The compression and explosion of Egyptian pyramids by numbers

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Abstract: Science fiction shows the aspiration of humans to break from our Universe and look into Multiverse. Although nowadays there are scientific theories for the existence of Multiverse, we do not have technical at our disposal to investigate Multiverse. Now, we offer a method to have a look into Multiverse by the mathematical explosion of real numbers. Let us start by compressing real numbers and applying the compression for Egyptian piramids. Finally, by exploding real numbers, we are able to explode pyramids, too. One of these exploded piramids will break from our universe. Moreover, Remarks 2 to 8 contain some hypothesis the future.

INTRODUCTION

Let us denote by \mathbb{R} the set of real numbers. The model we offer for our universe is very simple: an empty space, only. (Lacking mass, heat, entropy and so on.) It is the three – dimensional Euclidean space, that is

 $\mathbb{R}^3 = \{ The \ set \ of points \ P = (x, y, z) \ where \ x, y \ and z \ are \ real \ numbers \}.$

If the third coordinate is 0, we may use

 $\mathbb{R}^2 = \{ The \ set \ of \ points \ P = (x, y, 0) \ where \ x \ and \ y \ are \ real \ numbers \}.$

Let us consider the pyramid Δ_a^m with base edge a and height m where a, m are arbitrary positive numbers. The base \blacksquare_a is a closed square described by

(0.1)
$$\blacksquare_a = \left\{ P = (x, y) \in \mathbb{R}^2 | |x| + |y| \le \frac{a}{\sqrt{2}} \right\}.$$

So, for the pyramid

(0.2)
$$\Delta_a^m = \left\{ P = (x, y, z) \in \mathbb{R}^3 \middle| \begin{cases} (x, y) \in \blacksquare_a \\ 0 \le z \le m \left(1 - \frac{|x| + |y|}{a} \sqrt{2}\right) \end{cases} \right\}$$

is obtained.



Fig.1

Egyptian pyramids. The largest is pyramid $\Delta^{146,5\,metre}_{230,34\,metre}$ constructed between c. 2580-2560 B.C. (see the Great Pyramid of Giza – Wikipedia, 2019.08.28: 13.27) Pharaoh Cheops' Pyramid.

I. THE MOST IMPORTANT FORMULAE OF COMPRESSION VITH PARAMETER σ .

Let σ be a given positive number and x an arbitrary real number. Its σ – compressed is denoted and defined by

(1.1)
$$\underline{x}_{\sigma} = \sigma \tanh \frac{x}{\sigma} = \sigma \frac{e^{\frac{x}{\sigma}} - e^{-\frac{x}{\sigma}}}{e^{\frac{x}{\sigma}} + e^{-\frac{x}{\sigma}}},$$

where σ is the compression – parameter. Clearly, $-\sigma < x_{\sigma} < \sigma$.

Considering the real number ξ , such that $-\sigma < \xi < \sigma$ and $\xi = \underline{x}_{\sigma}$ by (1.1)

(1.2)
$$x = \sigma \tanh^{-1} \frac{\xi}{\sigma} = \frac{\sigma}{2} \ln \frac{\sigma + \xi}{\sigma - \xi}$$

is obtained. So, $x \leftrightarrow \underline{x}_{\sigma}$ is a mutual and unambiguous mapping between the set \mathbb{R} and the open interval $]-\sigma,\sigma[$. Therefore, this interval will be denoted by $\underline{\mathbb{R}}_{\sigma}$ and called the $\sigma-compressed$ of the set \mathbb{R} or the $\sigma-compressed$ set of the set of real numbers. Clearly, $\underline{\mathbb{R}}_{\sigma}$ is a proper subset of \mathbb{R} , so we can write

$$\mathbb{R}_{\sigma} \subset \mathbb{R}$$

With respect to the fact that the geometrical model of the set of real numbers is the number line, we may mention $\underline{\mathbb{R}}_{\sigma}$ as the σ -compressed number line. Here, the points $-\sigma$ and σ are discriminators, so they do not belong into the compressed number line. They play the role of $-\infty$, ∞ , there.

We define the basic – operations on $\underline{\mathbb{R}}_{\sigma}$. For any pair ξ , $\eta \in \underline{\mathbb{R}}_{\sigma}$ sub - addition is defined by

(1.4)
$$\xi \bigoplus_{\sigma} \eta = \frac{\xi + \eta}{1 + \frac{\xi \cdot \eta}{\sigma^2}}$$

and sub - multiplication by

(1.5)
$$\xi \odot_{\sigma} \eta = \sigma \tanh \left(\sigma \left(\tanh^{-1} \frac{\xi}{\sigma} \right) \left(\tanh^{-1} \frac{\eta}{\sigma} \right) \right).$$

It is already proved that for any pair $x, y \in \mathbb{R}$

$$(1.6) x + y_{\sigma} = \underline{x}_{\sigma} \oplus_{\sigma} y_{\sigma}$$

and

$$(1.7) \underline{x \cdot y_{\sigma}} = \underline{x_{\sigma}} \odot_{\sigma} \underline{y_{\sigma}}$$

are valid. (See [1], (4.1.24) and (4.1.25).)

With respect to (1.3) we are able to use the ordering relation $,\leq$ " (less than or equal to) in $\underline{\mathbb{R}}_{\sigma}$, too. It is well-known that $(\mathbb{R},+,\cdot,\leq)$ is an ordered field. Considering (1.1) to (1.7) we have that $(\underline{\mathbb{R}}_{\sigma},\oplus_{\sigma},\odot_{\sigma},\leq)$ is an ordered field, too. We remark that if x and y are arbitrary real numbers then

$$\lim_{\sigma \to \infty} \underline{x}_{\sigma} = x ,$$

(1.9)
$$\lim_{\sigma \to \infty} (\underline{x}_{\sigma} \oplus_{\sigma} y_{\sigma}) = x + y$$

and

(1.10)
$$\lim_{\sigma \to \infty} (\underline{x}_{\sigma} \odot_{\sigma} \underline{y}_{\sigma}) = x \cdot y . \quad \text{(See [1], Theorem 4.1..31.)}$$

Moreover, if real numbers |x + y| and $|x \cdot y|$ are quite small, then

$$\lim_{\sigma \to \infty} (x \bigoplus_{\sigma} y) = x + y$$

and

(1.12)
$$\lim_{\sigma \to \infty} (x \odot_{\sigma} y) = x \cdot y$$
. (See [1], Theorem 4.1.34*.)

II. THE EXPLOSION OF COMPRESSED NUMBERS

Having an arbitrary $\xi \in \mathbb{R}_{\sigma}$ the real number x given by (1.2) is called the σ – exploded of x and denoted by

(2.1)
$$\check{\xi}^{\sigma} = \sigma \tanh^{-1} \frac{\xi}{\sigma} , \quad -\sigma < \xi < \sigma.$$

Hence, by (1.1) the inversion formula

(2.2)
$$\underline{\left(\xi^{\sigma}\right)}_{\sigma} = \xi \qquad , \quad \xi \in \underline{\mathbb{R}}_{\sigma} ,$$

is obtained. Conversely, the other inversion formula

$$(2.3) (\underline{x}_{\sigma})^{\sigma} = x , x \in \mathbb{R}$$

is obtained, too

Assuming that we have the kinf of civilisation where the sub – operations are known, by (2.3) and (1.6) we can write

(2.4)
$$x + y = \underbrace{x_{\sigma} \oplus_{\sigma} y_{\sigma}}^{\sigma} , \quad x, y \in \mathbb{R}.$$

Moreover, if $x = \check{\xi}^{\sigma}$ and $y = \check{\eta}^{\sigma}$ then

(2.5)
$$\xi^{\sigma} + \check{\eta}^{\sigma} = \xi \bigoplus_{\sigma} \eta^{\sigma} , \quad \xi, \eta \in \underline{\mathbb{R}}_{\sigma},$$

is valid.

Similarly, by (2.3) and (1.7) we have

(2.6)
$$x \cdot y = \underline{x_{\sigma} \odot_{\sigma} y_{\sigma}}^{\sigma} \quad , \quad x, y \in \mathbb{R} .$$

Moreover, if
$$x=\check{\xi}^{\sigma}$$
 and $y=\check{\eta}^{\sigma}$ then
$$(2.7) \qquad \qquad \check{\xi}^{\sigma}\cdot\check{\eta}^{\sigma}=\widecheck{\xi \odot_{\sigma}}\eta^{\sigma} \quad , \qquad \xi\,,\eta\in\underline{\mathbb{R}}_{\sigma}\,,$$
 is valid.

III. THE COMPRESSION OF OUR UNIVERSE

We consider the σ – compressed of our universe as a pointwise compression of \mathbb{R}^3 , that is

$$(3.1) \qquad \underline{\mathbb{R}^{3}}_{\sigma} = \left\{ (\xi, \eta, \zeta) \middle| \begin{cases} \xi = \underline{x}_{\sigma} \\ \eta = \underline{y}_{\sigma} \text{ where } P = (x, y, z) \in \mathbb{R}^{3} \\ \zeta = \underline{z}_{\sigma} \end{cases} \right\}$$

such that σ – *compressed* of the point P is $\underline{P}_{\sigma} = (\xi, \eta, \zeta)$. Especially, the σ – *compressed* of origo \mathcal{O} in our universe is itself, that is $\underline{\mathcal{O}}_{\sigma} = (0,0,0)$). Casting a glance of (1.3), we have

$$\underline{\mathbb{R}^3}_{\sigma} = \left\{ (\xi, \eta, \zeta) \in \mathbb{R}^3 \middle| \begin{cases} -\sigma < \xi < \sigma \\ -\sigma < \eta < \sigma \\ -\sigma < \zeta < \sigma \end{cases} \right\}$$

which shows that the σ – *compressed* of our universe is an open cube in our universe. Its edge has the length 2σ . For any subset $\mathbb S$ of our universe the σ – *compressed* of $\mathbb S$ is

$$\underline{\mathbb{S}}_{\sigma} = \{ \underline{P}_{\sigma} | P \in \mathbb{S} \}.$$

We emphasize that the border of the σ – *compressed* universe do not belong to the compressed universe, They cannot be seen by the dwellers of \mathbb{R}^3_{σ} . Analoguously, standing in the point $\mathcal{O} = (0,0,0)$, the border of our universe is invisible for us. (Later on in the article it will be shown how a certain part of the bordering sheet becomes visible when taking another point as the origo.)

Our purpose is illuminated by the next figure where the spectator is in our universe and investigates the compressed universe. (Inspiration by Jonathan Swift's Gulliver's Travels.)

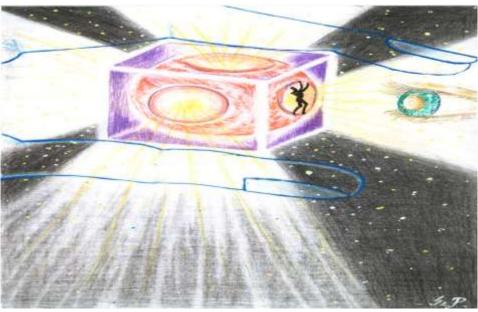


Fig. 2

Of course, the "compressed man" cannot see the viewer's eye. (Thanks for original picture of Fig.2 to Pálma Szalay, Vicenza, Italy, 2014.) Every object created by mathematical instruments is compressible by the method given under (3.1) - (3.3).

If we investigate the compressed universe, then we have, that the compressed lines are curved open passages

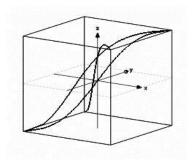


Fig.3

Fig 3. shows the compresseds of some cutting or parallel, moreover detouring lines. They represent the Euclidean geometry of the compressed universe. The compresseds of parallel lines have joint points on the border of compressed universe. Both compressed pairs of cutting or parallel lines determine compressed planes. (Compressed lines and planes are detailed in [1]. See Chapter 5, pages 61 - 99.)

As the pyramid Δ_a^m is defined by mathematical bacic –operators (see (0.2),) it is compressible.

First step. We compress the base \blacksquare_a , given by (0.1). We complete the σ – compression by the (mutual and unambiguous) mapping

$$(3.4) P = (x, y) \leftrightarrow \underline{P}_{\sigma} = (\underline{x}_{\sigma}, \underline{y}_{\sigma}).$$

Starting from (0.1) we have

$$\underline{|x|+|y|}_{\sigma} \leq \underline{\left(\frac{a}{\sqrt{2}}\right)}_{\sigma}, P = (x,y) \in \blacksquare_{a}.$$

So, by (1.6)

$$\underline{|x|}_{\sigma} \oplus_{\sigma} \underline{|y|}_{\sigma} \leq \underline{\left(\frac{a}{\sqrt{2}}\right)}_{\sigma}.$$

using that the function tanh is odd, by (1.1) we can write $|\underline{x}|_{\sigma} = |\underline{x}_{\sigma}|$ and $|\underline{y}|_{\sigma} = |\underline{y}_{\sigma}|$

Denoting $\xi = \underline{x}_{\sigma}$ and $\eta = \underline{y}_{\sigma}$ we have $|\xi| \oplus_{\sigma} |\eta| \leq \left(\frac{\underline{a}}{\sqrt{2}}\right)_{\sigma}$. Using (1.4) and on the left hand side, absolving the absolute values, we get that $\underline{\blacksquare}_{\sigma}$ is a closed set bordered by four hyperbolae

(3.5)
$$\frac{|\xi| + |\eta|}{1 + \frac{|\xi\eta|}{\sigma^2}} \le \frac{\left(\frac{a}{\sqrt{2}}\right)}{\sigma}, \quad \underline{P}_{\sigma} = (\xi, \eta) \in \underline{\blacksquare}_{a_{\sigma}}.$$

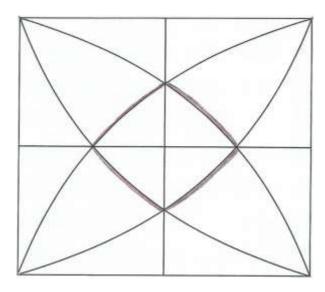


Fig. 4

Second step. We compress the mantle of the pyramid Δ_a^m given by (0.2). We complete the σ – compression by the (mutual and unambiguous) mapping

$$(3.6) P = (x, y, z) \leftrightarrow \underline{P}_{\sigma} = (\underline{x}_{\sigma}, \underline{y}_{\sigma}, \underline{z}_{\sigma}).$$

Having (0.1) and starting from (0.2) we have $z = m\left(1 - \frac{|x| + |y|}{a}\sqrt{2}\right)$, $P = (x, y, z) \in \mathbb{R}^3$. Denoting that $= \underline{x}_{\sigma}$, $\eta = \underline{y}_{\sigma}$ and $\zeta = \underline{z}_{\sigma}$ we have by (2.3) and (2.2) $\xi^{\sigma} = m\left(1 - \frac{|\xi^{\sigma}| + |\tilde{\eta}^{\sigma}|}{a}\sqrt{2}\right)$ and $\zeta = \underline{m}\left(1 - \frac{|\xi^{\sigma}| + |\tilde{\eta}^{\sigma}|}{a}\sqrt{2}\right)$.

Hence, (1.1) and (2.1) yield the equation of the mantle

$$m\left(1 - \frac{\left|\sigma \tanh^{-1}\frac{\xi}{\sigma}\right| + \left|\sigma \tanh^{-1}\frac{\eta}{\sigma}\right|}{a}\sqrt{2}\right)$$

$$(3.7) \qquad \zeta = \sigma \tanh \frac{\sigma}{\sigma} \qquad (\xi, \eta) \in \mathbb{R}^{2}_{\sigma}.$$

Third step. Finally, by (3.7) we get that the compressed pyramid is a closed set described by

$$(3.8) \quad \underline{\Delta}_{\underline{a}}^{m} = \left\{ P = (\xi, \eta, \zeta) \in \underline{\mathbb{R}}^{3} \middle| \begin{cases} (\xi, \eta) \in \underline{\underline{a}}_{\underline{a}} \\ m \left(1 - \frac{|\sigma \tanh^{-1} \frac{\xi}{\sigma}| + |\sigma \tanh^{-1} \frac{\eta}{\sigma}|}{a} \sqrt{2}\right) \end{cases} \right\}.$$
Of some $A^{m} \in \mathbb{R}^{3}$, if $\zeta = 0$, then the First 4 shows the base \overline{a} , which is a some

Of course, $\underline{\Delta_a^m}_{\sigma} \subset \underline{\mathbb{R}^3}_{\sigma}$. If $\zeta = 0$ then the Fig. 4 shows the base $\underline{\blacksquare}_a$ which is a compressed squre bordered by compressed lines. The mantle's sheets are compressed triangles bordered by the pairs of compressed cutting lines described by

$$\left\{ P = (\xi, \eta, \zeta) \in \mathbb{R}^{3} \atop \sigma \right| \left\{ m \left(1 - \frac{\left| \sigma \tanh^{-1} \frac{\xi}{\sigma} \right|}{a} \sqrt{2} \right) \atop \zeta = \sigma \tanh \frac{\sigma}{\sigma} \right\} \right\}$$

and

$$\begin{cases} P = (\xi, \eta, \zeta) \in \mathbb{R}^{3} \\ \zeta = \sigma \tanh \frac{m\left(1 - \frac{\left|\sigma \tanh^{-1} \frac{\eta}{\sigma}\right|}{a}\sqrt{2}\right)}{\sigma} \end{cases} \end{cases}$$

For $\underline{\Delta_a^m}_{\sigma}$ by (3.8) we have the figure (the base $\underline{\blacksquare_a}_{\sigma}$ is without distorsion on Fig. 4.)

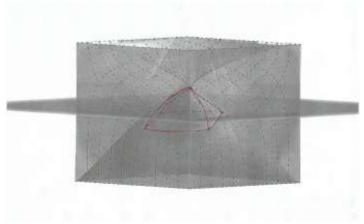


Fig 5.

To construct a compressed pyramid is a challenging task for arhitects in the compressed universe. They can compute by the sub – operations, only. So, for the compressed pyramid they have

$$(3.9) \ \underline{\Delta_{\underline{a}}^{m}}_{\sigma} = \left\{ P = (\xi, \eta, \zeta) \in \underline{\mathbb{R}^{3}}_{\sigma} \middle| \begin{cases} (\xi, \eta) \in \underline{\blacksquare_{\sigma}}_{\sigma} \\ 0 \le \zeta \le \underline{m}_{\sigma} \odot_{\sigma} \left(\underline{1}_{\sigma} \ominus_{\sigma} (|\xi| \bigoplus_{\sigma} |\eta|) \odot_{\sigma} \left(\underline{\frac{\sqrt{2}}{a}}_{\sigma}\right) \end{cases} \right\}. \text{ (See (0.2).)}$$

To prove our statement we use the concept of sub – subtraction. For any pair ξ , $\eta \in \mathbb{R}_{\sigma}$ we define by (1.4)

(3.10)
$$\xi \ominus_{\sigma} \eta = \xi \bigoplus_{\sigma} (-\eta) = \frac{\xi - \eta}{1 - \frac{\xi - \eta}{2}}.$$

Using (2.2) we have

$$(|\xi| \oplus_{\sigma} |\eta|) \odot_{\sigma} \underbrace{\left(\frac{\sqrt{2}}{a}\right)}_{\sigma} = \underbrace{\left(|\xi| \oplus_{\sigma} |\eta|\right]^{\sigma}}_{\sigma} \odot_{\sigma} \underbrace{\left(\frac{\sqrt{2}}{a}\right)}_{\sigma}.$$

Hence, by (1.7) and (2.3)

$$(|\xi| \oplus_{\sigma} |\eta|) \odot_{\sigma} \left(\frac{\sqrt{2}}{a}\right)_{\sigma} = \overline{|\xi| \oplus_{\sigma} |\eta|}^{\sigma} \cdot \frac{\sqrt{2}}{a}$$

is obtained.

Considering (3.10), (1.6) and (2.3)

$$(3.11) \qquad \underline{1}_{\sigma} \ominus_{\sigma} (|\xi| \bigoplus_{\sigma} |\eta|) \odot_{\sigma} \underbrace{\left(\frac{\sqrt{2}}{a}\right)}_{\sigma} = \underline{1}_{\sigma} \ominus_{\sigma} \underbrace{\overline{|\xi| \bigoplus_{\sigma} |\eta|}^{\sigma} \cdot \frac{\sqrt{2}}{a}}_{\sigma}$$

$$\underline{1}_{\sigma} \ominus_{\sigma} (|\xi| \bigoplus_{\sigma} |\eta|) \odot_{\sigma} \underbrace{\left(\frac{\sqrt{2}}{a}\right)}_{\sigma} = \underline{1} - \overline{|\xi| \bigoplus_{\sigma} |\eta|}^{\sigma} \cdot \frac{\sqrt{2}}{a}_{\sigma}$$

is obtained.

Having (3.11) we apply (1.7)
$$\underline{m}_{\sigma} \odot_{\sigma} \left(\underline{1}_{\sigma} \ominus_{\sigma} (|\xi| \oplus_{\sigma} |\eta|) \odot_{\sigma} \underbrace{\left(\frac{\sqrt{2}}{a} \right)_{\sigma}} \right) = \underline{m}_{\sigma} \odot_{\sigma} \underline{1 - |\xi| \oplus_{\sigma} |\eta|}^{\sigma} \cdot \frac{\sqrt{2}}{a}_{\sigma} \text{ and get}$$
(3.12) $\underline{m}_{\sigma} \odot_{\sigma} \left(\underline{1}_{\sigma} \ominus_{\sigma} (|\xi| \oplus_{\sigma} |\eta|) \odot_{\sigma} \underbrace{\left(\frac{\sqrt{2}}{a} \right)_{\sigma}} \right) = m \cdot \left(1 - \overline{|\xi| \oplus_{\sigma} |\eta|}^{\sigma} \cdot \frac{\sqrt{2}}{a} \right)_{\sigma}.$

Finally (3.12) and (2.5) with (1.1) and (2.1) yield

$$\underline{m}_{\sigma} \odot_{\sigma} \left(\underline{1}_{\sigma} \ominus_{\sigma} (|\xi| \oplus_{\sigma} |\eta|) \odot_{\sigma} \underbrace{\left(\frac{\sqrt{2}}{a} \right)}_{\sigma} \right) = \sigma \cdot \tanh \frac{m \cdot \left(1 - \left(\sigma \tanh^{-1} \frac{|\xi|}{\sigma} + \sigma \tanh^{-1} \frac{|\eta|}{\sigma} \right) \cdot \frac{\sqrt{2}}{a} \right)}{\sigma}.$$

Casting glances on (3.9) and (3.8) our statement is proved. (Istennek Hála, 2019. 08. 28 12.26. Szalay.István)

Remark 1. It is known that the original height of Pharaoh Cheops's Pyramid $\Delta^{146,5\,meter}_{230,04\,meter}$ (see Fig.1) was 146,5 meter. At present it is 138.8 meter, only. (See the Great Pyramid of Giza – Wikipedia, 2019.08.28:13.27.) Considering that $\underline{146.5}_{356\,meter} = 138.8\,meter$ we give our

(Strongly Naive) Hypothesis: Our universe is a σ – compressed universe with parameter σ = 356 metre. In this case our universe would be a very small cube. This contradicts our experience. (Of course, much more data vould be necessary.)

(Less Naive) Hypothesis: Accepting that the speed of light $c = 1 \frac{light\ year}{year}$ and the age of our universe is $t_{universe} = 13.7\ billion\ (thousand\ million)\ year$, then

$$\sigma = c \cdot t_{universe} = 13,7 \text{ billion light year.}$$

With respect to formulae (1.8) – (1.12) our $(\sigma - compressed)$ universe is large enough but there is a serious problem: parameter σ depends on c and $t_{universe}$, merely.

Third Bravely Naive Hypothesis: Accepting that our universe is unlimited for us, we suppose the existence of the Multiverse. From the Multiverse, our universe is a very very big cube such that the border of our universe is invisible for us. It is imaginable by Fig. 2: The cube represents our Universe here, and the man cannot see the borders of this cube. Our universe plays the role of the Multiverse, that is the hand and eye are in the Multiverse. Of course, we have to create the Multiverse by exploding real numbers. See the preparation is in point II. and the continuation is follows.

IV. THE EXPLOSION OF REAL NUMBERS

Unfotunately the explosion formula (2.1) is unsuited for the explosion of all real numbers, because it is able to explode the elements of the open interval $]-\sigma$, $\sigma[$. On the other hand the explodeds of the elements of the open interval $]-\sigma$, $\sigma[$ occupy the complete set of real numbers \mathbb{R} . Geometrically, the still imaginable explodeds of the elements of the set $\mathbb{R}\setminus \underline{\mathbb{R}}_{\sigma}=]-\infty, -\sigma] \cup [\sigma,\infty[$ are outside the number line. Temporally, they are called invisible exploded numbers. So, $-\sigma^{\sigma}$ and σ^{σ} are invisible exploded numbers. They are called potential negative and positive discriminators, respectively.

If we regard the number line as a model of a one - dimensional space, the invisible exploded numbers may be in the two - dimensional space \mathbb{R}^2 . So, the invisible expoded numbers are represented by a pair of real numbers. As the explodeds of the elements of the open interval $]-\sigma$, $\sigma[$ are real numbers, they can be consodered as visible exploded numbers such that in their pairs the second coordinate is 0. It is known that the complex number is also represented by a pair of real numbers. (First coordinate is the real part, second one is the imagined part.) So, for convenience's sake, we may use the complex number to represent the exploded number.

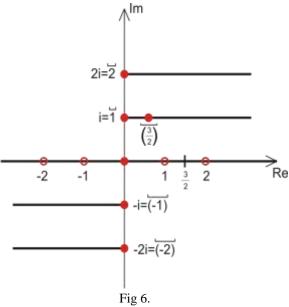
A very important task is to find a permanent extension of the explosion formula (2.1). A possibility is

(4.1)
$$\check{\mathbf{x}}^{\sigma} = \sigma \cdot (sgn \, \mathbf{x}) \left(\tanh^{-1} \left\{ \left| \frac{\mathbf{x}}{\sigma} \right| \right\} + i \left[\left| \frac{\mathbf{x}}{\sigma} \right| \right] \right) , \quad \mathbf{x} \in \mathbb{R},$$

where sgn, [], { } and | | mean the sign, entire part, fraction part, absolute value of the real number, respectively. Denoting by \mathbb{R}^{σ} the set of exploded numbers by (4.1) we have that \mathbb{R}^{σ} is a proper subset of the set of complex numbers \mathbb{C} . Its description is

(4.2)
$$\mathbb{R}^{\sigma} = \left\{ u \in \mathbb{C} \middle| \frac{\operatorname{Im} u}{\sigma} \text{ is integer number and } (\operatorname{Re} u) \cdot (\operatorname{Im} u) \geq 0 \right\}$$

The real number line is represented by the real axis. The set \mathbb{R}^{σ} forms a "flag" on the Gauss plane. $-\sigma^{\sigma} = -\sigma i$ and $\sigma^{\sigma} = \sigma i$.



In Fig.6 $\sigma = 1$, so, the open interval $]-\sigma$, $\sigma =]-1$, 1[and \check{x}^1 is denoted by \check{x} , simply.

Relation equality $,=^{in\mathbb{R}^{\sigma}}$ " is the same as used $,=^{in\mathbb{C}}$ ". For the sake of visible exploded numbers $\check{\chi}^{\sigma},\check{y}^{\sigma}\in\mathbb{R}$ we mention: $\check{\chi}^{\sigma}=^{in\mathbb{R}^{\sigma}}\check{y}^{\sigma}$ if and only if $\check{\chi}^{\sigma}=\check{y}^{\sigma}$ that is $,=^{in\mathbb{R}^{\sigma}}$ " is an extension of ,=". So, we may use ,=" instead of $,=^{in\mathbb{R}^{\sigma}}$ ". Moreover, we have the

Property of equality. If $x, y \in \mathbb{R}$ then $\check{x}^{\sigma} = \check{y}^{\sigma}$ if and only if = y. (See [1], Theorem 4.7) Reflexivity, symmetry and transitivity remain valid for the extended equality, too.

The ordedering relation in \mathbb{C} does not exist. So, for \mathbb{R}^{σ} we give a lexicographical sorting.

Relation ordering in \mathbb{R}^{σ} : we say that $\check{\chi}^{\sigma} <^{in} \mathbb{R}^{\sigma} \check{\chi}^{\sigma}$ if and only if

$$Im\ \check{x}^{\sigma} < Im\ \check{y}^{\sigma} \text{ or } Im\ \check{x}^{\sigma} = Im\ \check{y}^{\sigma} \text{ and } Re\ \check{x}^{\sigma} < Re\ \check{y}^{\sigma}.$$

This is the extension of the ordering of real numbers, so we may use "" instead of "<" instead of "<"...

Property of ordering. If $x, y \in \mathbb{R}$ then $\check{x}^{\sigma} < \check{y}^{\sigma}$ if and only if < y. (See [1], Theorem 4.15)

Consequently, irreflexivity, assimetry and transitivity remain valid for the extended ordering, too.

For the flag seen in Fig. 6 we mention

Property of completeness. For any $u \in \mathbb{R}^{\sigma}$

$$\left(\operatorname{Im} u + \sigma \cdot \tanh \frac{\operatorname{Re} u}{\sigma}\right)^{\sigma} = u.$$

(See [1], Theorem 4.11) By this property we introduce the generalized compression

$$(4.3) \underline{u}_{\sigma} = Im \ u + \sigma \cdot \tanh \frac{Re \ u}{\sigma} , u \in \mathbb{R}^{\sigma}.$$

Really, if $u \in \mathbb{R}$ the formula (4.3) is reduced to formula (1.1).

Without any proof we mention the

Monotonity property of explosion. Let $x \neq 0$ be an arbitrary real number

If
$$0 < x$$
 then $x < \check{x}^{\sigma}$ and if $x < 0$ then $\check{x}^{\sigma} < x$.

Monotonity property of compression. Let $u \neq 0$ be an arbitrary compressed number

If
$$0 < u$$
 then $0 < \underline{u}_{\sigma} < u$ and if $u < 0$ then $u < \underline{u}_{\sigma} < 0$.

Having the Property of completeness and using (4.3) we have the inversion formula

(4.4)
$$\widetilde{\left(\underline{u}_{\sigma}\right)}^{\sigma}=u \qquad , \qquad u\in\mathbb{R}^{\sigma} \ ,$$
 which is en extension of (2.3). On the other hand by (4.1) and (4.3)

$$(4.5) (\check{x}^{\sigma})_{\sigma} = x , x \in \mathbb{R}$$

is obtained. This is an extension of the inversion formula (2.2).

By the super – operators we create an algebraic structure in set \mathbb{R}^{σ} :

Super- addition

(4.6)
$$\check{x}^{\sigma} \overline{\bigoplus}^{\sigma} \check{y}^{\sigma} = x + y^{\sigma} , x, y \in \mathbb{R} , (\text{see } (2.5))$$

and

Super- multiplication

(4.7)
$$\check{x}^{\sigma} \overline{\bigcirc}^{\sigma} \check{y}^{\sigma} = \widecheck{x \cdot y}^{\sigma} , x, y \in \mathbb{R} , (\text{see } (2.7))$$

 $(4.7) \hspace{1cm} \breve{x}^{\sigma}\overline{\bigcirc}^{\sigma}\breve{y}^{\sigma}=\widecheck{x\cdot y}^{\sigma} \hspace{0.5cm},\hspace{0.5cm} x,y\in\mathbb{R} \hspace{0.5cm},\hspace{0.5cm} (\text{see }(2.7)).$ As $(\mathbb{R}\,,+\,,\cdot\,)$ is an algebraic field, $\left(\widecheck{\mathbb{R}}^{\sigma}\,,\hspace{0.5cm}\overline{\bigoplus}^{\sigma},\hspace{0.5cm}\overline{\bigcirc}^{\sigma}\right)$ is an algebraic field, too. This means that for super – operations associativity, commutativity, distributivity are valid. Moreover 0 is the additive unit element. For any \check{x}^{σ} ($x \in \mathbb{R}$) the $-\check{x}^{\sigma}$ is the additive inverse element. $\check{1}^{\sigma}$ is the multiplicative unit element.

For any \check{x}^{σ} $(x \neq 0, and x \in \mathbb{R})$ the $(\widetilde{\frac{1}{x}})^{\sigma}$ is the multiplicative inverse element. We may extend the meanings of the sign "minus" and the absolute value

$$-\check{x}^{\sigma} = \stackrel{def}{(-x)^{\sigma}}, \quad x \in \mathbb{R}$$

and

$$(4.9) |\check{\mathbf{x}}^{\sigma}| = {}^{def} |\widecheck{\mathbf{x}}|^{\sigma} , \quad \mathbf{x} \in \mathbb{R},$$

respectively.

Be careful! Although an exploded numbers is represented by a complex number for our absolute value defined under (4.9) $|\check{x}^{\sigma}| \neq \sqrt{(Re\ \check{x}^{\sigma})^2 + (Im\ \check{x}^{\sigma})^2}$, where the latter is a non-negative real number, but

$$|u| = \begin{cases} u & \text{if } u > 0 \\ 0 & \text{if } u = 0 \\ -u & \text{if } u < 0 \end{cases} , u \in \widetilde{\mathbb{R}}^{\sigma}$$

is true. This shows that (4.9) is a generalisation a of the notion 'absolute value' concerning real numbers.

For any real number x, y and z we have that if x < y then x + z < y + z. Hence by the Property of ordering and (4.6) we have $\check{x}^{\sigma} \overline{\bigoplus}^{\sigma} \check{z}^{\sigma} < \check{v}^{\sigma} \overline{\bigoplus}^{\sigma} \check{z}^{\sigma}$.

Moreover, for any (real) positive z we have that if x < y then $x \cdot z < y \cdot z$. Hence by the Property of ordering and (4.7) we have $\check{\chi}^{\sigma} \overline{\bigcirc}^{\sigma} \check{z}^{\sigma} < \check{\gamma}^{\sigma} \overline{\bigcirc}^{\sigma} \check{z}^{\sigma}$.

So, we have that field $(\mathbb{R}^{\sigma}, \overline{\bigoplus}^{\sigma}, \overline{\odot}^{\sigma}, \leq)$ is an ordered field.

V. EXPLOSION OF OUR UNIVERSE

We consider the σ – *exploded* of our universe as a pointwise exploded \mathbb{R}^3 , that is our Multiverse is

(5.1)
$$\widetilde{\mathbb{R}^3}^{\sigma} = \left\{ (u, v, w) \middle| \begin{cases} u = \check{x}^{\sigma} \\ v = \check{y}^{\sigma} \text{ where } P = (x, y, z) \in \mathbb{R}^3 \end{cases} \right\}$$

such that $\sigma - exploded$ of the point P is $\check{P}^{\sigma} = (u, v, w)$. Especially, the $\sigma - exploded$ of origo $\mathcal{O} = (0,0,0)$ is itself, that $\check{\mathcal{O}}^{\sigma} = \mathcal{O}$. The inversion formula (2.3) shows that the exploded of a compressed number is a real number. So, the exploded of the compressed universe (see Fig. 3) is our universe \mathbb{R}^3 . By the explosion (5.1) the sub – lines (compressed Euclidean lines) and sub – planes (compressed Euclidean planes) of \mathbb{R}^3 (see (3.1)) become lines and planes of our universe. This shows that explosion is not a simple enlargement and compression is not a simple reduced picture. But what happened if we explode Euclidean line? For example in the case of

$$\mathbb{L} = \begin{cases} x(t) = \frac{1}{\sqrt{6}} \cdot t \\ y(t) = \frac{1}{\sqrt{6}} \cdot t \\ z(t) = \frac{2}{\sqrt{6}} \cdot t \end{cases}, \quad -\infty < t < \infty,$$

 $\mathbb{L} = \begin{cases} x(t) = \frac{1}{\sqrt{6}} \cdot t \\ y(t) = \frac{1}{\sqrt{6}} \cdot t \\ z(t) = \frac{2}{\sqrt{6}} \cdot t \end{cases}, \quad -\infty < t < \infty,$ the exploded coordinate $\widetilde{x(t)}^{\sigma} = \sigma \cdot \left(sgn\,x(t)\right) \left(\tanh^{-1}\left\{\left|\frac{x(t)}{\sigma}\right|\right\} + i\left[\left|\frac{x(t)}{\sigma}\right|\right]\right)$ will be a real number if $-\sigma\sqrt{6} < t < \sigma\sqrt{6}$ and other two coordinates are similar. In our universe the so called $\sigma - box$ phenomenon of $\check{\mathbb{L}}^{\sigma}$ is

$$\left(\mathbb{L}^{\sigma}\right)^{\sigma-box} = \begin{cases} u(t) = \sigma \tanh^{-1} \frac{t}{\sigma\sqrt{6}} \\ v(t) = \sigma \tanh^{-1} \frac{t}{\sigma\sqrt{6}} \\ w(t) = \sigma \tanh^{-1} \frac{2t}{\sigma\sqrt{6}} \end{cases}, -\sigma\sqrt{6} < t < \sigma\sqrt{6}.$$



Is it possible that Cheops's exploded pyramid (see Fig.1) is the Eiffel Tower?



Fig. 8

It is time to explode the pyramid Δ_a^m .

First step. We explode the base \blacksquare_a , given by (0.1). We complete the σ - explosion by the (mutual and unanbiguous) mapping

$$(5.2) P = (x, y) \leftrightarrow \check{P}^{\sigma} = (u, v), where \ u = \check{x}^{\sigma} \text{ and } v = \check{y}^{\sigma}.$$

Having (0.1) and using the inversion formula (4.5) we have for points $\check{P}^{\sigma} = (u, v) \in \mathbf{r}_{a}^{\sigma}$

(5.3)
$$\left|\underline{u}_{\sigma}\right| + \left|\underline{v}_{\sigma}\right| \le \frac{a}{\sqrt{2}} \quad , \ u, v \in \widetilde{\mathbb{R}}^{\sigma}$$

To $\mathbf{m}_{a}^{\sigma} \subset \mathbb{R}^{2}$ is sufficient if the condition

$$(5.4) (0 <) \frac{a}{\sqrt{2}} < \sigma$$

is fulfilled because in this case
$$u$$
 and v are real numbers, so by (2.1) the inequality (5.3) has the form $\left|\sigma\tanh\frac{u}{\sigma}\right| + \left|\sigma\tanh\frac{v}{\sigma}\right| \le \frac{a}{\sqrt{2}}$, $u,v \in \mathbb{R}$,

or

$$\tanh \frac{|u|}{\sigma} + \tanh \frac{|v|}{\sigma} \le \frac{a}{\sigma\sqrt{2}}, \quad u, v \in \mathbb{R}$$
.

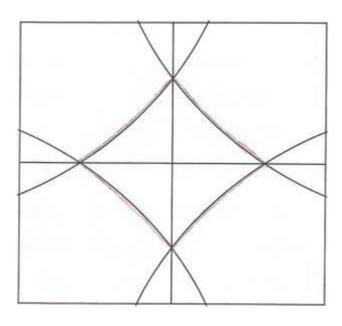


Fig. 9

Second step. We explode the mantle of the pyramid Δ_a^m given by (0.2). We complete the σ – explosion by the (mutual and unambiguous) mapping

$$(5.5) P = (x, y, z) \leftrightarrow \check{P}^{\sigma} = (u, v, w), where \ u = \check{x}^{\sigma}, v = \check{y}^{\sigma} \ and \ w = \check{z}^{\sigma}.$$

Having (0.1) and starting from (0.2) we have

$$z=m\left(1-\frac{|x|+|y|}{a}\sqrt{2}\right)\quad,\ P=(x,y,z)\in\mathbb{R}^3\,.$$

Since $x = \underline{u}_{\sigma}$, $y = \underline{v}_{\sigma}$ and $z = \underline{w}_{\sigma}$ we can write

$$\underline{w}_{\sigma} = m \left(1 - \frac{|\underline{u}_{\sigma}| + |\underline{v}_{\sigma}|}{a} \sqrt{2} \right) \quad , \quad \widecheck{P}^{\sigma} = (u, v, w) \in \widecheck{\mathbb{R}^{3}}.^{\sigma}$$

Hence

$$w = \left(m\left(1 - \frac{|\underline{u}_{\sigma}| + |\underline{v}_{\sigma}|}{a}\sqrt{2}\right)\right)^{\sigma}$$

Assuming (5.4) we have that u and v are real numbers, so, $\underline{u}_{\sigma} = \sigma \tanh \frac{u}{\sigma}$ and $\underline{v}_{\sigma} = \sigma \tanh \frac{v}{\sigma}$.

As the base $\widecheck{\blacksquare_a}^{\sigma}$ reclines on the coordinate plane "u , v" of \mathbb{R}^2 we will see $\widecheck{\Delta_a}^{m}$ in our universe under the condition

$$(5.6) 0 < m \left(1 - \frac{\left| \sigma \tanh \frac{u}{\sigma} \right| + \left| \sigma \tanh \frac{v}{\sigma} \right|}{a} \sqrt{2} \right) < \sigma.$$

So, under the conditions (0.1), (0.2), (5.4) and (5.6)

(5.7)
$$w = \sigma \tanh^{-1} \frac{m \left(1 - \frac{|\sigma \tanh \frac{u}{\sigma}| + |\sigma \tanh \frac{v}{\sigma}|}{a} \sqrt{2}\right)}{\sigma}$$

is obtained.

Third step. Finally, by (5.7) we get that the exploded pyramid is a closed set described by

$$(5.8) \qquad \widetilde{\Delta_a^m}^{\sigma} = \left\{ P = (u, v, w) \in \mathbb{R}^3 \mid \begin{cases} (u, v) \in \widecheck{\blacksquare_a}^{\sigma} \\ 0 \le w \le \sigma \tanh^{-1} \frac{m \left(1 - \frac{|\sigma \tanh \frac{u}{\sigma}| + |\sigma \tanh \frac{v}{\sigma}|}{\sigma} \sqrt{2}\right)}{\sigma} \right\}.$$

If w = 0 then Fig. 9 shows the base \mathbf{r}_a^{σ} which is an exploded square (super – square) bordered by exploded lines (super - lines). The mantle's sheets are super-triangles bordered by the pairs of cutting super - lines. Itself $\Delta_a^{\widetilde{m}^{\sigma}}$ is a super – pyramid, with visible with height \check{m}^{σ} .

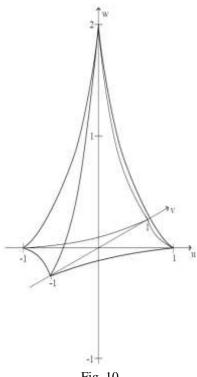


Fig. 10

This is an Eiffel Tower (see Fig.8.) or a radiating antenna?

If we use the super – addition, then the base \widecheck{a}^{σ} (exploding both sides of inequality (5.3) and using (4.6) with (4.9)) is given by the form

$$|u|\overline{\bigoplus}^{\sigma}|v| \leq \underbrace{\widetilde{\left(\frac{a}{\sqrt{2}}\right)}^{\sigma}}_{\sigma}, \quad (0 <) a \in \mathbb{R}, and \ u, v \in \widecheck{\mathbb{R}}^{\sigma}.$$

If the condition (5.4) fulfils then the base \widecheck{a}^{σ} is seen in Fig. 9. If the condition (5.4) is omitted then the base $\widecheck{\blacksquare}_a^{\sigma}$ steps out \mathbb{R}^2 . (Imaginable that $\mathbb{R}^2 \subset \widecheck{\blacksquare}_a^{\sigma}$,too.)

Using the super – subtraction

(5.10)
$$\check{x}^{\sigma} \overline{\bigcirc}^{\sigma} y^{\sigma} = \check{x - y}^{\sigma} , x, y \in \mathbb{R}$$

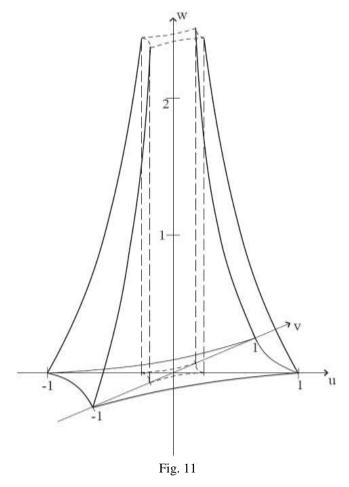
together the super – operations under (4.6) and (4.7) with (4.9) the equality

$$\left(m\left(1-\frac{|\underline{u_{\sigma}}|+|\underline{v_{\sigma}}|}{a}\sqrt{2}\right)\right)^{\sigma}=\check{m}^{\sigma}\overline{\odot}^{\sigma}\left(\check{1}^{\sigma}\overline{\ominus}^{\sigma}\left(\left(|u|\overline{\oplus}^{\sigma}|v|\right)\overline{\odot}^{\sigma}\left(\overline{\sqrt{2}}\right)^{\sigma}\right)\right)$$

is verified. Hence we have the description of super – pyramid by super – operators

$$(5.11)\ \ \widecheck{\Delta_{a}^{m}}^{\sigma} = \left\{ P = (u, v, w) \in \widecheck{\mathbb{R}^{3}}^{\sigma} \middle| \begin{cases} (u, v) \in \widecheck{\blacksquare_{a}}^{\sigma} \\ 0 \leq w \leq \widecheck{m}^{\sigma} \overline{\bigcirc}^{\sigma} \left(\widecheck{1}^{\sigma} \overline{\bigcirc}^{\sigma} \left(\left(|u| \overline{\bigoplus}^{\sigma} |v| \right) \overline{\bigcirc}^{\sigma} \left(\overline{\sqrt[3]{a}}^{\sigma} \right) \right) \right) \right\}$$

If (5.4) fulfils and $m \ge \sigma$ then the $\sigma - box\ phenomenon\ of\ \Delta_a^m{}^{\sigma}$ is visible in \mathbb{R}^3 , only.



VI. INSIGHT INTO MULTIVERSE

Turning back to the Fig. 7 we can see that the grapf of $(\mathbb{L}^{\sigma})^{\sigma-box}$ reaches the sky. This fact is proved by $\lim_{t\to \frac{\sigma\sqrt{6}}{2}}\sigma \tanh^{-1}\frac{2t}{\sigma\sqrt{6}}=\infty$. At the same time considering the parameter $t=\frac{\sigma\sqrt{6}}{2}$ we get that the point $t<\frac{\sigma\sqrt{6}}{2}$

 $\mathbb{P} = \left(\frac{\sigma}{2}, \frac{\sigma}{2}, \sigma\right) \in \mathbb{L}$. So, $\mathbb{P}^{\sigma} = \left(\left(\frac{\sigma}{2}\right)^{\sigma}, \left(\frac{\sigma}{2}\right)^{\sigma}, \check{\sigma}^{\sigma}\right) \in \mathbb{L}^{\sigma}$ and one of the points where the super – line \mathbb{L}^{σ} breaks through the border of our universe \mathbb{R}^3 . Consequently, \mathbb{P}^{σ} is invisible in our universe.

What can we do to see \mathbb{P}^{σ} ?

The inversion formula (4.4) gives that the compressed of the Multiverse is our universe and the compressed of super – line \mathbb{L}^{σ} is the line \mathbb{L} . The central cube (see Fig.12) is our compressed universe. The explodeds of other cubes are new parallel universes.

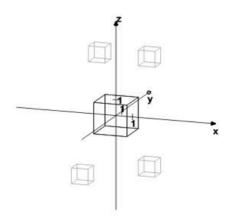
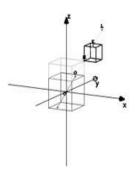


Fig. 12

The line
$$l = \left\{ (x, y, z) \in \mathbb{R}^5 : x = \frac{t}{\sqrt{6}}; y = \frac{t}{\sqrt{6}}; z = \frac{2t}{\sqrt{6}}; t \in \mathbb{R} \right\}$$



 $l \subset S$

Fig. 13

In Fig. 13 we can discover point \mathbb{P} in the border of the central cube. Moreover, we can see that point \mathbb{P} is inside another cube having the centre (σ, σ, σ) .

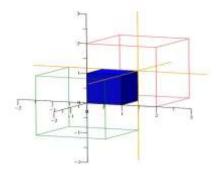


Fig. 14

Let an arbitrary fixed point $\mathcal{O}_0 = (x_0, y_0, z_0) \in \mathbb{R}^3$ be given. If we shift the Descartes coordinate – system ,, x, y, z'' in our universe \mathbb{R}^3 by the shift transformation

(6.1)
$$\begin{cases} \alpha = x - x_0 \\ \beta = y - y_0 \\ \gamma = z - z_0 \end{cases}, P = (x, y, z) \text{ and } \mathcal{O}_0 = (x_0, y_0, z_0) \in \mathbb{R}^3,$$

then each point $P=(x,y,z)\in\mathbb{R}^3$ will remain in its place, but (exception the case $\mathcal{O}=\mathcal{O}_0$) has new coordinates in the coordinate – system " α , β , γ ". So, \mathcal{O}_0 is the origo of the system ", α , β , γ ", while \mathcal{O} (the origo of the system ", x, y, z") remaining in its place has new coordinates $\alpha=-x_0$, $\beta=-y_0$ and $\gamma=-z_0$.

Using coordinate – sytem x, y, z'' we consider the open cube

(6.2)
$$\mathbb{S}_{\mathcal{O}_0} = \left\{ P = (x, y, z) \in \mathbb{R}^3 \middle| \begin{cases} x_0 - \sigma < x < x_0 + \sigma \\ y_0 - \sigma < y < y_0 + \sigma \\ z_0 - \sigma < z < z + \sigma \end{cases} \right\}.$$

Fig 14. shows the open cube $\mathbb{S}_{\mathcal{O}}$ (below) end the open cube $\mathbb{S}_{\mathcal{O}_0}$ with $\mathcal{O}_0 = (\sigma, \sigma, \sigma)$. The latter open cube contains the point $\mathbb{P} = \left(\frac{\sigma}{2}, \frac{\sigma}{2}, \sigma\right)$.

On the other hand using the shifted coordinate system ,, α , β , γ " for any \mathcal{O}_0 we have

$$\mathbb{S}_{\mathcal{O}_0} = \left\{ P = (\alpha, \beta, \gamma) \in \mathbb{R}^3 \middle| \begin{cases} -\sigma < \alpha < \sigma \\ -\sigma < \beta < \sigma \end{cases} \right\}.$$

We remark that in the system " α , β , γ " with $\mathcal{O}_0 = (\sigma, \sigma, \sigma)$ the point \mathbb{P} the coordinates $\alpha = -\frac{\sigma}{2}$, $\beta = -\frac{\sigma}{2}$ and $\gamma = 0$.

Denoting that

$$\check{\alpha}^{\sigma}=\kappa$$
, $\check{\beta}^{\sigma}=\lambda$, $\check{\gamma}^{\sigma}=\mu$, $\check{x}^{\sigma}=u$, $\check{y}^{\sigma}=v$, $\check{z}^{\sigma}=w$, $\check{x_0}^{\sigma}=u_0$, $\check{y_0}^{\sigma}=v_0$ and $\check{z_0}^{\sigma}=w_0$ by (6.1) and (5.10) we define the super – shift transformation

$$\begin{cases} \kappa = u \ \overline{\bigcirc}^{\sigma} \ u_{0} \\ \lambda = v \ \overline{\bigcirc}^{\sigma} \ v_{0} \ , \check{P}^{\sigma} = (u, v, w) \ and \ \widecheck{O_{0}}^{\sigma} \ = (u_{0}, v_{0}, w_{0}) \in \widetilde{\mathbb{R}^{3}}^{\sigma}. \\ \mu = w \ \overline{\bigcirc}^{\sigma} \ w_{0} \end{cases}$$

(The special case $\widecheck{O_0}^{\sigma} = \widecheck{O}^{\sigma} = \mathcal{O}$ is the identical super – shift transformation.)

Having (6.2) by (5.10) and (4.6) with respect to the Property of ordering we have the parallel universes

$$(6.5) \qquad \widetilde{\mathbb{S}_{\mathcal{O}_{0}}}^{\sigma} = \left\{ \widecheck{P}^{\sigma} = (u, v, w) \in \widetilde{\mathbb{R}^{3}}^{\sigma} \middle| \begin{cases} u_{0} \overline{\bigoplus}^{\sigma} \widecheck{\sigma}^{\sigma} < u < u_{0} \overline{\bigoplus}^{\sigma} \widecheck{\sigma}^{\sigma} \\ v_{0} \overline{\bigoplus}^{\sigma} \widecheck{\sigma}^{\sigma} < v < v_{0} \overline{\bigoplus}^{\sigma} \widecheck{\sigma}^{\sigma} \\ w_{0} \overline{\bigoplus}^{\sigma} \widecheck{\sigma}^{\sigma} < w < w_{0} \overline{\bigoplus}^{\sigma} \widecheck{\sigma}^{\sigma} \end{cases} \right\}.$$

The centre of universe $\mathfrak{S}_{\mathcal{O}_0}^{\sigma}$ is the point \mathfrak{O}_0^{σ} . If $\mathcal{O}_0 = \mathcal{O}$ then $\mathfrak{O}_0^{\sigma} = \mathfrak{O}_0^{\sigma} = \mathcal{O}_0$, so, we get back our universe

(6.6)
$$\mathbb{R}^{3} = \left\{ \widecheck{P}^{\sigma} = (u, v, w) \in \widetilde{\mathbb{R}}^{3}{}^{\sigma} \middle| \begin{cases} -\widecheck{\sigma}^{\sigma} < u < \widecheck{\sigma}^{\sigma} \\ -\widecheck{\sigma}^{\sigma} < v < \widecheck{\sigma}^{\sigma} \end{cases} \right\}.$$

(By (5.10) and (4.8) we compute that $0 \ \overline{\ominus}^{\sigma} \ \check{\sigma}^{\sigma} = \check{0}^{\sigma} \ \overline{\ominus}^{\sigma} \ \check{\sigma}^{\sigma} = \underbrace{\check{0}^{\sigma} \ \bar{\sigma}^{\sigma}} = -\check{\sigma}^{\sigma} = -\check{\sigma}^{\sigma}$, on the other hand 0 is the additive unit element of super – addition. Moreover, $-\check{\sigma}^{\sigma}$ and $\check{\sigma}^{\sigma}$ represent the symbols $-\infty$ and ∞ , respectively.)

Usig the super – shift transformation given under (6.4), then (6.3) yields

(6.7)
$$\widetilde{\mathbb{S}_{\mathcal{O}_0}}^{\sigma} = \left\{ \check{P}^{\sigma} = (\kappa, \lambda, \mu) \in \widetilde{\mathbb{R}^3}^{\sigma} \middle| \begin{cases} -\check{\sigma}^{\sigma} < \kappa < \check{\sigma}^{\sigma} \\ -\check{\sigma}^{\sigma} < \lambda < \check{\sigma}^{\sigma} \\ -\check{\sigma}^{\sigma} < \mu < \check{\sigma}^{\sigma} \end{cases} \right\}.$$

Despite that (excepting the case $\mathcal{O} = \mathcal{O}_0$) the universes \mathbb{R}^3 and $\mathfrak{S}_{\mathcal{O}_0}^{\sigma}$ are different but (6.6) and (6.7) show parallel universes have the same size.

For example the point $\check{\mathbb{P}}^{\sigma} = (\underbrace{\left(\frac{\sigma}{2}\right)}^{\sigma}, \underbrace{\left(\frac{\sigma}{2}\right)}^{\sigma}, \check{\sigma}^{\sigma})$ is invisible in our universe but having the shifted coordinates $\kappa = -\left(\frac{\sigma}{2}\right)^{\sigma}$, $\lambda = -\left(\frac{\sigma}{2}\right)^{\sigma}$ and $\mu = 0$, it is visible in the universe $\check{\mathbb{S}_{0_0}}^{\sigma}$ with the center $\check{\mathcal{O}_0}^{\sigma} = (u = \check{\sigma}^{\sigma}, v = \check{\sigma}^{\sigma})$, $v = \check{\sigma}^{\sigma}$. (Of course, $\check{\mathcal{O}_0}^{\sigma}$ descripted by the shifted coordinates $\check{\mathcal{O}_0}^{\sigma} = (\kappa = 0, \lambda = 0, \mu = 0)$.) So, the earlier question "What can we do to see $\check{\mathbb{P}}^{\sigma}$? "is answered."

By the above-mentioned example we can state that one (or more) suitable shift transformation is able to show us any "large" subset of the Multiverse. The exception is the Multiverse itself. Analogy is given by Fig. 14, such that the Multiverse is represented by our universe and the parallel universes are represented by cubes.

Another example is the super – pyramid $\widetilde{\Delta_a^m}^{\sigma}$ given under (5.11) with $a = \underline{1}_{\sigma} \cdot \sqrt{2}$ and $\widecheck{m}^{\sigma} = \overline{\left(\frac{3\sigma}{2}\right)}^{\sigma}$. The $\left(\widetilde{\Delta_a^m}^{\sigma}\right)^{\sigma-box}$ is visible in Fig. 11 but the remaining part of $\widetilde{\Delta_a^m}^{\sigma}$ is outside our universe. We will use the super – shift transformation

(6.8)
$$\begin{cases} \kappa = u \\ \lambda = v \\ \mu = w \ \Theta^{\sigma} \ \check{\sigma}^{\sigma} \end{cases}, \check{P}^{\sigma} = (u, v, w) \ and \ \widecheck{\mathcal{O}_{0}}^{\sigma} = (0, 0, \check{\sigma}^{\sigma}) \in \widetilde{\mathbb{R}^{3}}^{\sigma}.$$

Starting from (5.11) by (6.8) we consider

$$\check{\sigma}^{\sigma} \leq w \leq \check{m}^{\sigma} \overline{\odot}^{\sigma} \left(\check{1}^{\sigma} \overline{\ominus}^{\sigma} \left(\left(|u| \overline{\oplus}^{\sigma} |v| \right) \overline{\odot}^{\sigma} \left(\overline{\sqrt{2}} \overline{a} \right)^{\sigma} \right) \right)$$

and hence

$$0 \leq \mu \leq \widecheck{m}^{\sigma} \overline{\odot}^{\sigma} \left(\widecheck{1}^{\sigma} \overline{\ominus}^{\sigma} \left(\left(|\kappa| \overline{\oplus}^{\sigma} |\lambda| \right) \overline{\odot}^{\sigma} \left(\overline{\frac{\sqrt{2}}{a}} \right)^{\sigma} \right) \right) \overline{\ominus}^{\sigma} \widecheck{\sigma}^{\sigma}.$$

Computing with $\check{m}^{\sigma} = \underbrace{\left(\frac{\Im\sigma}{2}\right)}^{\sigma}$ and $a = \underline{1}_{\sigma} \cdot \sqrt{2}$ we write step by step

$$\begin{split} \widetilde{\left(\frac{3\sigma}{2}\right)}^{\sigma} \, \overline{\odot}^{\sigma} \left(\, \mathbf{I}^{\sigma} \, \overline{\odot}^{\sigma} \left(\, \left(|\kappa| \overline{\oplus}^{\sigma} |\lambda| \right) \overline{\odot}^{\sigma} \, \widetilde{\left(\frac{1}{1_{\sigma}}\right)}^{\sigma} \right) \right) \overline{\odot}^{\sigma}} \, \check{\sigma}^{\sigma} = \\ &= \widetilde{\left(\frac{3\sigma}{2}\right)}^{\sigma} \, \overline{\odot}^{\sigma} \, \left(\, \overline{\frac{3\sigma}{2}} \right)^{\sigma} \, \overline{\odot}^{\sigma} \, \left(|\kappa| \overline{\oplus}^{\sigma} |\lambda| \right) \overline{\odot}^{\sigma} \, \widetilde{\left(\frac{1}{1_{\sigma}}\right)}^{\sigma} \, \overline{\odot}^{\sigma} \, \check{\sigma}^{\sigma} = \\ &= \left(\, \overline{\frac{\sigma}{2}} \right)^{\sigma} \, \overline{\odot}^{\sigma} \, \widetilde{\left(\frac{3\sigma}{2}\right)}^{\sigma} \, \overline{\odot}^{\sigma} \, \left(\, \underline{|\kappa|_{\sigma}} + \underline{|\lambda|_{\sigma}} \right)^{\sigma} \, \overline{\odot}^{\sigma} \, \widetilde{\left(\frac{1}{1_{\sigma}}\right)}^{\sigma} = \\ &= \left(\, \overline{\frac{\sigma}{2}} \right)^{\sigma} \, \overline{\odot}^{\sigma} \, \left(\, \overline{\frac{3\sigma}{2 \cdot \underline{1}_{\sigma}}} \cdot \left(\, \underline{|\kappa|_{\sigma}} + \underline{|\lambda|_{\sigma}} \right) \right)^{\sigma} = \left(\, \overline{\frac{\sigma}{2}} - \frac{3\sigma \, \left(\, \underline{|\kappa|_{\sigma}} + \underline{|\lambda|_{\sigma}} \right)}{2 \cdot \underline{1}_{\sigma}} \right)^{\sigma} = \\ &= \sigma \, \tanh^{-1} \left(\, \overline{\frac{1}{2}} - \frac{3\left(\, \underline{|\kappa|_{\sigma}} + \underline{|\lambda|_{\sigma}} \right)}{2 \cdot \underline{1}_{\sigma}} \right) = \sigma \, \tanh^{-1} \left(\, \overline{\frac{1}{2}} - \frac{3\left(\tanh \frac{|\kappa|}{\sigma} + \tanh \frac{|\lambda|}{\sigma} \right)}{2\sigma \, \tanh \frac{1}{\sigma}} \right). \end{split}$$

Istennek Hála, 2019.10.01;10.56.

Szalay I.

So, the "fragment" is described by

$$(6.9) \quad \widetilde{\Delta_a^m}^{\sigma} \setminus \left(\widetilde{\Delta_a^m}^{\sigma}\right)^{\sigma - box} = \left\{ P = (\kappa, \lambda, \mu) \in \mathbb{R}^3 \middle| \begin{cases} \tanh \frac{|\kappa|}{\sigma} + \tanh \frac{|\lambda|}{\sigma} \le \frac{\sigma \tanh \frac{1}{\sigma}}{3} \\ 0 \le \mu \le \sigma \tanh^{-1} \left(\frac{1}{2} - \frac{3\left(\tanh \frac{|\kappa|}{\sigma} + \tanh \frac{|\lambda|}{\sigma}\right)}{2\sigma \tanh \frac{1}{\sigma}}\right) \end{cases} \right\}$$

or

$$(6.10)\ \widetilde{\Delta_a^m}^\sigma \setminus \left(\widetilde{\Delta_a^m}^\sigma\right)^{\sigma-box} = \left\{ P = (u,v,w) \in \widetilde{\mathbb{R}^3}^\sigma \middle| \begin{cases} |u| \overline{\bigoplus}^\sigma |v| \leq \left(\widecheck{1}^\sigma \overline{\bigcirc}^\sigma \left(\overline{\widecheck{\alpha}}\right)^\sigma\right) \overline{\bigcirc}^\sigma \left(\overline{\left(\frac{\sqrt{2}}{a}\right)}^\sigma \right) \\ \check{\sigma}^\sigma \leq w \leq \widecheck{m}^\sigma \overline{\bigcirc}^\sigma \left(\widecheck{1}^\sigma \overline{\bigcirc}^\sigma \left(\left(|\kappa| \overline{\bigoplus}^\sigma |\lambda|\right) \overline{\bigcirc}^\sigma \left(\overline{\left(\frac{\sqrt{2}}{a}\right)}^\sigma\right)\right) \right\}. \end{cases}$$

Descriptions under (6.9) and (6.10) are valid int he universe $\widetilde{\mathbb{S}_{\mathcal{O}_0}}^{\sigma}$ having the center $\widecheck{\mathcal{O}_0}^{\sigma} = (0,0,\check{\sigma}^{\sigma})$ and in the Multiverse (with center $\mathcal{O} = (0,0,0) \notin \widetilde{\mathbb{S}_{\mathcal{O}_0}}^{\sigma}$), respectively. The "fragment") $\widecheck{\Delta_a^m}^{\sigma} \setminus \bigl(\widecheck{\Delta_a^m}^{\sigma}\bigr)^{\sigma-box}$ is invisible in our universe. Its base is characterised by the inequality

$$|u|\overline{\oplus}^{\sigma}|v| \leq \left(\widecheck{1}^{\sigma}\overline{\ominus}^{\sigma}\right)\overline{\odot}^{\sigma}\widetilde{\left(\frac{\sigma}{m}\right)}^{\sigma}\widetilde{\left(\frac{\sqrt{2}}{a}\right)}^{\sigma}$$

and located on the "upper" border. (See Fig. 15 and Fig. 16. with $\sigma=1$.) Of course, the peak point of the super – pyramid $\widetilde{\Delta_a^m}^\sigma$ is $\widetilde{\mathbb{Q}}^\sigma=\left(0,0,\left(\frac{3\sigma}{2}\right)^\sigma\right)$, (see (5.11) with u=v=0 and $m=\frac{3}{2}$) is visible in $\widetilde{\mathbb{S}_{\mathcal{O}_0}}^\sigma$, because by shifted coordinates $\widetilde{\mathbb{Q}}^\sigma=\left(0,0,\left(\frac{\sigma}{2}\right)^\sigma\right)$, (see (6.9) with $\kappa=\lambda=0$ and σ tanh⁻¹ $\frac{1}{2}=\left(\frac{\sigma}{2}\right)^\sigma$). The next figure shows the "fragment". For the sake of simplicity we compute with the parameter =1. The border of the base has equation $\tanh|\kappa|+\tanh|\lambda|=\frac{\tanh 1}{3}$. The coordinate – system " κ,λ,μ " is in the universe $\widetilde{\mathbb{S}_{\mathcal{O}_0}}^\sigma$ with center $\widetilde{\mathcal{O}_0}^\sigma=(\kappa=0,\lambda=0,\mu=0)$.

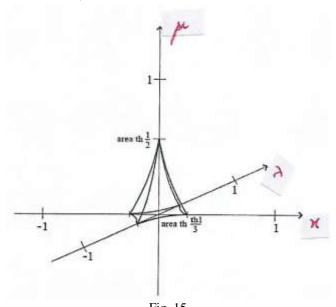


Fig. 15 ent" is seen (without distorsion) in

Moreover, the projection of the base of the "fragment" is seen (without distorsion) in the "u,v" coordinate plane of our Universe

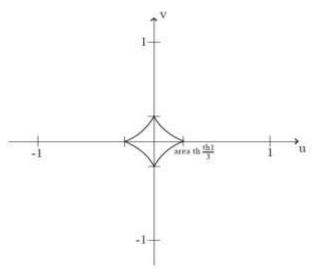


Fig. 16

The base of the "fragment" itself is invisible in our universe. This fact is showed by the dotted lines in Fig.11.

Let be $a = \underline{1}_{\sigma} \cdot \sqrt{2}$ and $m = \frac{3\sigma}{2}$. Considering (5.11) and (6.10) the decomposition $\widetilde{\Delta_a^m}^{\sigma} = \left(\widetilde{\Delta_a^m}^{\sigma}\right)^{\sigma-box} \cup \left(\widetilde{\Delta_a^m}^{\sigma} \setminus \left(\widetilde{\Delta_a^m}^{\sigma}\right)^{\sigma-box}\right)$, where $\left(\widetilde{\Delta_a^m}^{\sigma}\right)^{\sigma-box}$ is in our universe \mathbb{R}^3 and $\left(\widetilde{\Delta_a^m}^{\sigma} \setminus \left(\widetilde{\Delta_a^m}^{\sigma}\right)^{\sigma-box}\right)$ is situated in the universe $\widetilde{\mathbb{S}_{\mathcal{O}_0}}^{\sigma}$ having the center $\widetilde{\mathcal{O}_0}^{\sigma} = (0,0,\check{\sigma}^{\sigma})$. On the other hand $\left(\widetilde{\Delta_a^m}^{\sigma}\right)^{\sigma-box} \cap \left(\widetilde{\Delta_a^m}^{\sigma} \setminus \left(\widetilde{\Delta_a^m}^{\sigma}\right)^{\sigma-box}\right) = \{$ } proves that the super pyramid $\widetilde{\Delta_a^m}^{\sigma}$ has two pieces, situated in different universes.

As the super - pyramid Δ_a^m is a coherent building, we will show a universe that the super pyramid Δ_a^m is already visible in it. In other words, we will show what it looks like in one piece! We will use the super – shift transformation

(6.11)
$$\begin{cases} \kappa = u \\ \lambda = v \\ \mu = w \ \overline{\Theta}^{\sigma} \ (\overline{\frac{3\sigma}{4}})^{\sigma} \ , \check{P}^{\sigma} = (u, v, w) \ with \ center \ \widecheck{\mathcal{O}_*}^{\sigma} = \left(0, 0, \overline{\left(\frac{3\sigma}{4}\right)}^{\sigma}\right) \in \mathbb{R}^3 \ . \end{cases}$$

Starting from (5.11) by (6.11) we consider

$$0 \le w \le \widecheck{m}^{\sigma} \overline{\Theta}^{\sigma} \left(\widecheck{1}^{\sigma} \overline{\Theta}^{\sigma} \left(\left(|u| \overline{\oplus}^{\sigma} |v| \right) \overline{\Theta}^{\sigma} \left(\overline{\sqrt{2}} \overline{a} \right)^{\sigma} \right) \right)$$

and hence

$$-\widetilde{\left(\frac{3\sigma}{4}\right)}^{\sigma} \leq \mu \leq \widecheck{m}^{\sigma}\overline{\odot}^{\sigma}\left(\widecheck{1}^{\sigma}\overline{\Theta}^{\sigma}\left(\left(|\kappa|\overline{\oplus}^{\sigma}|\lambda|\right)\overline{\odot}^{\sigma}\widetilde{\left(\frac{\sqrt{2}}{a}\right)}^{\sigma}\right)\right)\overline{\Theta}^{\sigma}\widetilde{\left(\frac{3\sigma}{4}\right)}^{\sigma}.$$

Computing with $\widetilde{m}^{\sigma} = \underbrace{\left(\frac{3\sigma}{2}\right)^{\sigma}}^{\sigma}$ and omitting the details we write

$$\underbrace{\left(\overline{\frac{3\sigma}{2}}\right)^{\sigma}}_{\sigma} \overline{\bigcirc}^{\sigma} \left(\underbrace{1^{\sigma} \overline{\bigcirc}^{\sigma}}_{\sigma} \left(\left(|\kappa| \overline{\bigoplus}^{\sigma} |\lambda| \right) \overline{\bigcirc}^{\sigma} \left(\underbrace{\frac{\sqrt{2}}{\underline{1}_{\sigma} \cdot \sqrt{2}}}_{\sigma} \right)^{\sigma} \right) \right) \overline{\bigcirc}^{\sigma} \left(\underline{\frac{3\sigma}{4}} \right)^{\sigma} = \\
= \sigma \tanh^{-1} \left(\frac{3}{4} - \frac{3 \left(\tanh \frac{|\kappa|}{\sigma} + \tanh \frac{|\lambda|}{\sigma} \right)}{2\sigma \tanh \frac{1}{\sigma}} \right).$$

Moreover, we have that $-\widetilde{\left(\frac{3\sigma}{4}\right)}^{\sigma} = -\sigma \tanh^{-1} \frac{3}{4}$. So,

$$(6.12) \quad \widetilde{\Delta_a^m}^{\sigma} = \left\{ P = (\kappa, \lambda, \mu) \in \mathbb{R}^3 \middle| \begin{cases} \tanh \frac{|\kappa|}{\sigma} + \tanh \frac{|\lambda|}{\sigma} \leq \sigma \tanh \frac{1}{\sigma} \\ -\sigma \tanh^{-1} \frac{3}{4} \leq \mu \leq \sigma \tanh^{-1} \left(\frac{3}{4} - \frac{3(\tanh \frac{|\kappa|}{\sigma} + \tanh \frac{|\lambda|}{\sigma})}{2\sigma \tanh \frac{1}{\sigma}} \right) \right\}$$

is obtained.

The super – pyramid $\widetilde{\Delta_a^m}^\sigma(a=\underline{1}_\sigma\cdot\sqrt{2}\ and\ m=\frac{3\sigma}{2}\)$ offers a coherent sight in the universe $\widetilde{\mathcal{S}_{\mathcal{O}_*}}^\sigma$ with center $\widetilde{\mathcal{O}_*}^\sigma=\left(0,0,\overline{\left(\frac{3\sigma}{4}\right)}^\sigma\right)\in\mathbb{R}^3$.

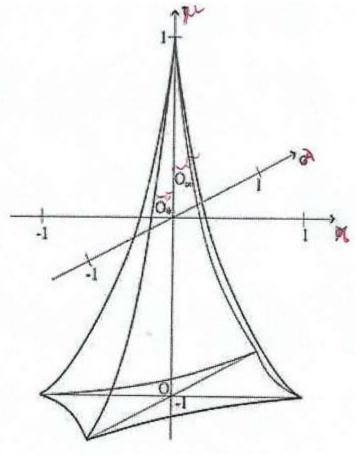


Fig. 17

(Be careful! For Fig.17 the parameter $\sigma=1$ is choosen. The peak point of the super – pyramid has the coordinates

$$\kappa=0, \lambda=0 \ and \ \mu=\left(\overline{\frac{3}{4}}\right)^1=\tanh^{-1}\frac{3}{4}\approx 0,9729550745<1$$
 but in Fig.17 the last coordinate seems as 1.)

Observe the following points given in the dual coordinate system!

$$\mathcal{O} = (u = 0, v = 0, w = 0) = \left(\kappa = 0, \lambda = 0, \mu = -\left(\frac{3\sigma}{4}\right)^{\sigma}\right)$$

$$\widetilde{\mathcal{O}_*}^{\sigma} = \left(u = 0, v = 0, w = \left(\frac{3\sigma}{4}\right)^{\sigma}\right) = (\kappa = 0, \lambda = 0, \mu = 0)$$

$$\widetilde{\mathcal{O}_{\infty}}^{\sigma} = (u = 0, v = 0, w = \check{\sigma}^{\sigma}) = \left(\kappa = 0, \lambda = 0, \mu = \left(\frac{\check{\sigma}}{4}\right)^{\sigma}\right)$$
and the peak point of $\widetilde{\Delta_a^m}^{\sigma}$ with $\kappa = 0, \lambda = 0$ and $\mu = \left(\frac{3\sigma}{4}\right)^{\sigma}$

Of course, $\mathcal{O}, \mathcal{O}_*^{\sigma}$ and $\mathcal{O}_{\infty}^{\sigma}$ are situated in the Multiverse \mathbb{R}^3 and in the universe $\mathcal{S}_{\mathcal{O}_*}^{\sigma}$ with center $\mathcal{O}_*^{\sigma} = \left(0,0,\left(\frac{3\sigma}{4}\right)^{\sigma}\right)$ but while \mathcal{O} and \mathcal{O}_*^{σ} is situated in our universe, the point $\mathcal{O}_{\infty}^{\sigma}$ is "over" \mathbb{R}^3 . Moreover, the passage $[\mathcal{O},\mathcal{O}_{\infty}^{\sigma}]$ is running in our universe but the closed passage beginning from the point $\mathcal{O}_{\infty}^{\sigma}$ to the top of super – pyramid is ouside. In brief, the super – passage comes out from our universe.

Looking back we can see the base of super - pyramid $\widetilde{\Delta_a^m}^{\sigma}$ by previous figures, too. Namely, it is seen in Fig.9, Fig. 11 and Fig. 17 with center \mathcal{O} . The cover of super – truncated pyramid $\left(\widetilde{\Delta_a^m}^{\sigma}\right)^{\sigma-box}$ with the center $\widetilde{\mathcal{O}_{\infty}}^{\sigma}$ is seen in Fig. 11, Fig 15 and Fig. 16.

VII. CONCLUSIONS

At the end of Remark 1 (see point 3.) there were three hypotheses. Among them the third (Bravely Naive Hypothesis) is very exiting. Now we give more remarks to them.

Remark 2. The sub-pyramide $\underline{\Delta}_{a}^{m}$ (see (3.8),(3.9) and Fig. 5) is forever locked into the σ -compressed universe $\underline{\mathbb{R}^{3}}$ (see (3.2)).

Remark 3. Using the real numbers and their usual operations (addition, multiplication and so on) the pyramid Δ_a^m (see (0.2) and Fig. 1) is forever locked into our three – dymensional universe \mathbb{R}^3 . (See the Babylonian tower in the Bible.)

Remark 4. If the super – pyramid $\widetilde{\Delta_a^m}^{\sigma}$ (see (5.11) is constructed by sufficiently small a and m (see the conditions (5.4) and (5.6)) then the super – pyramid remains in our universe \mathbb{R}^3 . (See (5.8) and Fig. 10) This fact may be interesting for architecture or for radiating–technology with respect to the Eiffel Tower and antennae.

Remark 5. The next figure shows that the pyramide Δ_a^m constructed by the real numbers and their usual operations is able to break through the border of the σ -compressed universe \mathbb{R}^3 .

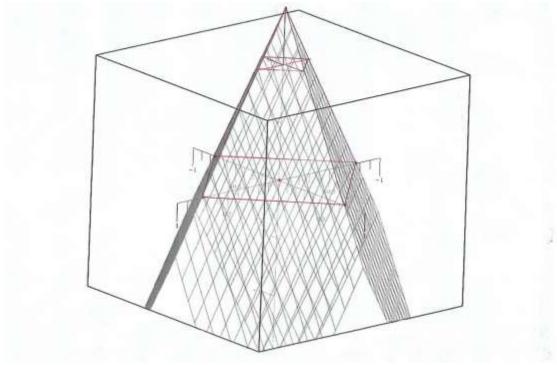


Fig. 18

Now we find an important analogy between our universe and the Multiverse.

Remark 6. If the super – pyramid Δ_a^m (see (5.11) is constructed by sufficiently small a (see the condition (5.4)) and m such that $m \ge \sigma$ then the super – pyramid reachs or breaks through the invisible border of our universe \mathbb{R}^3 (see Fig. 11 and Fig 15). Thus, we can say that the explosion is not a usual magnification, but much more. Because, the explosion is able to expel the bodies from our universe. Moreover, you will notice that the exploded numbers and their super - operations must be used, because \check{m}^{σ} is not a real number. (The familiar operations (addition, multiplication and so on) do not work for invisible exploded numbers.)

Remark 7. If humanity finds the tecnical realization of super – shift transformation (see (6.4) then it can build an antenna that connects us with other universe.

Remark 8. Any point in our universe can be the origin of a parallel universe. If this point is far from us then the new parts of the Multiverse will be discovered. (See Fig. 17)

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