On sg$\omega\alpha$ –homeomorphism in Topological Spaces

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Abstract

The aim of this paper is to introduce a new class of maps called sgωα-closed maps, sgωα-open maps ,sgωα-homeomorphism ,sgωα- homeomorphism in topological spaces. Using these new types of maps, several characterizations and properties have been obtained.*

Keywords: *sgωα-closed map ,sgωα-open map ,sgωα- homeomorphism ,sgωα*-homeomorphism .*

I. INTRODUCTION

Mappings plays an important role in the study of Mathematics. Closed and open mappings are such mappings which are studied for different types of closed sets by various mathematicians for the past many years. Devi et. al. [9] introduced and studied gα- closed maps and αg- closed maps. The concept of semi-generalized homeomorphism and α-homeomorphism were introduced and studied by Devi et. al. [11] in 1994. Benchalli et. al. [3] introduced and studied □α-continuous maps and g□α- continuous maps in topological spaces. Recently Rajeshwari K et.al.[8] introduced and studied the properties of sg□α-closed set.

In this paper we introduce the concept of new class of maps called sgωα-closed maps , sgωα-open maps in topological spaces and compare with other closed maps, open maps. Further we introduce sgωα-homeomorphism sgωα*-homeomorphism in topological spaces. Using these new types of maps, several characterizations and properties have been obtained.

II. PRELIMINARIES

Throughout this paper, the space (X, τ) (or simply X), (Y, σ) (or simply Y) always means a topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), cl(A), int(A) and A^C denote the closure of A, the interior of A and the compliment of A in X respectively.

Definition 2.1: A subset A of a topological space X is called

i) regular open [18] if A =int(cl(A)) and regular closed if A = cl(int(A)).

ii) semi-open set [15] if A$\subseteq$ cl(int(A)) and semi-closed set if int(cl(A)) $\subseteq$ A.

iii) pre-open set [16] if A $\subseteq$ int(cl(A)) and pre-closed set if cl(int(A)) $\subseteq$ A.

iv) α-open set [17] if A $\subseteq$ int(cl(int(A))) and α-closed set if cl(int(cl(A))) $\subseteq$ A.

v) semi-preopen set [20] (= β-open [21]) if A $\subseteq$ cl(int(cl(A))) and semi-pre closed set [20] (= β-closed [21]) if int(cl(int(A))) $\subseteq$ A.

The intersection of all semi-closed (resp. semi-open) subsets of (X, τ) containing A is called the semi-closure (resp. semi-kernel) of A and denoted by scl(A) (resp.sker(A)).Also the intersection of all pre-closed (resp. semi-pre-closed and α-closed) subsets of (X, τ) containing A is called the pre-closure (resp. semi-pre closure and α-closure) of A and is denoted by pcl(A) (resp. spcl(A) and α-cl(A)).

***Definition 2.2*:** A subset A of a topological space X is called a

i) generalized closed (briefly g-closed) set [19] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

ii) generalized semi-closed (briefly gs-closed) set[22] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

iii) α-generalized closed (briefly αg-closed) set [23] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

iv) generalized pre-closed (briefly gp-closed) set [32] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

v) generalized semi-pre-closed (briefly gsp-closed) set[25] if spcl(A) $\subseteq$ U whenever A$\subseteq$ U and U is open in X.

vi) generalized pre-regular-closed (briefly gpr-closed) set[26] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular-open in X.

vii) g^*- closed set [33] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.

viii) α-generalized regular closed (briefly αgr-closed) set [28] if αcl(A) $\subseteq$ U whenever A$\subseteq$U and U is regular open in X .

ix) g^*- pre closed (briefly g^*p- closed) set [29] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g- open in X.

x) ω-closed [31] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X.

xi) ωα-closed [2] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ω-open in X.

xii) generalized ωα-closed (briefly gωα-closed) set[30] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ωα-open in X.

xiii) semi generalized ωα-closed (briefly sgωα-closed) set [8] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ωα-open in X.

Definition 2.3: A function f: (X, τ) → (Y, σ) is said to be

i) semi-continuous [6] if f^{-1}(A) is semi-closed in (X, τ) for every closed set A of (Y, σ).

ii) g-continuous[1] if f^{-1}(A) is g-closed in (X, τ) for every closed set A of (Y, σ).

iii) α-continuous [4] if f^{-1} (A) is α-closed set in (X, τ) for every closed set A of (Y, σ).

iv) ωα-continuous [3] if f^{-1}(A) is ωα-closed set in (X, τ) for every closed set A of (Y, σ).

v) gωα-continuous [13] if f^{-1}(A) is gωα-closed set in (X, τ) for every closed set A of (Y, σ) .

vi) sgωα-continuous [14] if f^{-1}(A) is sgωα-closed set in (X, τ) for every closed set A of (Y, σ) .

vii) rgwα-continuous [27] if f^{-1}(A) is rgwα- closed in (X, τ) for every closed set A of (Y, σ) .

viii) sgωα-irresolute [14] if $f^{-1}(A)$ is sgωα-closed set in (X, τ) for every sgωα-closed set A of (Y, σ).

Definition 2.4: A function f: (X, τ) → (Y, σ) is called

i) g-closed [12] (g-open) if f(A) is g-closed (g-open) in (Y, σ) for every closed(open) set A in (X, τ).

ii) gs-closed [7] (gs-open) if f(A) is gs-closed (gs-open) in (Y, σ) for every closed(open) set A in (X, τ).

iii) αg-closed [9] (αg-open) if f(A) is αg-closed (αg-open) in (Y, σ) for every closed(open) set A in (X, τ).

iv) g□α-closed [5] (g□α-open) if f(A) is g□α-closed (g□α-open) in (Y, σ) for every closed (open) set in (X, τ).

Definition 2.5**:** A bijection function f: (X, τ) → (Y, σ) is called

i) g-homeomorphism [24] if f is both g-continuous and g-open.

ii) gs-homeomorphism [10] (sg-homeomorphism) if f is both gs-continuous(sg-continuous) and gs-open(sg-open).

iii) α-homeomorphism [11] if f is both α-continuous and α-open.

III. sgωα-CLOSED AND SGωA-OPEN FUNCTIONS IN TOPOLOGICAL SPACES

In this section we introduce semi generalized ωα-closed (briefly sgωα-closed) and semi generalized ωα-open (briefly sgωα-open) functions in topological spaces and obtained some of their properties.

Definition 3.1 **:** A function f : X → Y is called sgωα -closed (briefly sgωα-closed) function if the image of every closed set in X is sgωα -closed in Y.

Theorem 3.2**:** Every closed function (resp. α-closed function) is sgωα-closed function.

Proof: The proof follows from the definition.

The converse of the above theorem need not be true as seen from the following examples.

Example 3.3**:** Let X ={a, b,c}, τ = {X, ϕ,{a},{a, b}} and σ = {Y, ϕ, {a},{b, c}}.Then the identity function f: X → Y is sgωα-closed but not a closed function, since for the closed set {c} in X, f({c}) = {c} is not a closed set in Y but it is sgωα-closed set in Y.

Example 3.4**:** Let X ={a, b, c}, τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {a}, {b, c}}. Define a function f: X → Y to be identity function. Then the function f is sgωα-closed function but not a α- closed function, since for the closed set {c} in X, f({c}) = {c} is not α-closed in Y but it is sgωα-closed set in Y.

Theorem 3.5: Every sgωα-closed function is gs-closed(respectively gsp-closed) function but not conversely.

Proof: Since every sgωα-closed set is gs-closed(resp. gsp-closed) set it follows that every sgωα -closed function is gs-closed(resp. gsp-closed).

Example 3.6: Let X =Y={a, b, c}, τ = {X, ϕ,{a},{b}, {a, b}} and σ = {Y, ϕ,{a}}. Define a function $f : X \rightarrow Y$ to be identity function. Then the function f is gs-closed(resp. gsp-closed) but not sgωα-closed, since for the closed set{a,c} in X, f({a, c}) = {a, c} is not sgωα-closed in Y but it is gs-closed(resp. gsp- closed) set in Y.

Remark 3.7: The concept of sgωα-closed function is independent of the concept of functions namely pre-closed, semi-pre closed, g-closed, gp-closed, αg-closed, gpr-closed, αgr-closed, g^*-closed, g^*p-closed, $\omega\alpha$-closed functions as seen from the following examples.

Example 3.8: In Example 3.6, the function f is $\omega\alpha$-closed, gp-closed, g-closed, αgr-closed functions but not sgωα-closed function ,as the closed set {a, c} in X, f({a, c}) = {a, c} is ωα-closed ,gp-closed, g-closed, αgr-closed but not sgωα-closed in Y.

Example 3.9: Let X={a, b, c},Y={ a, b, c, d},τ = {X, ϕ, {a, c}, σ ={Y,ϕ,{b, c},{b, c, d},{a, b, c}}. Define a function $f : X \rightarrow Y$ to be identity function. Then the function f is pre-closed, semi-pre closed but not sgωα-closed function, as the closed set {b} in X , f({b}) ={b} is pre-closed, semi-pre closed but not sgωα-closed in Y.

Example 3.10 : Let X=Y ={a, b, c},τ = {X,ϕ,{c}}and σ ={Y,ϕ,{a},{a, c}}Define a $f : X \rightarrow Y$ to be identity function. Then the function f is g-closed , g^*-closed , g^*p- closed but not sgωα- closed , as the closed set {a ,b} in X, f({a ,b}) = {a ,b} is g-closed , g^*-closed , g^*p-closed but not sgωα-closed in Y.

Example 3.11: In the Example **3.24** an identity function $f : X \rightarrow Y$ is sgωα-closed but not pre-closed, semi-pre closed, g-closed, gpr-closed, $\omega\alpha$-closed ,αgr-closed , g^* -closed, g^*p-closed, gp-closed function.

Theorem 3.12: A function $f : X \rightarrow Y$ is sgωα-closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a sgωα-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Suppose f is sgωα-closed function. Let S be a subset of Y and U be an open set of X such that $f^{-1}(S) \subseteq U$. Then V= Y- f(X - U) is sgωα- open set containing S such that $f^{-1}(V) \subseteq U$.

Conversely, suppose that F is a closed set in X. Then $f^{-1}(Y-f(F)) = X - F$ is open. By hypothesis, there is a sg□α-open set V of Y such that $Y - f(F) \subseteq V$ and $f^{-1}(V) \subseteq X - F$. Therefore $F \subseteq X - f^{-1}(V)$. Hence $Y - V \subseteq f(F) \subseteq f(X - f^{-1}(V)) \subseteq Y - V$, which implies, f(F) = Y – V. Since Y–V is sgωα-closed, f(F) is sgωα- closed. Thus f is sgωα- closed function.

Remark 3.13: The composition of two sgωα-closed functions need not be a sgωα-closed function as seen from the following example.

Example 3.14: Let X = Y = Z = {a, b, c}, τ = {ϕ,{a},{a, c},X}, σ = {ϕ,{a},Y}and γ = {ϕ, {a, b}, Z}. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be identity functions.Then f and g are both sgωα-closed functions, but their composition $gof : X \rightarrow Z$ is not a sgωα-closed function, since for the closed set {b} in X, (gof)({b}) = g(f({b}) = g{b} = {b} is not sgωα-closed in Z.

Theorem 3.15: If $f : X \rightarrow Y$ is closed and $g : Y \rightarrow Z$ is sgωα-closed, then $gof : X \rightarrow Z$ is sgωα- closed function.
Proof: Let V be any closed set in X. Since f is closed, f(V) is closed in Y. Again since g is sgωα- closed function, g(f(V)) is sgωα-closed set in Z. But g(f (V)) = (gof) (V) is sgωα-closed in Z. Therefore $gof : X \rightarrow Z$ is sgωα-closed function.

***Theorem3.16*:** If f : X → Y and g : Y → Z are closed and sgωα-closed maps then their composition gof : X → Z is sgωα-closed function.

***Theorem 3.17* :** Let f : X → Y and g : Y → Z be two functions and gof : X → Z is sgωα-closed. Then the following statements are true.

i) If f is continuous and surjective, then g is sgωα-closed.

ii) If g is sgωα-irresolute and injective, then f is sgωα-closed.

Proof: i) Let F be a closed set in Y. Then $f^{-1}(F)$ is closed in X as f is continuous. Since gof is sgωα- closed and f is surjective, (gof) ($f^{-1}(F)$) is sgωα- closed set in Z. That is (gof)($f^{-1}(F)$) = g(f ($f^{-1}(F)$)) = g(F) is sgωα-closed set in Z. Hence g is sgωα-closed function.

ii) Let H be a closed set in X. Since gof is closed (gof)(H) is an sgωα- closed set in Z. Since g is sgωα -irresolute and injective, g^{-1}((gof(H)) = g^{-1}(g(f(H))) = f(H) is sgωα-closed in Y. Thus f is sgωα-closed function.

***Definition 3.18*:** A function f : X →Y is called sgωα-open function if the image of every open set in X is sgωα-open in Y.

***Definition 3.19*:** A function f : X →Y is called sgωα-closed function if the image of every closed set in X is sgωα-closed in Y.

***Theorem 3.20*:** Every open function(resp. α-open) is sgωα-open function but not conversely.

Proof: Let f : X → Y be an open(α-open) function. Let G be an open set in X. Then f(G) is open (α-open) in Y. Therefore f(G) is sgωα-open in Y. Hence f is sgωα-open function.

The converse of the above theorem need not be true as seen from the following example.

***Example 3.21*:** Let X =Y= {a,b, c}, τ ={X, ϕ,{a}, {a, b}}and σ = {Y, ϕ,{a, b}} Then the identity function f : X → Y is not an open function, since for the open set {a} in X, f({a}) = {a} is not open in Y. However f is sgωα-open.

***Example 3.22*:** Let X ={a, b, c}, τ = {X, ϕ, {a ,b}} and σ = {Y, ϕ,{a},{b, c}}. Define a function f : X → Y to be identity function. Then the function f is sgωα-open function but not an α-open function, since for the open set {a ,b} in X, f({a ,b}) = {a ,b} is not α- open in Y but it is sgωα - open set in Y.

***Theorem 3.23*:** Every gωα-open function is sgωα-open function but not conversely.

Proof : Let f : X → Y be a gωα- open function. Let G be an open set in X. Then f(G) is gωα- open in Y. So f (G) is sgωα- open in Y. Hence f is sgωα- open function.

The converse of the above theorem need not be true as seen from the following example.

***Example 3.24*:** Let X= Y={a ,b, c}, τ = {X, ϕ,{a, c}}and σ ={Y, ϕ,{a},{b},{a, b}}.Let f : X → Y be an identity function. Then f is sgωα-open function but not gωα-open function since for the open set {a,c} in X , f({a, c})={a, c} is sgωα-open but not gωα-open in Y.

***Theorem 3.25*:** Every sgωα-open function is gs-open (resp.gsp-open) function but not conversely.

***Proof* :** Let f : X → Y be a sgωα-open function. Let G be an open set in X. Then f(G) is sgωα - open in Y as f is sgωα-open function. Therefore, f(G) is gs-open (resp. gsp-open) in Y. Hence f is gs-open (resp. gsp-open) function. The converse of the above theorem need not be true as seen from the following example.

Example 3.26: Let X =Y={a, b, c}, τ = {X, ϕ,{a},{b}, {a, b}} and σ = {Y, ϕ,{a}}. Define a function f : X → Y as f(a) = c, f(b) = b, f(c) = a .Then the function f is gs-open(resp. gsp-open) but not sgωα-open.

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***Theorem 3.27*:** Let a function f : X → Y be bijection. Then the following statements are equivalent.

i) f^{-1}: Y → X is sgωα-continuous.

ii) f is sgωα-open .

iii) f is sgωα-closed.

Proof: (i) ⇒ (ii) : - Let G be an open set in X. By (i), $(f^{-1})^{-1}(G) = f(G)$ is sgωα- open in Y and so f is sgωα-open.
(ii) ⇒ (iii): - Let F be a closed set of X. Then X-F is open set of X. By hypothesis (ii), f (X-F) is sgωα-open in Y. That is f (X-F) = Y- f(F) is sgωα-open in Y. Therefore f(F) is sgωα-closed in Y. Hence f is sgωα-closed function.
(iii) ⇒ (i):- Let F be a closed set of X. Then by (iii), f(F) is sgωα-closed set in Y. But $f(F) = (f^{-1})^{-1}(F)$ is sgωα-closed set in Y. Therefore f^{-1}: Y → X is sgωα-continuous function.

***Theorem 3.28*:** A function f : X → Y is sgωα-open if and only if for any subset S of Y and for any closed set F containing $f^{-1}(S)$, there exists sgωα- closed set K of Y containing S such that $f^{-1}(K) \subseteq F$.
Proof : Suppose f : X → Y is sgωα-open function. Let S be a subset of Y and F be a closed set of X containing $f^{-1}(S)$. Then K = Y- f(X- F) is a sgωα-closed set containing S such that $f^{-1}(K) \subseteq F$.

Conversely, suppose that U is an open set of X. Then $f^{-1}(Y- f(U)) \subseteq X- f^{-1}[f(U)] \subseteq X - U$ and X -U is closed. By hypothesis, there is a sgωα- closed set K of Y such that $Y-f(U) \subseteq K$ and $f^{-1}(K) \subseteq X - U$. Therefore $U \subseteq X - f^{-1}(K)$. Hence $Y- K \subseteq f(U) \subseteq f [X - f^{-1}(K)] \subseteq Y-K$, which implies $f (U) \subseteq Y - K$. Since Y - K is sgωα-open, f(U) is sgωα-open and thus f is sgωα- open function.

IV. sgωα- HOMEOMORPHISMS IN TOPOLOGICAL SPACES

In this section we introduce the concept of sgωα-homeomorphisms in topological spaces and obtained some of their properties.

***Definition 4.1*:** A bijective function f : (X,τ) → (Y, σ) is called sgωα -homeomorphism if f and f^{-1} are sgωα-continuous functions.

Example 4.2: Let X = Y ={a, b, c}, τ = {X, ϕ,{a},{a, c}} and σ ={Y, ϕ, {a,b}},Define a function f : (X, τ) → (Y, σ) by f(a)= a, f(b)= c and f(c) = b. Then the function f and f^{-1} are sgωα - continuous functions. Therefore f is a sgωα-homeomorphism.

***Theorem 4.3* :** Let f : (X, τ) → (Y, σ) be a bijective function. Then following statements are equivalent:

i) f is sgωα-homeomorphism.

ii) f is sgωα-continuous and sgωα-open map.

iii) f is sgωα-continuous and sgωα-closed map.

Proof: Follows from the definition.

***Theorem 4.4* :** Every homeomorphism is sgωα-homeomorphism.

Proof: Let f : (X,τ) → (Y,σ) be a homeomorphism. Then f and f^{-1} are continuous functions. Since every continuous function is sgωα-continuous it follows that f and f^{-1} are sgωα-continuous functions. Hence f is sgωα-homeomorphism.

The converse of the above theorem need not be true as seen from the following example.

Example 4.5 **:** In Example 4.2, the function f is sgωα- homeomorphism but not a homeomorphism.

Theorem 4.6**:** If f : (X, τ) → (Y, σ) is sgωα- homeomorphism then f is gsp- homeomorphism.

Proof: Let f : (X,τ) → (Y, σ) be sg□α-homeomorphism. Then f and f^{-1} are sgωα-continuous functions. But every sgωα-continuous is gsp-continuous. Therefore, f and f^{-1} are gsp-continuous functions. Hence f is gsp-homeomorphism.

The converse of the above theorem need not be true as seen from the following example.

Example 4.7**:** Let X = Y = {a, b, c}, τ = {X, ϕ, {a}} and σ = {Y, ϕ, {a},{b, c}}. Define a function f : (X,τ) → (Y, σ) by f(a) = c, f(b) = a and f(c) = b. Then the function f is gsp-homeomorphism but not a sgωα- homeomorphism, since for the open set {a} in (Y, σ) , f^{-1}({a}) = {b} is not sgωα -open in (X, τ) but it is gsp-open in (X, τ).

Theorem 4.8 **:** Every sgωα-homeomorphism is gs- homeomorphism, but not conversely.

Proof: The proof follows from the definition.

Example 4.9**:** In the example 4.7 f is gs-homeomorphism but not sgωα- homeomorphism.

Theorem 4.10**:** Every α-homeomorphism is sgωα-homeomorphism but not conversely.

Proof : Let f : (X, τ) → (Y, σ) be a α-homeomorphism. Then f and f^{-1} are α-continuous functions. Since every α-continuous function is sgωα-continuous function, if follows that f and f^{-1} are sgwα - continuous functions. Hence f is sgωα-homeomorphism.

Example 4.11**:** Let X = Y = {a, b, c}, τ = {X, ϕ, {a}} and σ = {Y, ϕ, {a ,b} }. Define a function f : (X,τ) → (Y, σ) to be identity function. Then the function f is sgωα-homeomorphism but not a α- homeomorphism, since for the open set {a} in X , f{a} = {a} is not α-open in (Y, σ) but it is sgωα-open in (Y, σ) .

Theorem 4.12**:** Every semi homeomorphism is sgωα-homeomorphism but not conversely.

Proof **:** The proof follows from the definition and the fact that every semi closed set is sgωα-closed.

Example 4.13**:** In the example 4.11 f is sgωα-homeomorphism but not semi homeomorphism, since for the open set {a} in X, f{a = {a} is not semi open in (Y,σ) but it is sgωα- open in (Y, σ)

Remark 4.14 **:** The composition of two sg$\omega\alpha$- homeomorphism need not be sg$\omega\alpha$ -homeomorphism in general as seen from the following example.

Example 4.15 **:** Let X = Y = Z = {a ,b ,c} with topologies τ = {X, ϕ, {a ,b}} , σ = {Y, ϕ, {a}, {a ,b}} and η = { Z , ϕ, {a}, {a ,c}} . Let f : (X,τ) $\rightarrow$ (Y, σ) and g : (Y, σ) $\rightarrow$ (Z, η) be the identity maps. Then f and g are sg$\omega\alpha$-homeomorphisms. But gof : (X,τ) $\rightarrow$ (Z, η) is not sg$\omega\alpha$- homeomorphism . Here gof is sg$\omega\alpha$-open map, but not sg$\omega\alpha$-continuous since for the open set {a ,c} in Z,(gof)$^{-1}$({a ,c }) = {a ,c} is not sg$\omega\alpha$-open.

Definition 4.16**:** A bijection f :(X,τ) $\rightarrow$ (Y, σ) is said to be sg$\omega\alpha^*$- homeomorphism if both f and f^{-1} are sg$\omega\alpha$-irresolute.The spaces (X,τ) and (Y, σ) are sg$\omega\alpha^*$- homeomorphism if there exists sg$\omega\alpha^*$-homeomorphism from (X,τ) onto (Y, σ).We denote the family of all sg$\omega\alpha^*$-homeomorphism of a topological space (X,τ) onto itself by sg$\omega\alpha^*$-h(X,τ).

Theorem 4.17**:** Every sg$\omega\alpha^*$- homeomorphism is sg$\omega\alpha$- homeomorphism but not conversely.

Proof: Follows from the definition.

Example 4.18**:** Let X = Y = {a ,b ,c} **,** τ = {X, ϕ, {a}, {b}, {a ,b}} and σ = {Y, ϕ, {a}, {b ,c}} . Define an identity function f : (X,τ) $\rightarrow$ (Y, σ).Then f is sg$\omega\alpha$-homeomorphism but not sg$\omega\alpha^*$- homeomorphism .

Theorem 4.19 **:** Let f: (X,τ) $\rightarrow$ (Y, σ) and g: (Y, σ) $\rightarrow$ (Z, η) be sg$\omega\alpha^*$- homeomorphism. Then their composition gof : (X,τ) $\rightarrow$ (Z, η) is also sg$\omega\alpha^*$- homeomorphism.

Proof : Let f and g be sg$\omega\alpha^*$-homeomorphism. Then f and g are sg$\omega\alpha$- irresolute. Let U sg$\omega\alpha$ - closed set in (Z, η). Since g is sg$\omega\alpha$-irresolute , g^{-1} (U) is sg$\omega\alpha$-closed in (Y, σ) . This implies that f^{-1}(g^{-1}(U)) = (gof)$^{-1}$ (U) is sg$\omega\alpha$-closed in (X, τ), since f is sg$\omega\alpha$- irresolute. Hence gof is sg$\omega\alpha$-irresolute. Also for a sg$\omega\alpha$-closed set V in (X, τ), gof (V) = g(f(V)). By hypothesis ,f(V) is sg$\omega\alpha$-closed set in (Y, σ). This implies that g(f(V)) is sg$\omega\alpha$-closed set in (Z η) .Thus gof(V) is sg$\omega\alpha$-closed set in (Z, η) which implies that (gof)$^{-1}$ is sg$\omega\alpha$-irresolute. Also gof is bijection. This proves gof is sg$\omega\alpha^*$-homeomorphism.

Theorem 4.20**:** The set sg$\omega\alpha^*$-h(X, τ) from (X, τ) onto itself is a group under the law of composition of functions.

Proof **:** Let f and g $\in$ sg$\omega\alpha^*$-h(X, τ). By Theorem 4.18, gof $\in$ sg$\omega\alpha^*$-h(X, τ). We know that the composition of functions is associative. The identity function I : (X, τ) $\rightarrow$ (X, τ) also belongs to sg$\omega\alpha^*$-h(X, τ) and is an identity element. If f $\in$ sg$\omega\alpha^*$-h(X, τ) then f^{-1} $\in$ sg$\omega\alpha^*$-h (X, τ). Thus sg$\omega\alpha^*$-h(X, τ) is a group under the law of composition of functions.

V. CONCLUSION

The notion of sets and mappings in topological spacse are extremely advanced and used in many areas such as quantum mechanics, computer science, semantics. By researching generalizations of closed sets, some new functions have been founded and they seem to be useful in the study of topological data analysis, computer networks. Therefore, sg□α-open and closed maps, sg□α-homeomorphisms defined using sg□α-closed sets will have possibilities of many applications in computer networks, topological data analysis.

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REFERENCES

[1] K. Balachandran, P. Sundaram & H. Maki, On Generalized Continuous Maps in Topological Spaces, Mem. Fac. Sci. Kochi Univ. *Ser. A. Math.*, 12, 5-13, 1991.
[2] S. S. Benchalli, P. G. Patil and T. D. Rayanagoudar ,□ α-closed sets in topological spaces, *The Global J. Appl. Math . and Math.Sci*.,2, 53-63, 2009.
[3] S. S. Benchalli, P. G. Patil. Some New Continuous Maps in Topological Spaces, *J. Adv. Studies in Topology*, 1, 16-21, 2010.
[4] A. S. Mashour, M. E. Abd. EI- Monsef and S.N.EI-Deeb ,α -open mappings, *Acta.Math.Hungar.*, 41, 213-218, 1983.
[5] S. S. Benchalli, P. G. Patil and P. M. Nalwad , g□α- closed function in topological spaces, *Jl. of Scientific and Innovative Mathematical Research.*, 3, 11-17,2015.
[6] N. Biswas, On Characterization of Semi Continuous Mappings, Alti. Accad. Naz Lincei. *Rend. Cl. Sci. Fsi. Mat.Natur,* 48, 399-402, 1970.
[7] R. Devi, K Balachandran and H. Maki, Semi –Generalized Closed Maps and Generalized Semi- Closed Maps, Mem. Fac. Sci. Kochi Univ. *ser. A. Math.*, 14, 41-54, 1993.
[8] Rajeshwari K, Dr. T.D. Rayanagoudar and Sarika M. Patil, On sg□α- closed sets in topological spaces, *The Global Jl. of Pure Appl. Math.*, 13(9), 5491-5503, 2017.
[9] R. Devi , K .Balachandran and H .Maki, Generalized α -Continuous Maps and $\alpha -$ Generalized Continuous Maps, *Indian Jl. of Pure Appl. Math.*, 29(1), 37-49, 1998.
[10] R. Devi, K. Balachandran and H. Maki, Semi Generalized Homeomorphism and Generalized Semi Homeomorphism in topological spaces, *Indian Jl. of Pure Appl. Math,* 26(3), 271-284, 1995.
[11] R. Devi, Studies on generalizations of closed maps and homeomorphisms in topological spaces, Ph.D thesis, Bharathiar University, Coimbatore (1994).
[12] S. R. Malghan, Generalized Closed Maps, Jl. Karnatak Univ. Sci., 27, 82-88, 1982.
[13] S. S. Benchalli, P. G. Patil and P. M. Nalwad, Some weaker forms of continuous functions in topological spaces. *J. of Advanced Studies in Topology,* 7, 101-109,2016.
[14] Rajeshwari K., T. D. Rayanagoudar , On semi generalized ωα-continuous functions in topological Spaces (communicated).
[15] N. Levine, Semi-Open sets and Semi Continuity in topological spaces, *Amer. Math. Monthly*, 70, 36-41, 1963.
[16] A. S. Mashour, M. E. Abd. EI- Monsef and S.N. EI-Deeb, On pre-continuous and weak pre-continuous mappings, *Proc. Math and Phys. Soc. Egypt*, 53, 47-53, 1982.
[17] O. Njastad, On Some Classes of Nearly Open Sets, *Pacific. Jl. Math.*,15, 961-970, 1965.
[18] M. Stone, Application of the theory of Boolean rings to general topology, *Trans. Amer .Math. Soc*., 41, 374-481, 1937.
[19] N. Levine, generalized closed sets in Topology, Rend. Circ..Math. Palermo, 19(2), 89-96, 1970.
[20] D. Andrijivic, Semi pre- open sets, Mat. Vesnic.,24-32, 1986.
[21] M. E. Abd. EI- Monsef and S.N.EI-Deeb and R. A. Mahamoud, β-open sets and β -Continuous Mappings, Bull, Fac. Sci. Assint. Univ., 12, 77-90, 1983.
[22] S.P. Arya and T. M. Nour, Characterizations of s-normal spaces, Indian J. Pure. Math,28.
[23] H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized closed sets and generalized closed sets, Mem. Fac. Sci. Kochi Univ. *Ser. AMath.*, 15, 51-63, 1994.
[24] H. Maki, P.Sundaram and K. Balachandran, On Generalized Homeomorphism in Topological Spaces, Bull. Fukuoka Univ. Ed Part III, 40, 13-21, 1991.
[25] J. Dontchev, On Generalizing Semi- Preopen Sets, Mem, Fac. Sci. Kochi Univ. *Ser. A. Math.*, 35-48,1995.
[26] Y. Gnanambal, On Grnerelized Pre Regular Closed Sets in Topological Spaces, *Indian Jl. of Pure Appl. Math*, 28(3), 351-360, 1997.
[27] R.S. Wali and Vijayalaxmi R .Patil ,On rgwα- Continuous maps and rgwα- Irresolute maps in topological spaces ,International Jl .of Mathematics Trends and Technology,45(1),1-10,2017.
[28] M. K. R. S. Veera Kumar, On α- generalsed- regular closed sets, *Indian Journal of Mathematics*, 44(2), 165-181, 2002.
[29] M. K. R. S. Veera Kumar, g –pre-closed Sets, Actaciencia Indica, 28(1), 51-60,2002.
[30] S. S. Bnchalli, P. G. Patil and P. M. Nalwad. Generalized □α- closed sets in Topological Spaces, Jl. *New Results in Science*, 7, 7-19, 2014.
[31] P.Sundaram and M. Sheik John, On □- closed Sets in Topology, *Actaciencia. Indica*,4, 389-392, 2000.
[32] H. Maki, J. Umehara and T. Noiri. Every topological space is pre-$T_{1/2}$, Mem. Fac. Sci. Kochi Univ. *Ser. A. Math*., 17, 33-42, 1996
[33] M. K. R. S. Veera Kumar, between closed sets and g-closed sets, Mem. Fac. Sci. Kochi Univ (Math), 21, 1-19, 2000.