numbers. Though Astanassov indicated the development of theory of Intuitionstic fuzzy sets with the membership function and non-membership function taking values in an arbitrary Lattice L, so far, it seems no progress is made in that direction. However theory of L- fuzzy sets has been developed extensively by many authors like John N. Mordeson and D.S. Malik, K.L.N.Swamy and U.M.Swamy, V.Murali etc.

The Fuzzy multiset which is a generalization of fuzzysets was introduced by R.R.Yager [5] . Multi set [6] allows the repeated occurrences of any element and hence the fuzzy multi set can occur more than once with the possibly of the same or the different membership values. Sabu Sebastian and T.Ramakrishnan [ ] continued the study of Multi fuzzy sets. Ramakrishna N [10] introduced the Multi Vague sets of a set X, as a pair of functions (t_A, f_A) where where $t_A(x) = (t_{1A}(x), t_{2A}(x), .., t_{kA}(x))$ and $f_A(x) = (f_{1A}(x), f_{2A}(x), .., f_{kA}(x))$ and $t_{iA} : X \to [0, 1]$, $f_{iA} : X \to [0, 1]$,are mappings such that $0 \leq t_{iA}(x)+f_{iA}(x) \leq 1$, ,for all $x \in X$, for i=1,2,3…,k with dimension 'k'. Here $t_{1A}(x) \geq t_{2A}(x) \geq \ldots\ldots \geq t_{kA}(x)$, for all $x \in X$. The basic operations like union, intersection, complement, addition, multiplication and two new operation on Multi vague sets are introduced and studied some properties in his paper [10]. In this paper we introduced some operations like cross product, average on Multi Vaguesets and established various properties on multi vague sets.

II. Preliminaries

Deftnition 2.1: A vague set A in the universe of discourse U is a pair (t_A, f_A) where $t_A : U \to [0, 1]$,$f_A : U \to [0, 1]$,are mappings such that $t_A(u) + f_A(u) \leq 1$, ,for all u in U. The functions t_A and f_A are called true membership function and false membership function respectively.

Deftnition 2.2.: The Vague Set A of set X with $t_A(x) = 0$ and $f_A(x) = 1$ for all $x \in X$, is called the Zero Vague set. We denote it by $\bar{0}$. The Vague Set A of set X with $t_A(x) = 1$ and $f_A(x) = 0$ for all x in X , is called the unit Vague set. We denote it by $\bar{1}$.

Deftnition 2.3.: Let X be a non-empty set. A multi-fuzzy set A in X is defined as $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A(x) = (\mu_1(x), \mu_2(x), ..\mu_k(x))$ and $\mu_i : X \to [0, 1]$ for all i=1,2,3....k. Here k is called the dimension of A. Also we note that, for all i, $\mu_i(x)$ is a decreasing ordered sequence of elements, that is $\mu_1(x) \geq \mu_2(x) \geq \ldots \geq \mu_k(x)$, for all $x \in X$.

Deftnition 2.4.:Let X be a non-empty set. A vague set A is a pair (t_A, f_A) where $t_A(x) = (t_{1A}(x), t_{2A}(x), .., t_{kA}(x))$ and $f_A(x) = (f_{1A}(x), f_{2A}(x), .., f_{kA}(x))$ and $t_{iA} : X \to [0, 1]$, $f_{iA} : X \to [0, 1]$,are mappings such that $t_{iA}(x)+f_{iA}(x) \leq 1$,for all $x \in X$, for i=1,2,3. ,k is called Multi vagueset of X with dimension 'k'.

Here $t_{1A}(x) \geq t_{2A}(x) \geq$..............................$\geq t_{kA}(x)$, for all $x \in X$.

Note: We arranged the true membership sequence is decreeing order, then the corresponding false membership sequence need not be in decreasing or increasing order.

Deftnition 2.5:Let X be a non-empty set and $A = (t_A, f_A)$, $B = (t_B, f_B)$ be any two multi vaguesets having the same dimension 'k' of X then The union of two Multi vaguesets A and B is a Multi vagueset, written as $A \cup B$ and is defined by $A \cup B = (t_{A\cup B}, f_{A\cup B})$, where $t_{A\cup B}(x) = max\{t_A(x), t_B(x)\}$ =

$(max.\{t_{1A}(x), t_{1B}(x)\}, max.\{t_{2A}(x), t_{2B}(x)\}, \ldots\ldots, max.\{t_{kA}(x), t_{kB}(x)\})$

$=(max.\{t_{iA}(x), t_{iB}(x)\})_{i=1}^{k}$. and $f_{A\cup B}(x) = min\{f_A(x), f_B(x)\}$

$= (min.\{f_{1A}(x), f_{1B}(x)\}, min.\{f_{2A}(x), f_{2B}(x)\}, \ldots, min.\{f_{kA}(x), f_{kB}(x)\})$

$=(min.\{f_{iA}(x), f_{iB}(x)\})_{i=1}^{k}$.

Here $\{t_{iA}(x), t_{iB}(x)\}$ represents the corresponding i^th position true membership values of A and B respectively also $\{f_{iA}(x), f_{iB}(x)\}$ represents the corresponding i^th position false membership values of A and B respectively.

Deftnition 2.6:Let X be a non-empty set and $A = (t_A, f_A)$, $B = (t_B, f_B)$ be any two multi vaguesets having the same dimension 'k' of X then The intersection of two Multi vaguesets A and B is a Multi vagueset, written as $A \cap B$ and is defined by $A \cap B = (t_{A\cap B}, f_{A\cap B})$, where

$t_{A\cap B}(x) = min\{t_A(x), t_B(x)\} =(min.\{t_{iA}(x), t_{iB}(x)\})_{i=1}^{k}$. and

$f_{A\cap B}(x) = max\{f_A(x), f_B(x)\} =(max.\{f_{iA}(x), f_{iB}(x)\})_{i=1}^{k}$.

Deftnition 2.7. :A Multi Vagueset $A = (t_A, f_A)$, is contained in another Multi Vagueset $B = (t_B, f_B)$, $A \subset B$, if and only if, $t_{iA}(x) \leq t_{iB}(x)$ and $f_{iA}(x) \geq f_{iB}(x)$ for all $x \in X$., for i=1,2,…k.

Also $A \subset B$ and $B \subset A$ *then* $A = B$.

Deftnition 2.8.: If A is the Multi Vagueset (t_A, f_A), then (f_A, t_A), is also a Multi Vagueset and is defined as the complement of A and is denoted by A^C .

III. Some more operations on Multi Vaguesets

Now we will introduce the some new operations cross product and average operator on Multi Vaguesets.

Definition 3.1. : Let X be a non-empty set and $A = (t_A, f_A)$, $B = (t_B, f_B)$ be any two Multi Vaguesets having the same dimension 'k' of X then The cross product of two Multi vaguesets A and B is a Multi Vagueset, written as A X B and is defined by A X B = ($t_{AXB,}$ $f_{AXB,}$) , where

$t_{AXB,}(x,y) = ([(t_{1A}(x).t_{1B}(y)], [(t_{2A}(x).t_{2B}(y)], \ldots\ldots [(t_{kA}(x).t_{kB}(y)])$

$f_{AXB,}(x,y) = ([(f_{1A}(x).f_{1B}(y)], [(f_{2A}(x).f_{2B}(y)], \ldots\ldots [(f_{kA}(x).f_{kB}(y)])$

where x, y in X

Note: Clearly $t_{1A}(x) \geq t_{2A}(x) \geq \quad \geq t_{kA}(x)$, and $t_{1B}(y) \geq t_{2B}(y) \geq \ldots.. \quad \geq t_{kB}(y)$

$\Rightarrow (t_{1A}(x).t_{1B}(y) \geq (t_{2A}(x).t_{2B}(y) \geq ,\ldots\ldots \geq (t_{kA}(x).t_{kB}(y)$ as a,b,c,d in [0,1] with $a \geq b$ and $c \geq d$ then $ac \geq bd$.

Also $t_{iA}(x).t_{iB}(y) + f_{iA}(x).f_{iB}(y) \leq t_{iA}(x) + f_{iB}(x) \leq 1$

$\Rightarrow 0 \leq t_{AXB,}(x,y) + f_{AXB,}(x,y) \leq 1 \Rightarrow$ A X B is a Multi Vagueset.

Here $\{t_{iA}(x), t_{iB}(x)\}$ represents the corresponding i^{th} position true membership values of A and B respectively also $\{ f_{iA}(x), f_{iB}(x) \}$ represents the corresponding i^{th} position false membership values of A and B respectively.

Example 3.2. : Let X = { a, b }, and

A={ (a, [0.2,0.4,0.5], [0.4,0.3,0.2]), (b, [0.2,0.3,0.3], [0.5,0.4,0.3]) }

B={ (a, [0.4,0.4,0.3], [0.5,0.5,0.6]), (b, [0.4,0.4,0.3], [0.3,0.5,0.2]) } then

AXB ={ ((a,a), [0.08,0.16,0.15], [0.20,0.15,0.12]),

((a,b), [0.08,0.16,0.15], [0.12,0.15,0.04]),

((b, a), [0.08,0.12,0.09], [0.20,0.20,0.18]),

((b, b), [0.08,0.12,0.09], [0.15,0.20,0.06]) }.

Theorem 3.3. : If A,B,C are three Multi Vaguesets ofa non-empty set X then

(i) . A X B = B X A

(ii) (A X B) X C = A X (B X C)

(iii). (A $\cup$ B) X C = (A X C) $\cup$ (B X C)

(iv). (A $\cap$ B) X C = (A X C) $\cap$ (B X C)

Proof : Let X be a non-empty set and $A = (t_A, f_A)$, $B = (t_B, f_B)$, $C = (t_C, f_C)$, be any three Multi Vaguesets of X.

A X B = ($t_{AXB,}$ $f_{AXB,}$) where
$t_{AXB,}(x,y)$ = ([$(t_{1A}(x).t_{1B}(y))$], [$(t_{2A}(x).t_{2B}(y))$],…… [$(t_{kA}(x).t_{kB}(y))$])

= ([$(t_{1B}(x).t_{1A}(y))$], [$(t_{2B}(x).t_{2A}(y))$],…… [$(t_{kB}(x).t_{kA}(y))$])

= $t_{BXA,}(x,y)$ and
$f_{AXB,}(x,y)$ = ([$(f_{1A}(x).f_{1B}(y))$], [$(f_{2A}(x).f_{2B}(y))$],…… [$(f_{kA}(x).f_{kB}(y))$])

= ([$(f_{1B}(x).f_{1A}(y))$], [$(f_{2B}(x).f_{2A}(y))$],…… [$(f_{kB}(x).f_{kA}(y))$])

= $f_{BXA,}(x,y)$.

Thus A X B = B X A.

(ii). i^{th} position of true membership value of (A X B) X C is

$t_{i\,(AXB)\,X\,C}$ = (t_{iA} . t_{iB}). t_{iC} = t_{iA} .(t_{iB} . t_{iC}) = $t_{i\ AX\,(B\,X\,C\,)}$ is i^{th} position of true membership value of A X (B X C).

Also i^{th} position of false membership value of (A X B) X C is

$f_{i\,(AXB)\,X\,C}$ = (f_{iA} . f_{iB}). f_{iC} = f_{iA} .(f_{iB} . f_{iC}) = $f_{i\ AX\,(B\,X\,C\,)}$ is i^{th} position of false membership value of A X (B X C).

Thus (A X B) X C = A X (B X C).

(iii). (A ∪ B) X C = ($t_{(A \cup B)\,X\,C}$, $f_{(A \cup B)\,X\,C}$)

i^{th} position of true membership value of (A ∪ B) X C is

$t_{i(A \cup B)\,X\,C}(x,y) = \{\ t_{i(A \cup B)}(x).\ t_{iC}(y)\ \}^{k}_{i=1}$

$= \{\ \max.\{ t_{iA}(x).\ t_{iB}(x)\ \}.\ t_{iC}(y)\ \}^{k}_{i=1}$

$= \{\ \max.\{ t_{iA}(x).\ t_{iC}(y)\ \}.\max.\{ t_{iB}(x).\ t_{iC}(y)\ \}\ \}^{k}_{i=1}$

= i^{th} position of true membership value of (A X C) ∪ (B X C)

Similarly we have, i^{th} position of false membership value of (A ∪ B) X C

= = i^{th} position of false membership value of (A X C) ∪ (B X C)

Thus (A ∪ B) X C = (A X C) ∪ (B X C)

In a similar way we can get (iv).

Definition 3.4. : Let X be a non-empty set and $A = (t_A, f_A)$, $B = (t_B, f_B)$ be any two Multi Vaguesets having the same dimension 'k' of X then The average operator of two Multi vaguesets A and B is a Multi Vagueset, written as *A # B* and
is defined by A # B = ($t_{A\#B,}$ $f_{A\#B,}$) , where

$$t_{i(A\#B)}(x) = \frac{t_{iA}(x)+t_{iB}(x)}{2} \quad \text{and} \qquad f_{i(A\#B)}(x) = \frac{f_{iA}(x)+f_{iB}(x)}{2}$$

Clearly A#B is a Multi Vagueset as $t_{1A}(x) \geq t_{2A}(x) \geq \ \geq t_{kA}(x)$, and $t_{1B}(y) \geq t_{2B}(y) \geq$….. $\geq t_{kB}(y)$ then

$$\frac{t_{1A}(x)+t_{1B}(x)}{2} \geq \frac{t_{2A}(x)+t_{2B}(x)}{2} \geq \ldots\ldots.. \geq \frac{t_{kA}(x)+t_{kB}(x)}{2} \quad \text{and}$$

$$0 \leq \frac{t_{iA}(x)+t_{iB}(x)}{2} + \frac{f_{iA}(x)+f_{iB}(x)}{2} \leq 1.$$

Example 3.5: Let X = { a, b, c }, and

A={ (a, [0.8,0.7,0.6], [0.1,0.2,0.3]), (b, [0.7,0.6,0.5], [0.2,0.3,0.4]),

(c, [0.6,0.5,0.4], [0.3,0.2,0.5]) }

B={ (a, [0.8,0.6,0.6], [0.1,0.3,0.2]), (b, [0.6,0.5,0.4], [0.3,0.4,0.5]),

(c, [0.5,0.4,0.3], [0.3,0.5,0.6]) } then

A # B={ (a, [0.8,0.65,0.6], [0.1,0.25,0.25]),

(b, [0.65,0.55,0.45], [0.25,0.35,0.45]),

(c, [0.55,0.45,0.35], [0.3,0.35,0.55]) }

Theorem 3.6 : Let A,B,C be Multi Vaguesets in X then

(i). A # A=A

(ii) A # B = B # A

(iii). A # (B # C) = (A # B) # C

(iv) (A # B)' = A' # B'

(v) A # (B $\cup$ C) = (A # B) $\cup$ (A # C)

(vi) A # (B $\cap$ C) = (A # B) $\cap$ (A # C)

Proof: Let X be a non-empty set and A = (t_A, f_A), B = (t_B, f_B) , C = (t_C, f_C), be any three Multi Vaguesets of X.

Clearly the proofs of (i),(ii), (iii) follows from definition.

(iv) A = (t_A, f_A), then A' = (f_A, t_A),

$$A' \# B' = (f_A, t_A) \# (f_B, t_B) = \left(\frac{f_{iA}+f_{iB}}{2}, \frac{t_{iA}+t_{iB}}{2}\right)_{i=1}^{k}$$

$$A \# B = \left(\frac{t_{iA}+t_{iB}}{2}, \frac{f_{iA}+f_{iB}}{2}\right)_{i=1}^{k}$$

$$(A \# B)' = \left(\frac{f_{iA}+f_{iB}}{2}, \frac{t_{iA}+t_{iB}}{2}\right)_{i=1}^{k} = A' \# B'$$

(v) A # (B $\cup$ C) = (t_A, f_A) # (max.{ t_{iB}(x). t_{iC}(x) }, min.{ f_{iB}(x). f_{iC}(x) })

$$= \left(\frac{t_{iA} + \max.\{\ t_{iB}(x).\ t_{iC}(x)\ \},}{2}, \frac{f_{iA} + \min.\{\ f_{iB}(x).\ f_{iC}(x)\ \}}{2} \right)_{i=1}^{k}$$

$$(A \# B) \cup (A \# C) = \left(\frac{t_{iA} + t_{iB}}{2}, \frac{f_{iA} + f_{iB}}{2} \right)_{i=1}^{k} \cup \left(\frac{t_{iA} + t_{iC}}{2}, \frac{f_{iA} + f_{iC}}{2} \right)_{i=1}^{k}$$

$$= \left(\max\left\{ \frac{t_{iA} + t_{iB}}{2}, \frac{t_{iA} + t_{iC}}{2} \right\}, \min\left\{ \frac{f_{iA} + f_{iB}}{2}, \frac{f_{iA} + f_{iC}}{2} \right\} \right)_{i=1}^{k}$$

$$= \left(\frac{t_{iA} + \max.\{\ t_{iB}(x).\ t_{iC}(x)\ \},}{2}, \frac{f_{iA} + \min.\{\ f_{iB}(x).\ f_{iC}(x)\ \}}{2} \right)_{i=1}^{k}$$

Thus $A \# (B \cup C) = (A \# B) \cup (A \# C)$

In s similar way we have (vi).

We note that from[10] , $\Delta A = (t_A, t'_A)$, $\nabla A = (f'_A, f_A)$, , where $t'_A = 1- t_A$

And $f'_A = 1- f_A$.

Theorem 3.7 : Let A,B,C be Multi Vaguesets in X then

(i). $\Delta(A \# A) = \Delta A$

(ii) $\nabla(A \# A) = \nabla A$

(iii) $\Delta(A \# B) = \Delta A \# \Delta B$

(iv) $\nabla(A \# B) = \nabla A \# \nabla B$

(v). $\Delta(A \# B)' = [\nabla(A \# B)]'$

(vi) $\nabla(A \# B)' = [\Delta(A \# B)]'$

Proof: Let X be a non-empty set and $A = (t_A, f_A)$, $B = (t_B, f_B)$, be any two Multi Vaguesets of X.

Clearly the proofs of (i),(ii) follows from definition.

$$\text{(iii). } A \# B = \left(\frac{t_{iA} + t_{iB}}{2}, \frac{f_{iA} + f_{iB}}{2} \right)_{i=1}^{k}$$

$$\Delta (A \# B) = \left(\frac{t_{iA} + t_{iB}}{2}, \left(\frac{t_{iA} + t_{iB}}{2} \right)' \right)_{i=1}^{k}$$

$$= \left(\frac{t_{iA} + t_{iB}}{2}, \left(\frac{t'_{iA} + t'_{iB}}{2} \right) \right)_{i=1}^{k}$$

$$\Delta A \# \Delta B = (t_A, t'_A) + (t_B, t'_B) = \left(\frac{t_{iA}+t_{iB}}{2}, \left(\frac{t'_{iA}+t'_{iB}}{2}\right)\right)_{i=1}^{k}$$

Thus Δ (A # B) = ΔA # ΔB

Similarly we get (iv).

(v) $A \# B = \left(\frac{t_{iA}+t_{iB}}{2}, \frac{f_{iA}+f_{iB}}{2}\right)_{i=1}^{k}$

$$(A \# B)' = \left(\frac{f_{iA}+f_{iB}}{2}, \frac{t_{iA}+t_{iB}}{2}\right)_{i=1}^{k}$$

$$\Delta (A \# B)' = \left(\frac{f_{iA}+f_{iB}}{2}, \left(\frac{f_{iA}+f_{iB}}{2}\right)'\right)_{i=1}^{k} = \left(\frac{f_{iA}+f_{iB}}{2}, \left(\frac{f'_{iA}+f'_{iB}}{2}\right)\right)_{i=1}^{k}$$

$$\nabla (A \# B) = \left(\left(\frac{f_{iA}+f_{iB}}{2}\right)', \frac{f_{iA}+f_{iB}}{2}\right)_{i=1}^{k} = \left(\left(\frac{f'_{iA}+f'_{iB}}{2}\right), \frac{f_{iA}+f_{iB}}{2}\right)_{i=1}^{k}$$

$$\Delta (A \# B)' = \left(\frac{f_{iA}+f_{iB}}{2}, \left(\frac{f'_{iA}+f'_{iB}}{2}\right)\right)_{i=1}^{k}$$

Thus $\Delta (A \# B)' = [\nabla (A \# B)]'$

Similarly we have (vi).

Theorem 3.8 : Let X be any non-empty set and A,B,C be Multi Vaguesets in X such that A $\subset$ B then A # C $\subset$ B # C.

Proof: Let X be a non-empty set and A = (t_A, f_A), B = (t_B, f_B) , C = (t_C, f_C), be any three Multi Vaguesets of X with A $\subset$ B

i.e. , $t_{iA}(x) \leq t_{iB}(x)$ and $f_{iA}(x) \geq f_{iB}(x)$ for all $x \in X$, i=1,2,….k.----(*)

$$A \# C = \left(\frac{t_{iA}+t_{iC}}{2}, \frac{f_{iA}+f_{iC}}{2}\right)_{i=1}^{k}$$

$B \# C = \left(\frac{t_{iB}+t_{iC}}{2}, \frac{f_{iB}+f_{iC}}{2}\right)_{i=1}^{k}$; but by (*) we have A # C $\subset$ B # C.

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