A Comparative Study for Solving Interval Linear Assignment Problem

P.Rajarajeswari[1], D.Maheswari[2]

[1]*Assistant Professor of Mathematics,*
Chikkanna Government Arts College, Tamilnadu, India.
[2]*Associate Professor of Mathematics,*
CMS College of Science and Commerce, Tamilnadu, India.

Abstract: *In this paper, the interval integer Assignment problem is considered. First IAP is converted to octagonal numbers using fuzzification technique and secondly it is converted to crisp numbers using different ranking techniques. Now different methods are applied to find the optimal solution of the assignment problem To illustrate this, numerical examples are solved and the results are compared.*

Keywords: *Interval numbers, Octagonal fuzzy numbers, assignment problem, ranking techniques.*

I. INTRODUCTION

The optimization problem of minimizing total cost in assigning available jobs to different men/machines in an organization/manufacturing unit under the condition that one machine has to take only one job, is generally referred to as an Assignment Problem. Different methods have been presented for an assignment problem and various articles have been published for finding the optimal solution of the problem. A.Seethalakshmy, N. Srinivasan [2] by using Zero Reduction Method solved Assignment Problem. A. Thiruppathi and D. Iranian [3] developed an innovative method for finding the optimal solution to assignment problems. M. Khalid et al [11] proposed new improved one assignment method. H.W. Kuhn [5] developed the Hungarian method for solving the assignment problem. J.G. Kotwal and T.S. Dhope [10] solved the unbalanced assignment problem by using the modified approach technique. S.T. Nizam et al [9] applied a genetic algorithm technique for finding the cost of the assignment problems.

The purpose of this paper is to know about the different effective algorithms are available to find the optimal solution of an assignment problem which aims to reduce computational cost in less time.

II. PRELIMINARIES

2.1 Definition

An interval number A is defined as A= [a, b] $=\{x \,/\, a \le x \le b,\ x \in \Re\}$. Here a, b$\in\Re$ are the lower and upper bound of the intervals.

2.2 Definition

Arithmetic operations on Interval Numbers
Let A = [a, b] and B =[c , d] are two interval numbers.
Addition: A + B = [a ,b] + [c , d] =[a + c , b + d].
Subtraction : A + B = [a ,b] - [c , d] = [a - d , c – b].
Multiplication: A * B = [x, y] where x = min {ac, ad, bc, bd } and
y = max {ac , ad , bc , bd}.

2.3 Definition

A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$, where $a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0\,, & x < a_1 \\ k\left(\dfrac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ k & a_2 \le x \le a_3 \\ k+(1-k)\left(\dfrac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ 1 & a_4 \le x \le a_5 \\ k+(1-k)\left(\dfrac{a_6-x}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ k & a_6 \le x \le a_7 \\ k\left(\dfrac{a_8-x}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ 0 & x \ge a_8 \end{cases}$$

where 0 < k < 1.

2.4 Definition

If ã is a convex fuzzy number, the Robust ranking index is defined by R(ã)= $\int_0^1 0.5\ [\ \alpha_\alpha{}^{L}, \alpha_\alpha{}^{U}\ ]\ d\alpha$,

Where $(\alpha_\alpha{}^{L}, \alpha_\alpha{}^{U}) = \{(\ b-a)\ \alpha + a,\ d-(d-c)\alpha\ \},\{\ (f-e)\ \alpha + e,\ \mathrm{h}-(\mathrm{h}-g)\alpha\ \}$ is a α-level cut of a fuzzy number ã.

2.5 Definition

Russell method: For a octagonal fuzzy number ã = $(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ a ranking method is given by a formula R(ã) = (1/8)* $(a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8)$

2.7 Fuzzification method

A new approach is used to fuzzily the given interval data into a octagonal fuzzy number. Consider an interval number [L , U] . The difference of this interval is d = $\frac{U-L}{7}$ .The required octagonal l fuzzy number will be in arithmetic progression.

Mathematical formulation of an assignment problem

Consider an assignment problem of assigning n jobs to n machines (one job to one machine). Let C_{ij} be the unit cost of assigning i^{th} machine to j^{th} job and let

$$x_{ij} = \begin{cases} 1, & if\ \ j^{th}\ job\ is\ assigned\ to\ i^{th}\ machine \\ 0 & otherwise \end{cases}$$

The assignment model is then given by the following LPP

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1,2,\ldots\ldots n$$

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2,\ldots\ldots. n$$

and $x_{ij} = 0$ (or) 1.

III. NUMERICAL EXAMPLE

Consider a minimization assignment problem where cost matrix are given in interval entries.

	1	2	3	4	5
A	[-2,8]	[-4,10]	[-3,11]	[4,11]	[-3,6]
B	[1,13]	[9,16]	[2,13]	[6,13]	[-6,36]
C	[-3,6]	[2,9]	[3,13]	[1,10]	[0,7]
D	[5,15]	[-1,10]	[9,16]	[-3,10]	[-3,11]
E	[5,17]	[-2,5]	[-1,6]	[2,11]	[2,11]

Solution:

Conversion of interval numbers to octagonal numbers using (2.7)

	1	2	3	4	5
A	(-2,-0.57,0.86,2.29,3.72,5.15,6.58,8)	(-4,-2,0,2,4,6,8,10)	(-3,-1,1,3,5,7,9,11)	(4,5,6,7,8,9,10,11)	(-3,-1.72,-0.44,0.84,2.12,3.4,4.68,6)
B	(1,2.7,4.4,6.1,7.8,9.5,11.2,13)	(9,10,11,12,13,14,15,16)	(2,3.57,5.14,6.71,8.28,	(6,7,8,9,10,11,12,13)	(-6,0,6,12,18,24,30,36)
C	(-3,-1.72,-0.44,0.84,2.12,3.4,4.68,6)	(2,3,4,5,6,7,8,9)	(3,4.43,5.86,7.29,8.72,10.15,11.58,13)	(1,2.28,3.56,4.,86,6.12,7.4,8.68,10)	(0,1,2,3,4,5,6,7)
D	(5,6.43,7.86,9.29,10.72,12.15,13.58,15)	(-1,0.58,2.16,3.74,5.32,6.9,8.48,10)	(9,10,11,12,13,14,15,16)	(-3,-1.15,0.7,2.55,4.4,6.25,8.1,10)	((-3,-1,1,3,5,7,9,11)
E	(5,6.7,8.4,10.1,11.8,13.5,15.2,17)	(-2,-1,0,1,2,3,4,5)	(-1,0,1,2,3,4,5,6)	(2,3.28,4.56,5.84,7.12,8.4,9.68,11)	(2,3.28,4.56,5.84,7.12,8.4,9.68,11)

Apply ranking technique (2.4) to convert into crisp numbers

	1	2	3	4	5
A	6	6	8	15	3
B	14	25	15	19	30
C	3	11	16	11	7
D	20	9	25	7	8
E	22	3	5	13	13

Method [5]

Row reduction

	1	2	3	4	5
A	3	3	5	12	0
B	0	11	1	5	16
C	0	8	13	8	4
D	13	2	18	0	1
E	19	0	2	10	10

Column reduction

	1	2	3	4	5
A	3	3	4	12	**0**
B	~~0~~	11	**0**	5	16
C	**0**	8	12	8	4
D	13	2	17	**0**	1
E	19	**0**	1	10	10

The optimum assignment is A→5 ,B→3,C→1,D→4,E→2

Total cost = 3+15+3+7+3 ⇒31

Method [1]

	1	2	3	4	5	
A	3	3	4	12	53 0	
B	67 0	11	66 0	5	16	
C	67 0	8	12	8	4	
D	13	2	17	68 **0**	1	
E	19	64 0	1	10	10	

	1	~~2~~	3	5
A	3	~~3~~	4	40 0
B	49 0	~~11~~	44 0	16
C	46 0	~~8~~	12	4
~~E~~	~~19~~	~~52~~ ~~**0**~~	~~1~~	~~10~~

Finally we have,

	1	2	3	4	5
A	6	6	8	15	**3**
B	14	25	**15**	19	30
C	**3**	11	16	11	7
D	20	9	25	**7**	8
E	22	**3**	5	13	13

The optimum assignment is A→5, B→3, C→1, D→4, E→2
Total cost = 3+15+3+7+3 ⇒31

Method [2]

Row minimum

	A	B	C	D	E	Row	value	Column
A	3	3	5	12	**0**	**A**	**0**	**E**
B	0	11	~~1~~/**0**	5	16	B	0	~~A/~~c
C	**0**	8	13	8	4	C	0	A
D	13	2	18	**0**	1	**D**	**0**	**D**
E	19	**0**	2	10	10	**E**	**0**	**B**

The optimum assignment is A→5, B→3, C→1, D→4, E→2
Total cost = 3+15+3+7+3 ⇒31

Method [7]

	A	B	C	D	E	ROC			
A	6	6	8	15	**3**	3	3	3	**12**
B	14	25	**15**	19	30	1	**4**	-	-
C	**3**	11	16	11	7	**4**	-	-	-
D	20	9	25	**7**	8	1	1	1	**1**
E	22	**3**	5	13	13	2	2	**10**	-

The optimum assignment is A→5, B→3, C→1, D→4, E→2
Total cost = 3+15+3+7+3 ⇒31

Method [6]

Row minimum

	1	2	3	4	5	Position of 0's	value
A	3	3	5	12	**0**	5	1.5
B	0	11	~~1~~/**0**	5	16	1	~~7~~/7.5
C	**0**	8	13	8	4	1	1.5
D	13	2	18	**0**	1	4	3.5
E	19	**0**	2	10	10	2	1.5

The optimum assignment is A→5, B→3, C→1, D→4, E→2
Total cost = 3+15+3+7+3 ⇒31

Comparison Table: 1

Method	Method [5]	Method [1]	Method [2]	Method [7]	Method [6]
Optimum Cost	31	31	31	31	31

Russell's ranking technique.

	1	2	3	4	5
A	3	3	4	7.5	1.5
B	7	12.5	7.5	9.5	15
C	1.5	5.5	8	5.5	3.5
D	10	4.5	12.5	3.5	4
E	11	1.5	2.5	6.5	6.5

Final solution:

	1	2	3	4	5
A	3	3	4	7.5	**1.5**
B	7	12.5	**7.5**	9.5	15
C	**1.5**	5.5	8	5.5	3.5
D	10	4.5	12.5	**3.5**	4
E	11	**1.5**	2.5	6.5	6.5

The optimum assignment is A→5, B→3, C→1, D→4, E→2
Total cost = 3+15+3+7+3 ⇒15.5

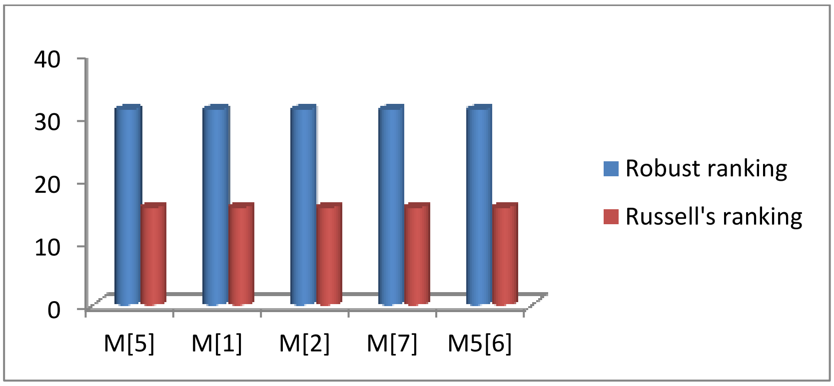
Comparison Table: 2

Method	Method [5]	Method [1]	Method [2]	Method [7]	Method [6]
Optimum Cost	15.5	15.5	15.5	15.5	15.5

IV. RESULTS AND DISCUSSION

The different methods for solving the assignment method are implemented in the numerical examples the obtained results are 31 and 15.5. These results of the numerical examples are equal to that of the results by the Hungarian assignment method. We see that the results obtained by Russell's ranking technique is very less when compared to Robust ranking. The achieved results are also represented graphically in the figure plotted below.

The above methods of assignment provide the optimal solutions when applied to the assignment problems of types balanced, minimization, maximization, and unbalanced assignment problems.



V. CONCLUSION

In this paper, different methods are applied for solving assignment problems. All the methods have a systematic procedure and very easy to understand. From this paper, it can be concluded that different method provides an optimal solution directly in a few steps for the assignment problem. The methods consume less time and it is very easy to understand. The optimal solution obtained by different methods is the same as that of the Hungarian method.

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