New Class of Topological spaces

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Abstract : *The aim of this paper is to introduce and study two new classes of spaces,namelyRegular Generalized Weaklynormal and Regular Generalized Weaklyregular spaces and obtained their properties by utilizing Regular Generalized Weaklyclosed sets.*

Keywords: *Regular Generalized Weaklyclosed set,Regular Generalized Weaklyopen sets, Regular Generalized Weakly regular space and Regular Generalized Weakly normal space.*

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I. Introduction

Recently, Benchalli et al [2,11] introduced and studied the properties of regular weakly closed sets and regular weakly continuous functions. It was further studied by Noiri and Popa[10],Dorsett[6] andArya[1]. Munshi[9], introduced g-regular and g- normal spaces using g-closed sets ofLevine[7].Maheshwari and Prasad[8] introduced the new class of spaces called s-normal spacesusing semi-open sets. Later, Benchalli et al [3] and ShikJohn[12] studied the concept of g*-pre-regular, g*-pre normal and w- normal, w-regular spaces in topological spaces.

II. Preliminaries

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c, and α-Cl(A), denote the Closure of A, Interior of A and Compliment of A and α-closure of A in X respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called

(i)Generalized closed set(briefly g-closed) [7] if cl(A)$\subseteq$U whenever A $\subseteq$ U and U is open in X.

(ii) Regular Generalized Weakly closed set(briefly rgw-closed) [39] if cl(int(A)$\subseteq$U whenever A $\subseteq$ U and U is Regular semi-open in X.

(ii)W-closed set[12] if cl(A) $\subseteq$ U whenever A$\subseteq$ U and U is semi-open in X.

Definition 2.2 :A topological space X is said to be a

(1)g-regular[10], if for each g-closed set F of X and each point x $\notin$F,there exists disjoint open sets U and V such that F$\subseteq$U and x ϵ V.

(2) w-regular[12], if for each closed set F of X and each point x ∉ F, there existsdisjoint w-open sets U and V such that F⊆U and xϵV.

(3) α - regular [4], if for each α - closed set F of X and each point x ∉ F, there exists disjoint α - open sets U and V such that F ⊆ V and x ϵ U.

Definition 2.3.A topological space X is said to be a

(1) α-normal [4], if for any pair of disjoint$\alpha -$ closed sets A and B, there exists dis-joint α-open sets U and V such that A⊆U and B⊆V .

(2) w-normal [12], if for any pair of disjoint w -closed sets A and B, there existsdisjoint open sets U and V such that A⊆ U and B⊆V.

(3) g- normal [10], if for any pair of disjoint g-closed sets A and B, there exists disjoint open sets U and V such that A⊆U and B⊆V.

Definition 2.4: [2] A topological space X is called $T_{\text{regular weakly}}$ -space if every Regular Generalized Weaklyclosed setis closed set.

Definition 2.5:A map f: (X, τ) ⟶ (Y, τ) is said to be

(i)Regular Generalized Weaklycontinuous map[19]if f^{-1}(V)is a Regular Generalized Weaklyclosed set of (X, τ) for every closed set V of (Y, τ).

(ii)Regular Generalized Weaklyirresolute map[20]if f^{-1}(V)is a Regular Generalized Weaklyclosed set of (X, τ) for everyRegular Generalized Weaklyclosed set V of (Y, τ).

III. REGULAR GENERALIZED WEAKLY REGULAR SPACE

In this section, we introduce a new class of space called Regular Generalized Weaklyregular space using Regular Generalized Weaklyclosed set and obtain some of their characterizations.

Definition 3.1. A topological space Xis said to be Regular Generalized Weaklyregular space if for each Regular Generalized Weaklyclosedset F and a point x∉F, there exist disjoint open sets G and H such that F⊆G andxϵH.

We have the following interrelationship between Regular Generalized Weaklyregularity and regularity.

Theorem 3.2. Every Regular Generalized Weaklyregular space is regular.

Proof: Let X be a Regular Generalized Weaklyregular space. Let F be any closed set in X and a point x∉Xsuch that x∉F. By [2], F is Regular Generalized Weakly topological space-closed and x ∉F. Since X is a Regular Generalized Weaklyregular space, thereexists a pair of disjoint open sets G and H such that F ⊆ G and x ϵH. Hence X is aregular space.

Remark 3.3: If X is a regular space and $T_{\text{Regular Generalized Weakly topological space}}$, then X is Regular Generalized Weakly regular space then we have the following characterization.

Theorem 3.4. The following statements are equivalent for a topological space X.

(i) X is a Regular Generalized Weakly regular space
(ii) For each x ϵX and each Regular Generalized Weakly topological spacesopen neighbourhoodU of x,there exists an openneighbourhoodN of x such that cl(N)⊆U.

Proof: (i)implies(ii): Suppose X is a Regular Generalized Weaklyregular space. Let U be any Regular Generalized Weaklyneighbourhood of x. Then there exists Regular Generalized Weaklyopen set G such that x ϵ G ⊆U. Now X –Gis Regular Generalized Weakly closed set and x ∉ X - G. Since X is pre generalized pre regular weakly regular space, then there exist open sets Mand N such that X -G⊆M, x ϵ N and M ∩ N = $\varphi$and so N ⊆X-M. Nowcl(N) ⊆ cl(X -M) = X-M and X -M ⊆M. This implies X -M⊆ U. Thereforecl(N)⊆U.

(ii)implies (i): Let F be any Regular Generalized Weakly topological space closed set in X and x ϵX -F and X - F is aRegular Generalized Weakly topological space open and so X - F is a Regular Generalized Weakly topological space neighbourhood of x. By hypothesis, there existsan open neighbourhoodN of x such that x ϵN and cl(N) ⊆X - F. This impliesF ⊆X - cl(N) is an open set containing F and N ∩ f(X - cl(N)= φ . Hence X isRegular Generalized Weakly regular space.

We have another characterization of Regular Generalized Weaklyregularity in the following.

Theorem 3.5: A topological space X is Regular Generalized Weaklyregular if and only if for each Regular Generalized Weakly topological space closedset F of X and each x ϵX - F there exist open sets G and H of X such that x ϵ G,F⊆H and cl(G) ∩ cl(H) = ∅.
Proof: Suppose X is Regular Generalized Weaklyregular space. Let F be a Regular Generalized Weakly topological space closed set in X with x ∉F.Then there exists open sets M and H of X such that x ϵ M, F ⊆H and M∩H =∅.This implies M∩cl(H) = ∅.As X is Regular Generalized Weaklyregular, there exist open sets U and V suchthat x ϵ U, cl(H)⊆V and U∩V = ∅.socl(U)∩V = ∅.Let G = M ∩U, then G andH are open sets of X such that xϵG, F ⊆ H and cl(H) ∩cl(H) =∅.

Conversely, if for each Regular Generalized Weaklyclosed set F of X and each x ϵ X -F there exists opensets G and H such that x ϵ G, F⊆ H and cl(H)∩cl(H) =∅.This implies x ϵG,F⊆H and G ∩ H = ∅. Hence X is Regular Generalized Weaklyregular.

Now we prove that Regular Generalized Weakly topological spaces- regularity is a heriditary property.

Theorem 3.6. Every subspace of a Regular Generalized Weaklyregular space is Regular Generalized Weaklyregular.

Proof: Let X be a Regular Generalized Weakly regular space. Let Y be a subspace of X. Let x ϵ Y and F bea Regular Generalized Weaklyclosed set in Y such that x∉F. Then there is a closed set and so Regular Generalized Weaklyclosedset A of X with F = Y ∩ A and x ∉A. Therefore we have x ϵ X, A is Regular Generalized Weaklyclosedin X such that x∉A. Since X is Regular Generalized Weaklyregular, then there exist open sets G and H suchthat x ϵG, A⊆H and G∩H = φ. Note that Y ∩G and Y∩H are open sets in Y.Alsox ϵ G and x ϵY, which implies x ϵ Y ∩G and A ⊆ H implies Y∩ G⊆Y ∩H,F⊆Y∩H. Also (Y∩G)∩(Y∩H) = φ. Hence Y is Regular Generalized Weaklyregular space.

We have yet another characterization of Regular Generalized Weakly topological spaces-regularity in the following.

Theorem 3.7 : The following statements about a topological space X are equivalent:

(i) X is Regular Generalized Weaklyregular
(ii) For each x ϵ X and each Regular Generalized Weakly topological space open set U in X such that x ϵ U there exists anopen set V in X such that x ϵ V⊆cl(V)⊆U.
(iii) For each point x ϵX and for each Regular Generalized Weakly topological space closed set A with x ∉ A, then there exists anopen set V containing x such that cl(V)∩A = φ.

Proof: (i) implies(ii): Follows from Theorem 3.5.

(ii) implies(iii): Suppose (ii) holds. Let x ϵ X and A be anRegular Generalized Weakly topological spaceclosed set of X suchthat x ∉ A.Then X - A is a Regular Generalized Weakly topological spaceopen set with xϵX -A. By hypothesis, thereexists an open set V such that x ϵV ⊆ cl(V) ⊆ X - A. That is xϵV ,V⊆ cl(A) andcl(A) ⊆ X - A. So x ϵV and cl(V)∩A = φ.

(iii) implies(i): Let x ϵ X and U be an Regular Generalized Weaklytopological space open set in X such that x ϵ U. ThenX - U is anRegular Generalized Weakly topological spaceclosed set and x∉ X - U. Then by hypothesis, there exists an openset V containing x such that cl(A) ∩(X -U) = Á. Therefore x ϵV ,cl(V)⊆U sox ϵ V⊆ cl(V)⊆U.

The invariance of Regular Generalized Weakly topological space regularity is given in the following.

Theorem 3.8: Let f :X →Y be a bijective, Regular Generalized Weakly topological space irresolute and open map from a Regular Generalized Weakly topological space regular space X into a topological space Y , then Y is Regular Generalized Weakly topological spaces-regular.

Proof: Let y ϵ Y and F be a Regular Generalized Weakly topological space closed set in Y with y ∉ F. Since F is Regular Generalized Weakly topological spaceirresolute, $f^{-1}(F)$ is Regular Generalized Weakly topological space closed set in X. Let f(x) = y so that x = f^{-1} (y) andx ∉ $f^{-1}(F)$. Again X is Regular Generalized Weakly-regular space, then there exist open sets U and V such thatx ϵ U and $f^{-1}(F)$⊆ G, U ∩ V = φ. Since f is open and bijective, we have y ϵ f(U),F ⊆ f(V) and f(U) ∩f(V) = f(U∩V) = f(φ) = φ . Hence Y is Regular Generalized Weaklyregular space.

Theorem 3.9. Let f :X→Y be a bijective, Regular Generalized Weaklyclosed and open map from atopological space X into a Regular Generalized Weaklyregular space Y . If X is $T_{\text{Regular Generalized Weakly topological spaces}}$, then X is Regular Generalized Weaklyregular.

Proof: Let x ϵX and F be an Regular Generalized Weaklyclosed set in X with x ∉F. Since X is $T_{\text{Regular Generalized Weakly topological spaces}}$,Fis closed in X. Then f(F) is Regular Generalized Weaklyclosed set with f(x)∉ f(F) in Y , since f is Regular Generalized Weakly closed. As Y is Regular Generalized Weaklyregular,then there exist open sets U and V such that x ϵ Uandf(x) ϵ U and f(F) ⊆V . Therefore x ϵ f^{-1}(U) and F⊆ f^{-1}(V). Hence X is Regular Generalized Weaklyregular space.

Theorem 3.10. If f : X→ Y is w-irresolute, continuous injection and Y is Regular Generalized Weakly topological spaces-regular space,thenX is Regular Generalized Weakly topological spaces-regular.

Proof: Let F be any closed set in X with x∉F. Since f is w-irresolute, f is Regular Generalized Weakly topological space closed set in Yand f(x) ϵ f(F). Since Y is Regular Generalized Weaklyregular,thenthere exists open sets U and V such that f(x) ϵU and f(F) ⊆ V . Thus x ϵf $^{-1}$(U),F⊆ f $^{-1}$(V) and f^{-1} (U) ∩ f^{-1}(V) = φ. Hence X is Regular Generalized Weaklyregular space.

IV. Regular Generalized Weaklynormal Spaces

In this section, we introduce the concept of Regular Generalized Weaklynormal spaces and study some of theircharacterizations.

Definition 4.1. A topological space X is said to be Regular Generalized Weaklynormal if for each pair ofdisjoint Regular Generalized Weakly topological spacesclosed sets A and B in X,thenthere exists a pair of disjoint open sets U andV in X such that A ⊆ U and B ⊆V

We have the following interrelationship.

Theorem 4.2. Every Regular Generalized Weaklynormal space is normal.

Proof: Let X be a Regular Generalized Weaklynormal space. Let A and B be a pair of disjoint closed sets inX. From [2], A andB are Regular Generalized Weakly topological spacesclosed sets in X. Since X is Regular Generalized Weaklynormal, then there existsa pair of disjoint open sets G and H in X such that A⊆ G and B ⊆ H. Hence X is normal.

Remark 4.3. The converse need not be true in general as seen from the following example.

Example 4.4. Let X = Y ={a,b,c,d},τ ={X, ∅,{a},{c},{a,c},{b,c,d}}.Then the space X is normal but not Regular Generalized Weaklynormal, since the pair of disjoint Regular Generalized Weakly topological spacesclosed setsnamely, A = {a,d}and B = {b,c}for which there do not exists disjoint open sets Gand H such that A ⊆ G and B⊆ H.

Remark 4.5.:IfX is normal and $T_{\text{Regular Generalized Weakly topological spaces}}$, then X is Regular Generalized Weakly-normal.

Hereditary property of Regular Generalized Weaklynormality is given in the following.

Theorem 4.6.A Regular Generalized Weaklyclosed subspace of a Regular Generalized Weaklynormal space is Regular Generalized Weaklynormal.We have the following characterization.

Theorem 4.7. The following statements for a topological space X are equivalent:

(i) X is Regular Generalized Weakly topological spacesis normal
(ii) For each Regular Generalized Weaklyclosed set Aand each pre generalized pre regular weaklytopological space open set U such that A⊆U, there existsan open set V such that A⊆V⊆cl(V)⊆U
(iii) For any Regular Generalized Weaklyclosed sets A, B, there exists an open set V such thatA⊆V andcl(V)∩B = φ.

(iv) For each pair A, B of disjoint Regular Generalized Weaklyclosed sets then there exist open sets U and V suchthat A ⊆ U,B ⊆V and cl(U) ∩cl(V) = φ.

Proof: (i)implies (ii): Let A be a Regular Generalized Weaklyclosed set and U be a Regular Generalized Weaklyopen set such that A⊆U.ThenA and X - U are disjoint Regular Generalized Weaklyclosed sets in X. Since X is Regular Generalized Weaklynormal , then thereexists a pair of disjoint open sets V and W in X such that A ⊆ V and X -U⊆W.Now X -W ⊆ X -(X -U), so X -W⊆ Ualso V∩W = φ.implies V ⊆ X -W, so
cl(V)⊆cl(X -W) which implies cl(V) ⊆ X -W. Therefore cl(V)⊆X -W⊆U. So cl(V) ⊆ U. Hence A⊆V⊆cl(V)⊆U.

(ii)implies(iii): Let A and B be a pair of disjoint Regular Generalized Weaklyclosed sets in X. Now A∩B = φ,so A⊆X -B, where A is Regular Generalized Weaklyclosed and X - B is Regular Generalized Weaklyopen . Then by (ii) thereexists an open set V such that A⊆V⊆cl(V)⊆X-B. Now cl(V)⊆X - B impliescl(V)∩B = φ. Thus A⊆V and cl(V)∩B = φ.

(iii) implies (iv): Let A and B be a pair of disjoint Regular Generalized Weaklyclosed sets in X.Then from (iii)there exists an open set U such that A⊆U and cl(U)∩B = φ. Since cl(V) is closed, soRegular Generalized Weaklyclosed set.Thereforecl(V) and B are disjoint Regular Generalized Weaklyclosed sets in X. By hypothesis,thentheir exists an open set V , such that B⊆V and cl(U) ∩cl(V) = φ.

(iv) implies (i): Let A and B be a pair of disjoint Regular Generalized Weaklyclosed sets in X.Then from (iv)then there exist an open sets U and V in X such that A⊆U, B⊆V and
cl(U)∩cl(V) = φ.So A ⊆U ,B⊆V and U∩V = φ.Hence X is Regular Generalized Weaklynormal.

Theorem 4.8. Let X be a topological space. Then X is Regular Generalized Weaklynormal if and only if forany pair A, B of disjoint Regular Generalized Weaklyclosed setthen there exist open sets U and V of X such thatA⊆U,B⊆V and cl(U) ∩cl(V) = φ.

Theorem 4.9. Let X be a topological space. Then the following are equivalent:
(i) X is normal
(ii) For any disjoint closed sets A and B,then there exist disjoint Regular Generalized Weakly topological spaces- open sets U and Vsuch that A⊆U,B⊆V .
(iii) For any closed set A and any open set V such that A⊆ V, there exists an Regular Generalized Weaklyopen set U of X such that A⊆U⊆αcl(U)⊆ V .

Proof: (i) implies(ii): Suppose X is normal. Since every open set is Regular Generalized Weaklyopen [2], (ii)follows.

(ii) implies(iii): Suppose (ii) holds. Let A be a closed set and V be an open set containing A. Then A andX - V are disjoint closed sets. By (ii), then there exist disjoint Regular Generalized Weakly open sets U and W such that A⊆U and X - V ⊆ W, since X -V is closed, soRegular Generalized Weaklyis closed. From [2], we have X -V⊆α-int(W) and U ∩ α-int(W) =φ.and so wehave α-cl(U) ∩ α-int(W) = φ. Hence A ⊆ U ⊆α-cl(U) ⊆ X – α-int(W)⊆V . ThusA ⊆ U ⊆α-cl(U) ⊆ V .

(iii) implies (i): Let A and B be a pair of disjoint closed sets of X.ThenA ⊆X - B andX -B is open. There exists a Regular Generalized Weaklyopen set G of X such that A ⊆ G⊆α-cl(G) ⊆X-B.Since A is closed, it is w- closed, we have A ⊆α-int(G). Take U = int(cl(int(α-int(G))))and V = int(cl(int(X –α-cl(G)))). Then U and V are disjoint open sets of X such thatA ⊆U and B ⊆ VHence X is normal.

We have the following characterization of Regular Generalized Weakly topological spaces-normality and Regular Generalized Weakly topological spaces- normality.

Theorem 4.10. Let X be a topological space. Then the following are equivalent:
(i) X is α-normal.
(ii) For any disjoint closed sets A and B, there exist disjoint Regular Generalized Weakly topological space- open sets U and Vsuch that A⊆U,B⊆V and U∩ V = φ.

Proof: (i) implies (ii): Suppose X is α- normal. Let A and B be a pair of disjoint closedsets of X. Since X is α-normal,there exist disjoint $\alpha -$ open sets U and V such thatA⊆U and B⊆V and U ∩ V = φ.
(ii) implies(i):Let A and B be a pair of disjoint closed sets of X.The by hypothesis thereexist disjoint Regular Generalized Weaklyopen sets U and V such that A⊆U and B ⊆ V and U ∩V = φ .Sincefrom [2], A⊆α-intUand B ⊆$\alpha -$ int(V)and α –intU∩ α-intV= φ. Hence X is α-normal.

Theorem 4.11. Let X beaα- normal, then the following hold good:

(i)For each closed set A and every Regular Generalized Weakly open set B such that A⊆B ther exists aαopen set U such that A⊆U⊆α-cl(U)⊆ B.
(ii) For every Regular Generalized Weaklyclosed set A and every open set B containing A, there exist aα-open set U such that A⊆U⊆ α-cl(U)⊆B.

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