Maximum and Minimum 3-Equitable Labeling For Some Graphs

V J Kaneria[#1], M J Khoda[#2]

[#1]Associate Professor, Department of Mathematics, Saurashtra University, Rajkot-360005, Gujarat, India
[#2]Assistant Professor, V.V.P. Engineering College, Rajkot-360005, Gujarat, India.

Abstract — *In the research paper, we have defined maximum and minimum 3-equitable labeling. Also, we have figured out the labeling for path, cycle and star graph.*

Keywords — *Maximum 3-equitable labeling, Minimum 3-euitable labeling, Path, Cycle, Star graph.*

I. INTRODUCTION

Throughout the paper, we consider simple, finite and undirected graph $\mathbf{G}(\mathbf{V},\mathbf{E})$. The idea of 3-equitable labeling was brought by I. Cahit [1]. The vertex set and edge set of $\mathbf{G}$ are denoted by $\mathbf{V}(\mathbf{G})$and $\mathbf{E}(\mathbf{G})$ respectively. Kaneria, Meghpara and Khoda proved that Caterpillar $\mathbf{S}(\mathbf{x_1},\mathbf{x_2},\ldots\ldots,\mathbf{x_t})$ is a geometric mean 3-equitable [4]. Here, we have given brief summary of definitions which would be helpful for the present research paper.

For all standard terminology and notations, we follow Harry [3]. For detail survey of graph labeling, we referred Gallian [2].

Definition 1.1 — Let $G = (V, E)$ be a graph, $f: V(G) \rightarrow \{0, 1, 2\}$ is called ternary vertex labeling of G and f(v) is called the label of the vertex G under f.

For an edge e = uv this induced edge labeling function $f^*: E(G) \rightarrow \{0, 1, 2\}$ is given by $f^*(e) = \text{maximum}\{f(u), f(v)\}$. Let $v_f(0), v_f(1), v_f(2)$ be the numbers of vertices of G having labels 0, 1, 2 respectively under f and $e_f(0), e_f(1), e_f(2)$ be the numbers of edges of G having labels 0, 1, 2 respectively under f^*.

Definition 1.2 — The ternary vertex labeling of graph G is called maximum 3-equitable labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|ef(i) - ef(j)| \leq 1,\ \forall\, i, j \in \{0, 1, 2\}$. A graph G is called maximum 3-equitable if it admits maximum 3-equitable labeling.

Definition 1.3 — Let $G = (V, E)$ be a graph, $f: V(G) \rightarrow \{0, 1, 2\}$ is called ternary vertex labeling of G and f(v) is called the label of the vertex G under f.

For an edge e = uv this induced edge labeling function $f^*: E(G) \rightarrow \{0, 1, 2\}$ is given by $f^*(e) = \text{minimum}\{f(u), f(v)\}$. Let $v_f(0), v_f(1), v_f(2)$ be the numbers of vertices of G having labels 0, 1, 2 respectively under f and $e_f(0), e_f(1), e_f(2)$ be the numbers of edges of G having labels 0, 1, 2 respectively under f^*.

Definition 1.4 — The ternary vertex labeling of graph G is called minimum 3-equitable labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1,\ \forall\, i, j \in \{0, 1, 2\}$. A graph G is called maximum 3-equitable if it admits maximum 3-equitable labeling.

II. MAIN RESULTS

Theorem 2.1 — Path P_n, $\forall\, n \geq 2$ is maximum 3-equitable, where $n \in N$.

Proof — Let P_n be the path with n vertices. So,$n - 1$ be the number of edges of path P_n. Let $V(P_n) = \{v_1, v_2, \ldots.v_n\}$.
Case 1 — Let $n \equiv 0 (mod\ 3)$ which means that $n = 3t$ for some $t \in N$. The vertex labeling function $f: V(P_n) \rightarrow \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f(v_i) = \begin{cases} 0\,;if \quad 1 \le i \le t \\ 1\,;if \;\; t+1 \le i \le 2t \\ 2\,;if \;\; 2t+1 \le i \le 3t \end{cases} ,\forall\, i = 1,2,\ldots\ldots,n.$$

Hence, induced edge labeling function $f^*: E(P_n) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f^*(v_i, v_{i+1}) = \begin{cases} 0\,;if \quad 1 \le i \le t-1 \\ 1\,;if \;\; t \le i \le 2t-1 \\ 2\,;if \;\; 2t \le i \le 3t-1 \end{cases} ,\forall\, i = 1,2,\ldots\ldots,n.$$

Case 2 — Let $n \equiv 1(mod\ 3)$ which means that $n = 3t+1$ for some $t \in N$. The vertex labeling function $f: V(P_n) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f(v_i) = \begin{cases} 0\,;if \quad 1 \le i \le t+1 \\ 1\,;if \;\; t+2 \le i \le 2t+1 \\ 2\,;if \;\; 2t+2 \le i \le 3t+1 \end{cases} ,\forall\, i = 1,2,\ldots\ldots,n.$$

Hence, induced edge labeling function $f^*: E(P_n) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f^*(v_i, v_{i+1}) = \begin{cases} 0\,;if \quad 1 \le i \le t \\ 1\,;if \;\; t+1 \le i \le 2t \\ 2\,;if \;\; 2t+1 \le i \le 3t \end{cases} ,\forall\, i = 1,2,\ldots\ldots,n.$$

Case 3 — Let $n \equiv 2(mod\ 3)$ which means that $n = 3t{+}2$ for some $t \in N$. The vertex labeling function $f: V(P_n) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f(v_i) = \begin{cases} 0\,;if \quad 1 \le i \le t+1 \\ 1\,;if \;\; t+2 \le i \le 2t+2 \\ 2\,;if \;\; 2t+3 \le i \le 3t+2 \end{cases} ,\forall\, i = 1,2,\ldots\ldots,n.$$

Hence, induced edge labeling function $f^*: E(P_n) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f^*(v_i, v_{i+1}) = \begin{cases} 0\,;if \quad 1 \le i \le t \\ 1\,;if \;\; t+1 \le i \le 2t+1 \\ 2\,;if \;\; 2t+2 \le i \le 3t+1 \end{cases} ,\forall\, i = 1,2,\ldots\ldots,n.$$

In all these cases, we have $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$, $\forall\, i,j \in \{0,1,2\}$. Hence, Path P_n admits maximum 3-equitable labeling. Therefore, Path P_n , $\forall\, n \ge 2$ is maximum 3-equitable, where $n \in N$. ■

Theorem 2.2 — Path P_n , $\forall\, n \ge 2$ is minimum 3-equitable, where $n \in N$.

Proof — Let P_n be the path with n vertices. So,n − 1 be the number of edges of path P_n. Let $V(P_n) = \{v_1, v_2, \ldots.\, v_n\}$.

Case 1 — Let $n \equiv 0(mod\ 3)$ which means that $n = 3t$ for some $t \in N$. The vertex labeling function $f: V(P_n) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f(v_i) = \begin{cases} 0\,;if \quad 1 \le i \le t \\ 1\,;if \;\; t+1 \le i \le 2t \\ 2\,;if \;\; 2t+1 \le i \le 3t \end{cases} ,\forall\, i = 1,2,\ldots\ldots,n.$$

Hence, induced edge labeling function $f^*: E(P_n) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f^*(v_i, v_{i+1}) = \begin{cases} 0\,; if \quad 1 \le i \le t \\ 1\,; if \quad t+1 \le i \le 2t \\ 2\,; if \; 2t+1 \le i \le 3t-1 \end{cases}, \forall\, i = 1,2, \ldots\ldots, n.$$

Case 2 — Let $n \equiv 1(mod\ 3)$ which means that $n = 3t + 1$ for some $t \in N$. The vertex labeling function $f: V(P_n) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f(v_i) = \begin{cases} 0\,; if \quad 1 \le i \le t \\ 1\,; if \quad t+1 \le i \le 2t \\ 2\,; if \; 2t+1 \le i \le 3t+1 \end{cases}, \forall\, i = 1,2, \ldots\ldots, n.$$

Hence, induced edge labeling function $f^*: E(P_n) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f^*(v_i, v_{i+1}) = \begin{cases} 0\,; if \quad 1 \le i \le t \\ 1\,; if \; t+1 \le i \le 2t \\ 2\,; if \; 2t+1 \le i \le 3t \end{cases}, \forall\, i = 1,2, \ldots\ldots, n.$$

Case 3 — Let $n \equiv 2(mod\ 3)$ which means that $n = 3t + 2$ for some $t \in N$. The vertex labeling function $f: V(P_n) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f(v_i) = \begin{cases} 0\,; if \quad if\ 1 \le i \le t \\ 1\,; if \; t+1 \le i \le 2t+1 \\ 2\,; if \; 2t+2 \le i \le 3t+2 \end{cases}, \forall\, i = 1,2, \ldots\ldots, n.$$

Hence, induced edge labeling function $f^*: E(P_n) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f^*(v_i, v_{i+1}) = \begin{cases} 0\,; if \quad 1 \le i \le t \\ 1\,; if \; t+1 \le i \le 2t+1 \\ 2\,; if \; 2t+2 \le i \le 3t+1 \end{cases}, \forall\, i = 1,2, \ldots\ldots, n.$$

In all these cases, we have $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$, $\forall\, i, j \in \{0, 1, 2\}$. Hence, Path P_n admits minimum 3-equitable labeling. Therefore, Path P_n, $\forall\, n \ge 2$ is minimum 3-equitable, where $n \in N$. ■

Theorem 2.3 — The cycle C_n, $\forall\, n \ge 3$ is maximum 3-equitable, for $n \equiv 1(mod\ 3)$ and $n \equiv 2(mod\ 3)$ and $n \in N$.

Proof — Let C_n be the cycle with n vertices and n edges. Let the set of vertices be $V(C_n) = \{v_1, v_2, \ldots . v_n\}$.

Case 1 — Let $n \equiv 1(mod\ 3)$ which means that $n = 3t + 1$ for some $t \in N$. The vertex labeling function $f: V(C_n) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f(v_i) = \begin{cases} 0\,; if \quad 1 \le i \le t+1 \\ 1\,; if \; t+2 \le i \le 2t+1 \\ 2\,; if \; 2t+2 \le i \le 3t+1 \end{cases}, \forall\, i = 1,2, \ldots\ldots, n.$$

Hence, induced edge labeling function $f^*: E(C_n) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f^*(v_i, v_{i+1}) = \begin{cases} 0\,;if \quad 1 \le i \le t \\ 1;if \;\; t+1 \le i \le 2t \\ 2;if \;\; 2t+1 \le i \le 3t \end{cases} \text{ and } f^*(v_n, v_1) = 2, \forall\, i = 1,2,\ldots\ldots,n.$$

Case 2 — Let $n \equiv 2(mod\, 3)$ which means that $n = 3t + 2$ for some $t \in N$. The vertex labeling function $f: V(C_n) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f(v_i) = \begin{cases} 0\,;if \quad 1 \le i \le t+1 \\ 1\,;if \;\; t+2 \le i \le 2t+2 \\ 2\,;if \;\; 2t+3 \le i \le 3t+2 \end{cases}, \forall\, i = 1,2,\ldots\ldots,n.$$

Hence, induced edge labeling function $f^*: E(C_n) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f^*(v_i, v_{i+1}) = \begin{cases} 0;if \quad 1 \le i \le t \\ 1\,;if \;\; t+1 \le i \le 2t+1 \\ 2;if \;\; 2t+2 \le i \le 3t+1 \end{cases} \text{ and } f^*(v_n, v_1) = 2, \forall\, i = 1,2,\ldots\ldots,n.$$

In both the cases, we have $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$, $\forall\, i,j \in \{0,1,2\}$. Hence, Cycle C_n admits maximum 3-equitable labeling. Therefore, Cycle C_n , $\forall\, n \ge 3$ is maximum 3-equitable, for $n \equiv 1(mod\; 3)$ and $n \equiv 2(mod\; 3)$ where $n \in N$. ■

Theorem 2.4 — The cycle C_n, $\forall\, n \ge 3$ is minimum 3-equitable, for $n \equiv 1(mod\; 3)$ and $n \equiv 2(mod\; 3)$ and $n \in N$.

Proof — Let C_n be the cycle with n vertices and n edges. Let the set of vertices be $V(C_n) = \{v_1, v_2, \ldots.\, v_n\}$.

Case 1 — Let $n \equiv 1(mod\, 3)$ which means that $n = 3t + 1$ for some $t \in N$. The vertex labeling function $f: V(C_n) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f(v_i) = \begin{cases} 0\,;if \quad 1 \le i \le t \\ 1;if \quad t+1 \le i \le 2t \\ 2\,;if \;\; 2t+1 \le i \le 3t+1 \end{cases}, \forall\, i = 1,2,\ldots\ldots,n.$$

Hence, induced edge labeling function $f^*: E(C_n) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f^*(v_i, v_{i+1}) = \begin{cases} 0\,;if \quad 1 \le i \le t \\ 1;if \;\; t+1 \le i \le 2t \\ 2;if \;\; 2t+1 \le i \le 3t \end{cases} \text{ and } f^*(v_n, v_1) = 0, \forall\, i = 1,2,\ldots\ldots,n.$$

Case 2 — Let $n \equiv 2(mod\, 3)$ which means that $n = 3t + 2$ for some $t \in N$. The vertex labeling function $f: V(C_n) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f(v_i) = \begin{cases} 0\,;if \quad 1 \le i \le t \\ 1\,;if \;\; t+1 \le i \le 2t+1 \\ 2\,;if \;\; 2t+2 \le i \le 3t+2 \end{cases}, \forall\, i = 1,2,\ldots\ldots,n.$$

Hence, induced edge labeling function $f^*: E(C_n) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f^*(v_i, v_{i+1}) = \begin{cases} 0\,; if & 1 \le i \le t \\ 1\,; if & t+1 \le i \le 2t+1 \\ 2\,; if & 2t+2 \le i \le 3t+1 \end{cases} \text{ and } f^*(v_n, v_1) = 0, \forall\, i = 1,2, \ldots\ldots, n.$$

In both the cases, we have $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$, $\forall$ i, j $\in \{0, 1, 2\}$. Hence, Cycle C_n admits minimum 3-equitable labeling. Therefore, Cycle C_n , $\forall\, n \ge 3$ is minimum 3-equitable, for $n \equiv 1(mod\ 3)$ and $n \equiv 2(mod\ 3)$ where $n \in N$. ■

Theorem 2.5 — The star graph $K_{1,n}$,$\forall\, n \ge 2$, is maximum 3-equitable, where $n \in N$.

Proof — Let $K_{1,n}$ be the star graph with the apex vertex u and $\{v_1, v_2, \ldots . v_n\}$ be the n pendent vertices. Cleary $n+1$ and n be the number vertices and edges respectively of star graph $K_{1,n}$.

Case 1 — Let $n \equiv 0(mod\ 3)$ which means that $n = 3t$ for some $t \in N$. The vertex labeling function $f: V(K_{1,n}) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

For the apex vertex u, $f(u) = 0$ and for the pendent vertices we have,

$$f(v_i) = \begin{cases} 0\,; if & 1 \le i \le t \\ 1\,; if & t+1 \le i \le 2t \\ 2\,; if & 2t+1 \le i \le 3t \end{cases}, \forall\, i = 1,2, \ldots\ldots, n.$$

Hence, induced edge labeling function $f^*: E(k_{1,n}) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f^*(u, v_i) = \begin{cases} 0\,; if & 1 \le i \le t \\ 1; if & t+1 \le i \le 2t \\ 2; if & 2t+1 \le i \le 3t \end{cases}, \forall\, i = 1,2, \ldots\ldots, n.$$

Case 2 — Let $n \equiv 1(mod\ 3)$ which means that $n = 3t+1$ for some $t \in N$. The vertex labeling function $f: V(K_{1,n}) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

For the apex vertex u, $f(u) = 0$ and for the pendent vertices we have,

$$f(v_i) = \begin{cases} 0\,; if & 1 \le i \le t \\ 1; if & t+1 \le i \le 2t+1 \\ 2\,; if & 2t+2 \le i \le 3t+1 \end{cases}, \forall\, i = 1,2, \ldots\ldots, n.$$

Hence, induced edge labeling function $f^*: E(k_{1,n}) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f^*(u, v_i) = \begin{cases} 0; if & 1 \le i \le t \\ 1\,; if & t+1 \le i \le 2t+1 \\ 2; if & 2t+2 \le i \le 3t+1 \end{cases}, \forall\, i = 1,2, \ldots\ldots, n.$$

Case 3 — Let $n \equiv 2(mod\ 3)$ which means that $n = 3t+2$ for some $t \in N$. The vertex labeling function $f: V(K_{1,n}) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

For the apex vertex u, $f(u) = 0$ and for the pendent vertices we have,

$$f(v_i) = \begin{cases} 0\,; if \quad 1 \le i \le t \\ 1\,; if \;\; t+1 \le i \le 2t+1 \\ 2\,; if \;\; 2t+2 \le i \le 3t+2 \end{cases} , \forall\, i = 1,2, \ldots\ldots, n.$$

Hence, induced edge labeling function $f^*: E(k_{1,n}) \to \{0,1,2\}$ for maximum 3-equitable labeling is defined as follows.

$$f^*(u, v_i) = \begin{cases} 0\,; if \quad 1 \le i \le t \\ 1; if \;\; t+1 \le i \le 2t+1 \\ 2; if \;\; 2t+2 \le i \le 3t+2 \end{cases} , \forall\, i = 1,2, \ldots\ldots, n.$$

In all these cases, we have $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$, $\forall$ i, j $\in \{0, 1, 2\}$. Hence, Star graph $k_{1,n}$ admits maximum 3-equitable labeling. Therefore, Star graph $k_{1,n}$, $\forall\, n \ge 2$ is maximum 3-equitable, where $n \in N$. ■

Theorem 2.6 — The star graph $K_{1,n}$,$\forall\, n \ge 2$, is minimum 3-equitable, where $n \in N$.

Proof — Let $K_{1,n}$ be the star graph with the apex vertex u and $\{v_1, v_2, \ldots . v_n\}$ be the n pendent vertices. Cleary $n+1$ and n be the number vertices and edges respectively of star graph $K_{1,n}$.

Case 1 — Let $n \equiv 0(mod\, 3)$ which means that $n = 3t$ for some $t \in N$. The vertex labeling function $f: V(K_{1,n}) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

For the apex vertex u, $f(u) = 2$ and for the pendent vertices we have,

$$f(v_i) = \begin{cases} 2\,; if \quad 1 \le i \le t \\ 1\,; if \;\; t+1 \le i \le 2t \\ 0\,; if \;\; 2t+1 \le i \le 3t \end{cases} , \forall\, i = 1,2, \ldots\ldots, n.$$

Hence, induced edge labeling function $f^*: E(k_{1,n}) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f^*(u, v_i) = \begin{cases} 2\,; if \quad 1 \le i \le t \\ 1; if \;\; t+1 \le i \le 2t \\ 0; if \;\; 2t+1 \le i \le 3t \end{cases} , \forall\, i = 1,2, \ldots\ldots, n.$$

Case 2 — Let $n \equiv 1(mod\, 3)$ which means that $n = 3t+1$ for some $t \in N$. The vertex labeling function $f: V(K_{1,n}) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

For the apex vertex u, $f(u) = 2$ and for the pendent vertices we have,

$$f(v_i) = \begin{cases} 2\,; if \quad 1 \le i \le t \\ 1\,; if \;\; t+1 \le i \le 2t+1 \\ 0\,; if \;\; 2t+2 \le i \le 3t+1 \end{cases} , \forall\, i = 1,2, \ldots\ldots, n.$$

Hence, induced edge labeling function $f^*: E(k_{1,n}) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f^*(u, v_i) = \begin{cases} 2; if \quad 1 \le i \le t \\ 1\,; if \;\; t+1 \le i \le 2t+1 \\ 0; if \;\; 2t+2 \le i \le 3t+1 \end{cases} , \forall\, i = 1,2, \ldots\ldots, n.$$

Case 3 — Let $n \equiv 2(mod\, 3)$ which means that $n = 3t + 2$ for some $t \in N$. The vertex labeling function $f: V(K_{1,n}) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

For the apex vertex u, $f(u) = 2$ and for the pendent vertices we have,

$$f(v_i) = \begin{cases} 2\,;if & 1 \le i \le t \\ 1\,;if & t+1 \le i \le 2t+1 \\ 0\,;if & 2t+2 \le i \le 3t+2 \end{cases}, \forall\, i = 1,2,\ldots\ldots,n.$$

Hence, induced edge labeling function $f^*: E(k_{1,n}) \to \{0,1,2\}$ for minimum 3-equitable labeling is defined as follows.

$$f^*(u, v_i) = \begin{cases} 2\,;if & 1 \le i \le t \\ 1;if & t+1 \le i \le 2t+1 \\ 0;if & 2t+2 \le i \le 3t+2 \end{cases}, \forall\, i = 1,2,\ldots\ldots,n.$$

In all these cases, we have $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$, $\forall$ i, j $\in \{0, 1, 2\}$. Hence, Star graph $k_{1,n}$ admits minimum 3-equitable labeling. Therefore, Star graph $k_{1,n}$, $\forall\, n \ge 2$ is minimum 3-equitable, where $n \in N$. ■

REFERENCES

[1] I. Cahit, *On cordial and 3-equitable labeling of graphs, Until. Math., 37*,(1990), pp. 189 – 198.

[2] J A Gallian, *A dynamic survey of graph labeling, The electronics J. of combin.*, 18(2011), *#Ds6.*

[3] F Harry, Graph Theory, *Addision wisely*, New Delhi (1972).

[4] V J Kaneria, Meera Meghpara and Maulik Khoda, *Geometric mean 3-equitable labeling of some graphs, Int. J. of Scientific Research and Reviews*, 7(1)2018, pp. 245 – 250.