Effect of Unsteadiness On Blood Flow Through A Stenosed Artery Using A Third Grade Fluid Model With Slip Conditions

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ABSTRACT

Effect of unsteadiness on blood flow through a stenosed artery using a third grade fluid model with slip conditions was investigated in this paper. Externally applied magnetic field was also taken into consideration. The solutions of the steady and unsteady non-linear dimensionless momentum equations are obtained using Galerkin's Weighted residual, Newton Raphson and Fourth order Runge- Kutta methods. Comparative analysis as regards effects of slip velocity, magnetic field, shear thinning, pressure gradient and Reynold number on the flow characteristics of steady and unsteady third grade blood flow models were carried out and the results were presented graphically. It was reveals from the graphs that, slip velocity and shear thinning increases with velocity profiles, flow rate but reduces resistance to fluid flow for both steady and unsteady blood flow models. Also, magnetic field parameter increases with resistance to fluid flow but reduces velocity profiles and shear stress for both steady and unsteady blood flow models. Other parameters that can influences both steady and unsteady blood flow are pressure gradient and Reynold number. Finally, for the constant values of the parameters such as slip velocity, magnetic field, shear thinning, pressure gradient and Reynold number, the velocity profile of unsteady blood model is higher than that of the steady blood flow model.

Keywords: *Steady blood flow, unsteady blood flow, magnetic field, slip conditions, stenosed artery, shear thinning, shear thickening, third grade fluid models and Galerkin's Weighted residual methods.*

INTRODUCITON

Blood flow circulation disorders are well known to be responsible in most cases of death globally Shanthi *et al* [1] and stenosis is one of such cases. Stenosis can be viewed as the abnormal and unnatural growth in the arterial wall thickness that develops at various locations of the cardiovascular system Young and Tsai [2]. The presence of stenosis in the cardiovascular system can cause a serious circulatory disorders by occluding the normal blood supply which may result in myocardial infarction or cerebral strokes Miller [3].

Many researchers Biswas and Chakraborty [4], Narendra *et al* [5], Devajyoti and Uday [6] to mention but a few have carried out investigation theoretically and experimentally and taking blood behaving as a Newtonian fluid. But at low shear rate and in small diameter arteries, blood which contains predominantly suspension of erythrocytes in plasma behaves as a non-Newtonian fluid. It was reveals in the literature that, shear rate of blood is very low in the stenotic region. Some of the researchers that considered blood as non-Newtonian fluid in their studies includes Devajyoti and Rezia [7], Mallik *et al* [8], Misra and Shit [10].

It may be worthwhile to notice that all the aforementioned researchers did not considered unsteady blood flow in their studies. This is due to the fact that in an unsteady fluid flow, the conditions at any point of the flow depends on time and thus rendered the non-linear governing equation describing the fluid flow cumbersome to handle as the equation now have time derivative. Inview of this, Venkateswarlu and Rao [11] used difference finite method to solved numerically the unsteady flowof blood through an indented tube with atherosclerosis in the presence of mild Stenosis. Geeta and Siddiqui [12] used an appropriate perturbation scheme to obtained solutions to analysis of unsteady blood flow through stenosed artery with slip effects. They considered small womerseley frequency parameter and their results reveals that as the time along the axial distance increases, the axial velocities, wall shear stress, flow rate decreases. Some of the other researchers that considered unsteady blood flow models in their studies includes Srikanth *et al* [13], Ikbal *et al* [14], Islam [15], and Mandal [16].

In a very recent development Jimoh *et al* [17] investigated computational analysis of unsteady flow of blood and heat transfer through a stenosed artery in a third grade fluid model with slip conditions. They used Galerkin's weighted residual method to obtained solutions and shows the results graphically. Their results reveals that velocity profile, volumetric flow rate and shear stress increases while temperature profile and resistance to flow decreases with increasing values of the slip velocity. Effect of unsteadiness on blood flow through a stenosed artery remain outstanding in the literatures.

Thus, the present investigation has been devoted to the problem of effect of unsteadiness on blood flow through a stenosed artery in a third grade fluid model with slip conditions. Also incorporated in to the model is an externally applied magnetic field.

II. Mathematical Models

The momentum equations describing the fluid flow as obtained by Mohammed [18] is given as

$$\frac{\partial w}{\partial t}=\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+\frac{6\beta_3}{\rho}\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2}+\frac{2\beta_3}{\rho r}\left(\frac{\partial w}{\partial r}\right)^2+\frac{\alpha_1}{\rho r}\frac{\partial^2 w}{\partial r\partial t}+\frac{\alpha_1}{\rho}\frac{\partial^3 w}{\partial r^2\partial t}-\frac{\partial \bar{P}}{\rho\partial z}-\frac{\sigma\beta_0^2}{\rho}w \qquad (2.1)$$

From equation (2.1)the term$\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)$ can be re-written as

$$\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)=\frac{\mu}{\rho r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) \qquad (2.2)$$

When we substituted (2.2) into (2.1), we obtained

$$\frac{\partial w}{\partial t}=\frac{\mu}{\rho r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{6\beta_3}{\rho}\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2}+\frac{2\beta_3}{\rho r}\left(\frac{\partial w}{\partial r}\right)^2+\frac{\alpha_1}{\rho r}\frac{\partial^2 w}{\partial r\partial t}+\frac{\alpha_1}{\rho}\frac{\partial^3 w}{\partial r^2\partial t}-\frac{\partial \bar{P}}{\rho\partial z}-\frac{\sigma\beta_0^2}{\rho}w \qquad (2.3)$$

As a result of the constricted artery as shown in figure1, we employed slip velocity and the slip conditions to equation(2.3) is given as

$$\left.\begin{array}{lll} w = w_s \text{ at} & r = R(z) \\ \frac{\partial w}{\partial r} = 0 & \text{at} & r = 0 \end{array}\right\} \quad (2.4)$$

To non-dimensionalize equations (2.3) and (2.4), we introduce the following parameters and variables

$$\left.\begin{array}{ll} \bar{w} = \frac{w}{d/t_0}, & y = r/R_0 \\ \bar{t} = \frac{t}{t_0}, & V_0 = \frac{w_s t_0}{d} \end{array}\right\} \quad (2.5)$$

When equation (2.5) is substituted into (2.3) and (2.4), after simplifying we obtained

$$\frac{\partial \bar{w}}{\partial \bar{t}} = \frac{1}{\mathrm{RE}} \cdot \frac{\partial}{\partial y}\left(y \frac{\partial \bar{w}}{\partial y}\right) + \Omega\left(6\left(\frac{\partial \bar{w}}{\partial y}\right)^2 \frac{\partial^2 \bar{\omega}}{\partial y^2} + \frac{2}{y}\left(\frac{\partial \bar{w}}{\partial y}\right)^3\right) + \Omega_1\left(\frac{1}{y}\frac{\partial^2 \bar{w}}{\partial y \partial \bar{t}} + \frac{\partial^3 \bar{w}}{\partial y^2 \partial \bar{t}}\right) + \mathrm{G} - \mathrm{M}\bar{w} \quad (2.6)$$

where

$$\left.\begin{array}{l} \mathrm{RE} = \frac{R_0^2}{V t_0}, \mathrm{G} = -\frac{t_0^2}{d\rho}\frac{\partial \hat{p}}{\partial z}, \Omega = \frac{\beta_3 d^2}{t_0 \rho R_0^4} \\ \Omega_1 = \frac{\alpha_1}{R_0^2 \rho}, \ \mathrm{M} = \frac{t_0 \sigma \beta_0^2}{\rho} \end{array}\right\} \quad (2.7)$$

with dimensionless slip conditions simplified as

$$\left.\begin{array}{lll} \bar{w} = V_{01} \text{ at} & y = \frac{R}{R_0} = R_b \\ \frac{\partial \bar{w}}{\partial y} = 0 & \text{at} & y = 0 \end{array}\right\} \quad (2.8)$$

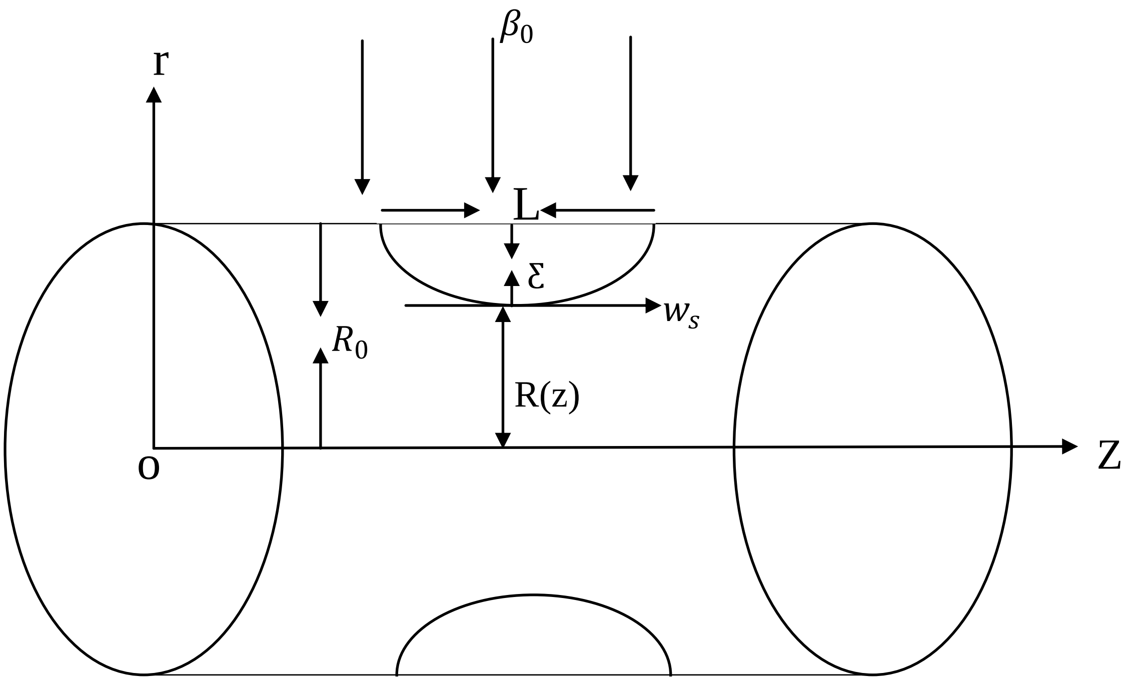

Figure1.Geometry of the stenosis

and has been described by Young [19] and Biswas [20]

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \frac{\varepsilon}{2R_0}\left[1 + \frac{cos\pi z}{L}\right], & \text{for} \quad |z| \leq L \\ R_0, & \text{for} \quad |z| > L \end{cases} \tag{2.9}$$

Equation (2.6) is the dimensionless momentum equation describing the flow while equation (2.8) is the corresponding dimensionless slip conditions. In the next section one considered two special cases of equation (2.6).

III. Solution of the Transformed Dimensionless Equation

(i) case 1

Setting $\frac{\partial \overline{w}}{\partial \bar{t}} = 0$in equation (2.6) one obtained

$$\frac{1}{\mathrm{RE}} \cdot \frac{\partial}{\partial y}\left(y\frac{\partial \overline{w}}{\partial y}\right) + \Omega\left(6\left(\frac{\partial \overline{w}}{\partial y}\right)^2 \frac{\partial^2 \overline{\omega}}{\partial y^2} + \frac{2}{y}\left(\frac{\partial \overline{w}}{\partial y}\right)^3\right) + \mathrm{G} - \mathrm{M}\overline{w} = 0 \tag{3.1}$$

with the associated dimensionless slip conditions simplified as

$$\left.\begin{aligned} &\bar{w} = V_0 \quad \text{at} \quad y = R_b \\ &\frac{\partial \bar{w}}{\partial y} = 0 \quad \text{at} \quad y = 0 \end{aligned}\right\} \tag{3.2}$$

In order to obtain solution to equation (3.1), one use Galerkin weighted residual method by taken approximate solution of the form

$$\bar{w}(y) = a_0 + a_1 y + a_2 y^2 \tag{3.3}$$

Subjecting (3.3) to the slip conditions (3.2) and after simplifying yields

$$\bar{w}(y) = \frac{V_0 y^2}{Rb^2} + a_0\left(1 - \frac{y^2}{Rb^2}\right) + a_2 \frac{y^2}{Rb^2}\left(1 - \frac{y^2}{Rb^2}\right) \tag{3.4}$$

Now,

$$\text{Let } \bar{r} = \frac{y}{Rb} \tag{3.5}$$

Putting (3.5) into (3.4) to obtain

$$w(\bar{r}) = V_0\bar{r}^2 + a_0(1 - \bar{r}^2) + a_2\bar{r}^2(1 - \bar{r}^2) \tag{3.6}$$

For convenience sake, the bar is dropped and write (3.6) as

$$w(r) = V_0 r^2 + a_0(1 - r^2) + a_2 r^2(1 - r^2) \tag{3.7}$$

The residual for equation (3.1) can be written as

$$RR_1(a_0, a_2, r) = G + \frac{1}{RE}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \Omega\left(6\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2} + \frac{2}{r}\left(\frac{\partial w}{\partial r}\right)^3\right) - Mw \tag{3.8}$$

By substituting (3.7) into (3.8) and simplified in full, one obtained

$$RR_1(a_0, a_2, r) = G + \frac{1}{RE}(4V_0 + 4a_2 - 4a_0 - 16a_2r^2) + \Omega(48V_0^2r^2 - 144V_0^2a_0r^2 + 144V_0^2a_2r^2 -$$
$$480V_0^2a_2r^4 + 144V_0a_0^2r^2 - 40V_0a_0a_2r^2 + 960V_0a_0a_2r^2 + 144V_0a_2^2r^2 - 144V_0a_2^2r^4 + 1344V_0a_2^2r^6 +$$
$$144a_0^2a_2r^2 - 480a_0^2a_2r^4 - 144a_0a_2^2r^2 + 960a_0a_2^2r^4 - 1344a_0a_2^2r^6 - 480a_2^3r^4 + 1344a_2^3r^6 - 48a_0^3r^2 +$$
$$48a_2^3r^2 - 1152a_2^3r^8 - 128a_2^3r^8 + 192V_0a_2^2r^6 - 192a_0a_2^2r^6 + 192a_2^3r^6 - 96V_0^2a_2r^4 + 192V_0a_0a_2r^4 -$$
$$192V_0a_2^2r^4 - 96a_0^2a_2r^4 + 192a_0a_2^2r^4 - 96a_2^3r^4 + 16V_0^3r^2 - 48V_0^2a_0r^2 + 48V_0^2a_2r^2 + 48V_0a_0^2r^2 -$$

$96V_0a_0a_2r^2 + 48V_0a_2^2r^2 - 16a_0^3r^2 + 48a_0^2a_2r^2 - 48a_0a_2^2r^2 + 16a_2^3r^2) - M(V_0r^2 + a_0 - a_0r^2 + a_2r^2 - a_2r^4)$ (3.9)

Taking the derivatives of $w(r)$ in (3.7) with respect to a_0 and a_2 respectively, one obtained $(1-r^2)$ and $r^2(1-r^2)$ as the weight functions.

By taking the orthogonality of the residue $RR_1(a_0, a_2, r)$ with respect to the weight functions $(1-r^2)$ and $r^2(1-r^2)$and after simplifying in full, the following systems were obtained

$$327520\Omega a_2^3 - 29568\Omega a_0^3 + 191136\Omega V_0a_2^2 + 88704\Omega V_0a_0^2 - 280896\Omega a_2^2a_0 - 62304\Omega a_0^2a_2 - 341088\Omega V_0a_0a_2 - \left(1848M + 88704\Omega V_0^2 + \frac{9240}{RE}\right)a_0 + \left(-264M + 25344\Omega V_0^2 - \frac{44352}{RE}\right)a_2 = -2310G - \frac{9240V_0}{RE} - 22176\Omega V_0^2 + 462MV_0 \quad (3.10)$$

and

$$-164736\Omega a_0^3 - 6720\Omega a_2^3 + 4944208\Omega V_0a_0^2 + 1331616\Omega V_0a_2^2 - 244608\Omega a_0a_2^2 - 329472\Omega a_2a_0^2 + 1194336\Omega V_0a_0a_2 - \left(3432M + 494208\Omega V_0^2 + \frac{24024}{RE}\right)a_0 - \left(1144M + 329472\Omega V_0^2 - \frac{54912a_2}{RE}\right)a_2$$

$$= -6006G - \frac{24024V_0}{RE} - 123552\Omega V_0^2 + 2574MV_0 \quad (3.11)$$

By substituting the appropriate values of the parameters $\Omega, V_{0,} M, G, and\ RE$ into (3.10) and (3.11) and solve, one obtained the values for a_0 anda_2 which are shown in table 1 below.

(ii) case II

Unsteady fluid flow model were obtained when the conditions at any point of the fluid flow depends on time, that is, when $\frac{\partial\overline{w}}{\partial\bar{t}} \neq 0$and the whole equation (2.6) will be considered.

Thus, to obtain solution to equation (2.6) using Galerkin weighted residual method, we assume a trial solution of the form

$$\overline{w}(y,t) = a_0(t) + a_1(t)y + a_2(t)y^2 \quad (3.12)$$

Subjecting (3.12) to the slip conditions (2.8) and after simplifying yields

$$\overline{w}(y,t) = \frac{V_{01}y^2}{Rb^2} + a_0(t)\left(1 - \frac{y^2}{Rb^2}\right) + a_2(t)\frac{y^2}{Rb^2}\left(1 - \frac{y^2}{Rb^2}\right) \quad (3.13)$$

using (3.5) in (3.13) and after dropping the bars to obtain

$$w(r,t) = V_{01}r^2 + a_0(t)(1-r^2) + a_2(t)r^2(1-r^2) \tag{3.14}$$

The residual for equation (2.6) can be written as

$$RR_2\big(r, a_0(t), a_2(t)\big) = \frac{\partial w}{\partial t} - G_1 - \frac{1}{REI}\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - \Omega\left(6\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2} + \frac{2}{r}\left(\frac{\partial w}{\partial r}\right)^3\right) - \Omega_1\left(\frac{1}{r}\frac{\partial^2 w}{\partial r\partial t} + \frac{\partial^3 w}{\partial r^2\partial t}\right) + M_1 w$$

(3.15) substituting (3.14) into (3.15) and simplifying to obtain

$RR_1\big(r, a_0(t), a_2(t)\big) = -G - \frac{1}{REI}(4V_{01} + 4a_2(t) - 4a_0(t) - 16a_2(t)r^2) + \dot{a}_0(t)(1-r^2) + \dot{a}_2(t)r^2(1-$

r2+4Ω1a0t−a2t+4r2a2t−Ω48V012r2−144V012a0tr2+144V012a2tr2−480V012a2tr4+144V01a02tr2−40V01a0ta2tr2+960V01a0ta2tr4+144V01a22tr2−144V01a22tr44V01a22tr6+144a02a2tr2−480a02ta2tr4−144a0ta22tr2+960a0ta22tr4−1344a0ta22tr6−480a23tr4+1344a23tr6−48a03tr2+48a23tr2−1152a23tr8−128a23tr8+192V01a22tr6−192a0ta22tr6+192a23tr6−96V012a2tr4+192V01a0ta2tr4−192V01a22tr4−96a02ta2tr4+192a0ta22tr4−96a23tr4+16V013r2−48V012a0tr2+48V01a02tr2−96V01a0ta2tr2+48V01a22tr2−16a03t+48a02ta2tr2−48a0ta22tr2+16a23tr2−M1V01r2+a0t−a0tr2+a2tr2−a2tr4

(3.16)

The following system of nonlinear first order differential equations are obtained by following the same procedure just likes in case I above.

$(1848 + 9240\Omega_1)\dot{a}_0(t) + (264 - 1848\Omega_1)\dot{a}_2(t) + 29568\Omega a_0^3(t) + 23600\Omega a_2^3(t) - 88704\Omega V_{01}a_0^2(t) - 191136\Omega V_{01}a_2^2(t) + 29568\Omega a_0(t)a_0^2(t) + 25344\Omega a_2(t)a_0^2(t) - 146784\Omega V_{01}a_0(t)a_0^2(t) - \big(1848M_1 +$

354816ΩV012−9240REIa0t−264M1−25344ΩV012−1848REIa2t=2310G1+9240V01REI+22176ΩV012+462M1V01 (3.17)

and

$(3432 + 24024\Omega_1)\dot{a}_0(t) + (1144 + 17160\Omega_1)\dot{a}_2(t) + 164736\Omega a_0^3(t) + 4872384\Omega a_2^3(t) - 494208\Omega V_{01}a_0^2(t) - 690768\Omega V_{01}a_0^2(t) - 6877728\Omega a_0^2(t)a_2(t) - 1677312\Omega a_2^2(t)a_0(t) - 1194336\Omega V_{01}a_0(t)a_2(t) - \left(3432M_1 - 1111968\Omega V_{01}^2 + \frac{24024}{REI}\right)a_0(t) - \big(429M_1 - 329472\Omega V_{01}^2 +$

$$\left.\frac{17160}{REI}\right) a_2(t) = 6006G_1 + \frac{24024}{REI} + 123552\Omega V_{01}^2 + 2574M_1V_{01}$$

(3.18)

By substituting the appropriate values of the parameters G_1, V_{01}, REI, Ω, Ω_1, M_1 and t into (3.17) and (3.18) and solved, one obtained the values for $a_0(t)$ and $a_2(t)$ which are shown in table 2 below.

Table 1: Values of the Parameters Used in the Numerical Results and the corresponding Velocity Profilefor the Steady Blood Flow Model with constant viscosity.

Figures	G	V_0	RE	Ω	M	w(r)
	1.5	0.25	0.9	10	0.35	$0.5746 – 0.3755r^2 – 0.0041r^2(1-r^2)$
2a	2.0	0.25	0.9	10	0.35	$0.7056 – 0.5056r^2 – 0.0473r^2(1-r^2)$
	2.5	0.25	0.9	10	0.35	$0.7489 – 0.5489r^2 – 0.0670r^2(1-r^2)$
	1.5	0.25	0.9	10	0.35	$0.3188 – 0.1188r^2 – 0.0169r^2(1-r^2)$
3a	1.5	0.25	0.9	10	0.65	$0.2850 – 0.0850r^2 – 0.0009r^2(1-r^2)$
	1.5	0.25	0.9	10	0.95	$0.2517 – 0.0517r^2 – 0.0058r^2(1-r^2)$
	1.5	0.25	0.9	10	0.35	$0.5769 – 0.3269r^2 – 0.1192r^2(1-r^2)$
4a	1.5	0.35	0.9	10	0.35	$0.6744 – 0.3244r^2 – 0.1179r^2(1-r^2)$
	1.5	0.45	0.9	10	0.35	$0.7718 – 0.3218r^2 – 0.1167r^2(1-r^2)$
	1.5	0.25	0.9	10	0.35	$0.3385 – 0.1385r^2 – 0.0483r^2(1- r^2)$ 0.3539 –
5a	1.5	0.25	0.9	20	0.35	$0.1539r^2 – 0.0526r^2(1-r^2)$
	1.5	0.25	0.9	30	0.35	$0.3823 – 0.1823r^2 – 0.0558r^2(1-r^2)$
	1.5	0.25	0.3	10	0.35	$0.2669 – 0.0669r^2 – 0.0028r^2(1-r^2)$
6a	1.5	0.25	0.6	10	0.35	$0.3118 – 0.1118r^2 – 0.0179r^2(1-r^2)$
	1.5	0.25	0.9	10	0.35	$0.3338 – 0.1375r^2 – 0.0333r^2(1-r^2)$

Table 2: Values of the Parameters Used in the Numerical Results and the corresponding Velocity Profilefor the Unsteady Blood Flow Model with constant viscosity.

Figures	G_1	V_{01}	REI	Ω	Ω_1	M_1	t	w(r, t)
	1.5	0.25	0.9	10	1	0.35	0.5	$-0.1437r^2 + 0.3437 +0.0335r^2(1-r^2)$
2b	2.0	0.25	0.9	10	1	0.35	0.5	$-0.1858r^2 + 0.3858 +0.0296r^2(1-r^2)$
	2.5	0.25	0.9	10	1	0.35	0.5	$-0.2290r^2 + 0.4290 +0.0262r^2(1-r^2)$

	1.5	0.25	0.9	10	1	0.35	0.5	$-0.2261r^2 + 0.4261 +0.0260r^2(1-r^2)$
3b	1.5	0.25	0.9	10	1	0.65	0.5	$-0.2174r^2 + 0.4174 +0.0249r^2(1-r^2)$
	1.5	0.25	0.9	10	1	0.95	0.5	$-0.2090r^2 + 0.4090 +0.0237r^2(1-r^2)$
	1.5	0.25	0.9	10	1	0.35	0.5	$-0.0572r^2 + 0.2572 +0.0009r^2(1-r^2)$
4b	1.5	0.35	0.9	10	1	0.35	0.5	$-0.1538r^2 + 0.3538 +0.0019r^2(1-r^2)$
	1.5	0.45	0.9	10	1	0.35	0.5	$-0.2510r^2 + 0.4510 +0.0030r^2(1-r^2)$
	1.5	0.25	0.9	10	1	0.35	0.5	$-0.3246r^2 + 0.5246 +0.0499r^2(1-r^2)$
5b	1.5	0.25	0.9	20	1	0.35	0.5	$-0.3108r^2 + 0.5108 +0.0538r^2(1-r^2)$
	1.5	0.25	0.9	30	1	0.35	0.5	$-0.2848r^2 + 0.4848 +0.0554r^2(1-r^2)$
	1.5	0.25	0.3	10	1	0.35	0.5	$-0.0053r^2 + 0.2053 +0.000Ir^2(1-r^2)$
6b	1.5	0.25	0.6	10	1	0.35	0.5	$-0.0094r^2 + 0.2094 +0.0001r^2(1-r^2)$
	1.5	0.25	0.9	10	1	0.35	0.5	$-0.0091r^2 + 0.2091 +0.0005r^2(1-r^2)$

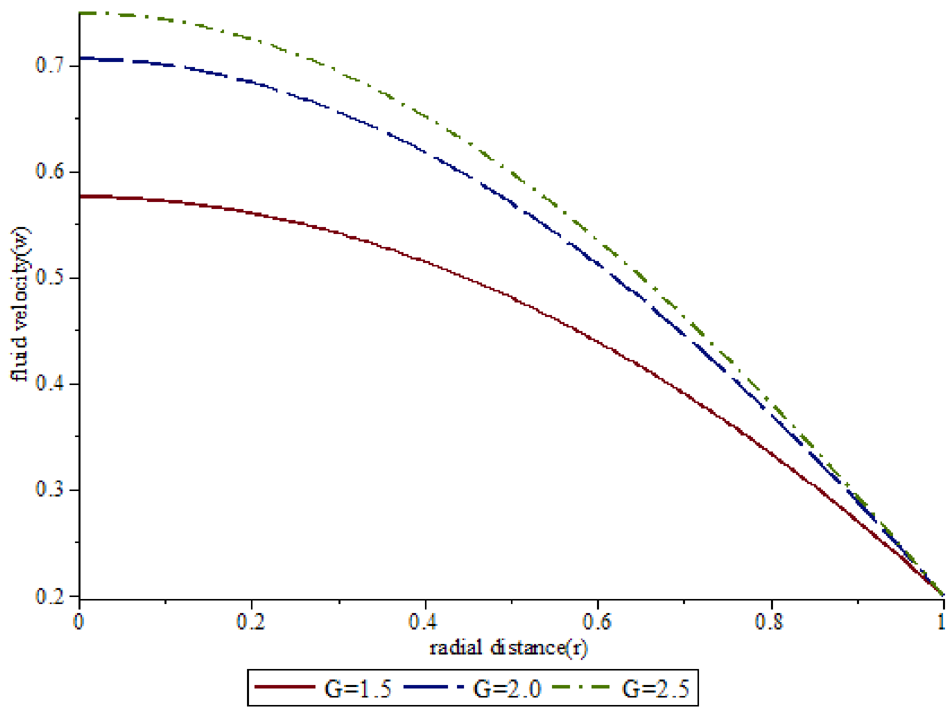

Figure 2a: Variation of Velocity Profile of the Steady Blood Flow Model with increasing values of the Pressure Gradient in the radial direction.

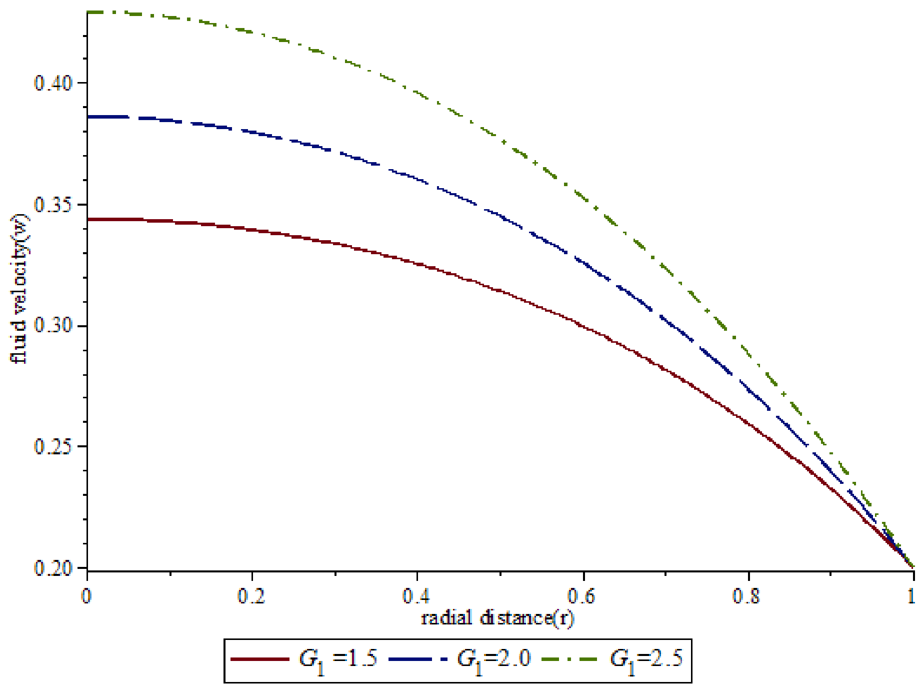

Figure 2b: Variation of Velocity Profile of the Unsteady Blood Flow Model with increasing values of the Pressure Gradient in the radial direction.

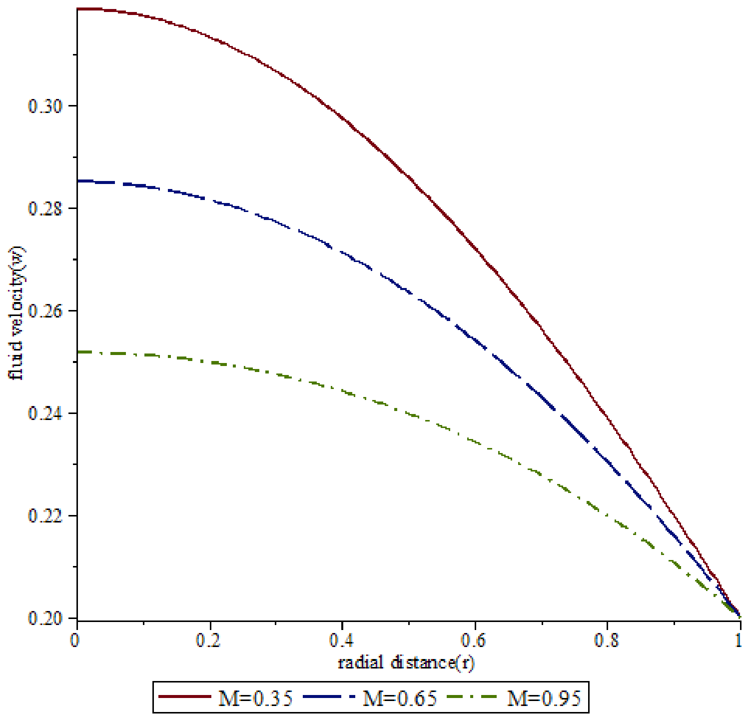

Figure 3a: Variation of Velocity Profile of the Steady Blood Flow Model with increasing values of the Magnetic Field Parameter in the radial direction.

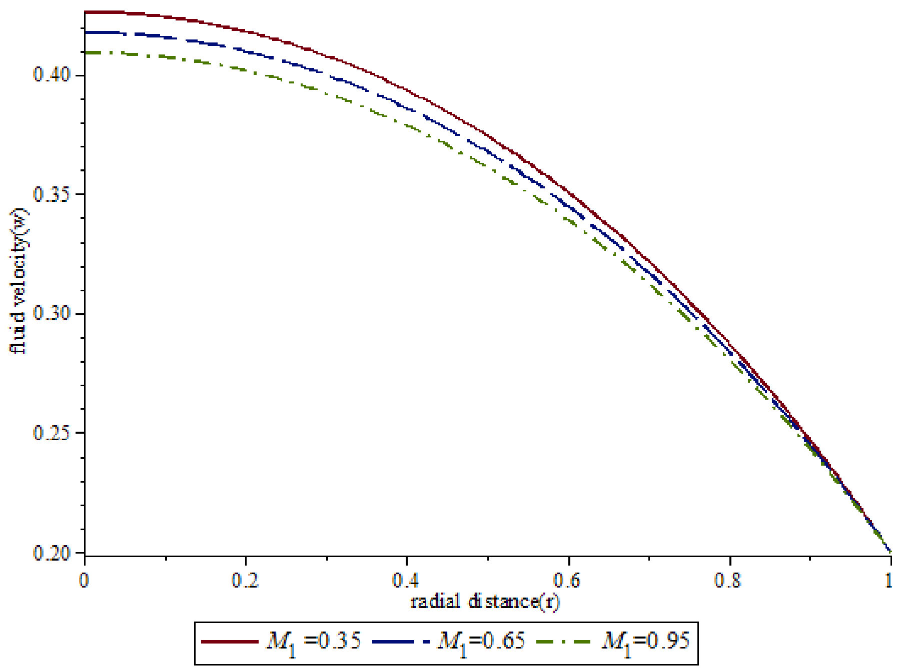

Figure 3b: Variation of Velocity Profile of the Unsteady Blood Flow Model with increasing values of the Magnetic Field Parameter in the radial direction.

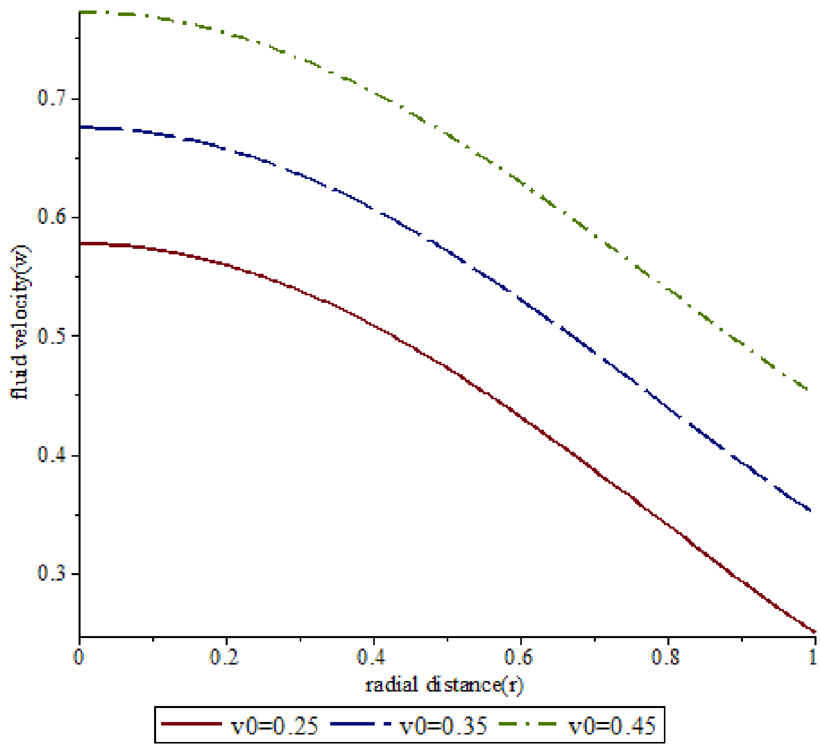

Figure 4a: Variation of Velocity Profile of the Steady Blood Flow Model with increasing values of the Slip Velocity in the radial direction.

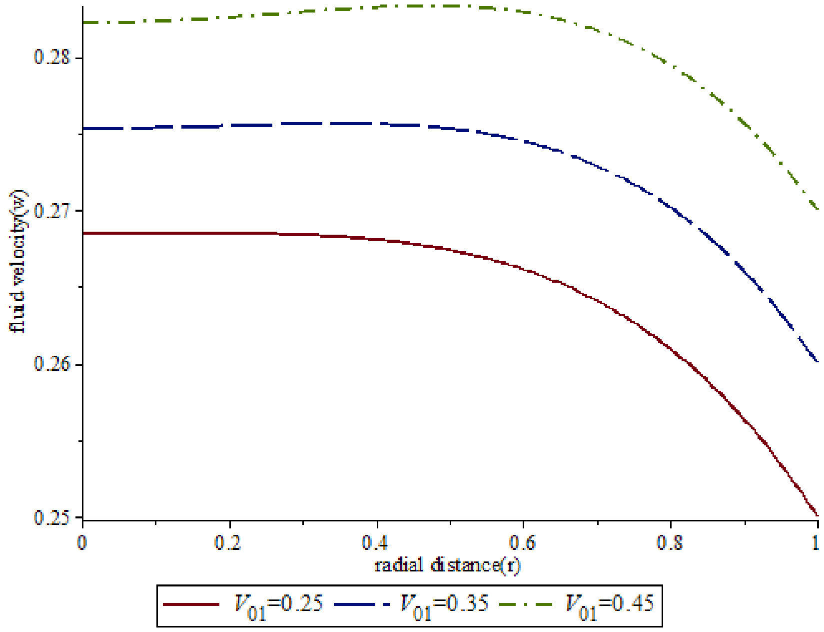

Figure 4b: Variation of Velocity Profile of the Unsteady Blood Flow Model with increasing values of the Slip Velocity in the radial direction.

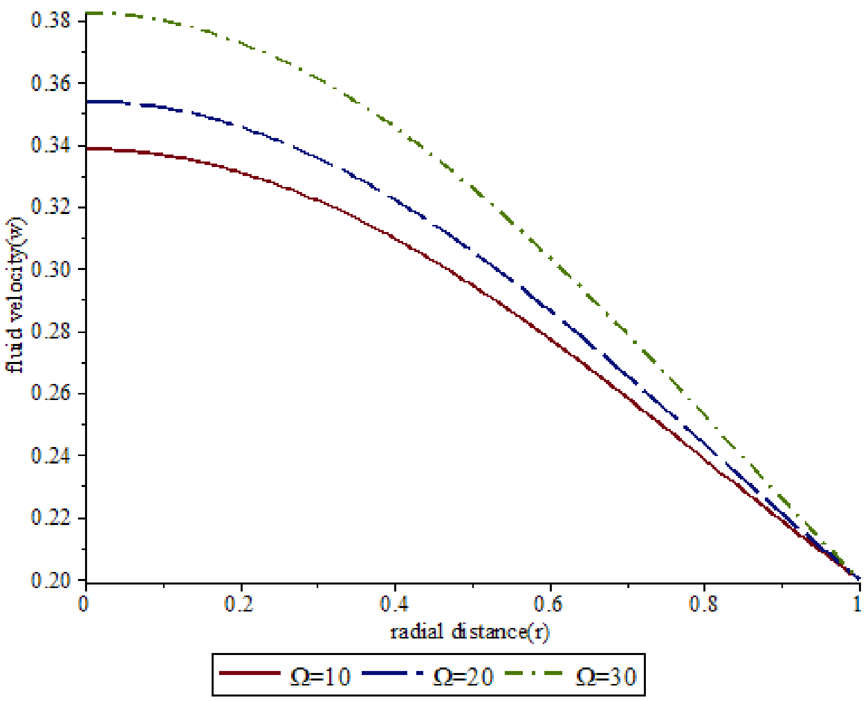

Figure 5a: Variation of Velocity Profile of the Steady Blood Flow Modelwith increasing values of the Shear Thinning in the radial direction.

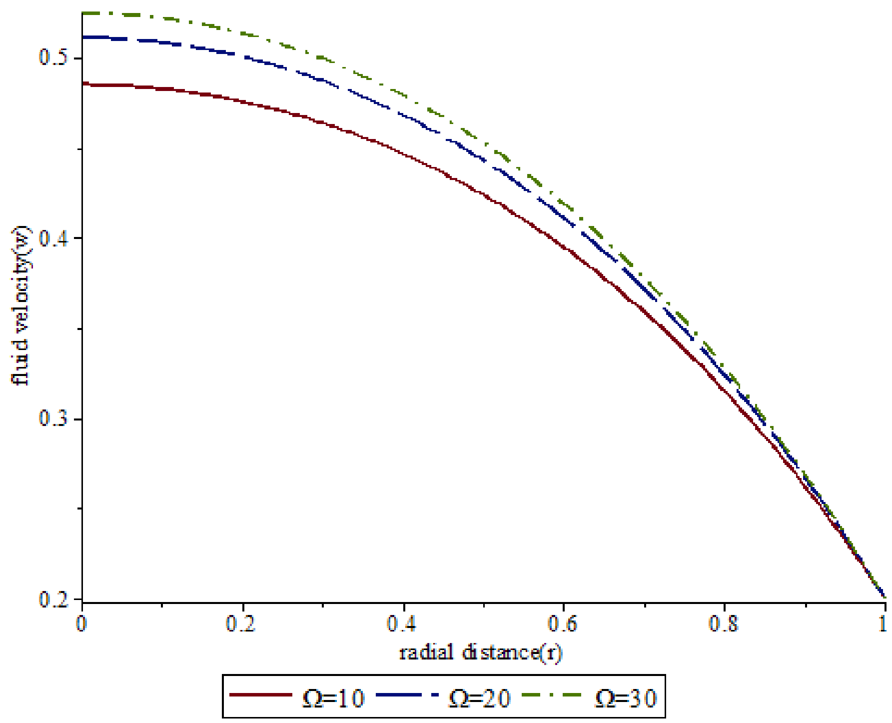

Figure 5b: Variation of Velocity Profile of the Unsteady Blood Flow Modelwith increasing values of the Shear Thinning in the radial direction.

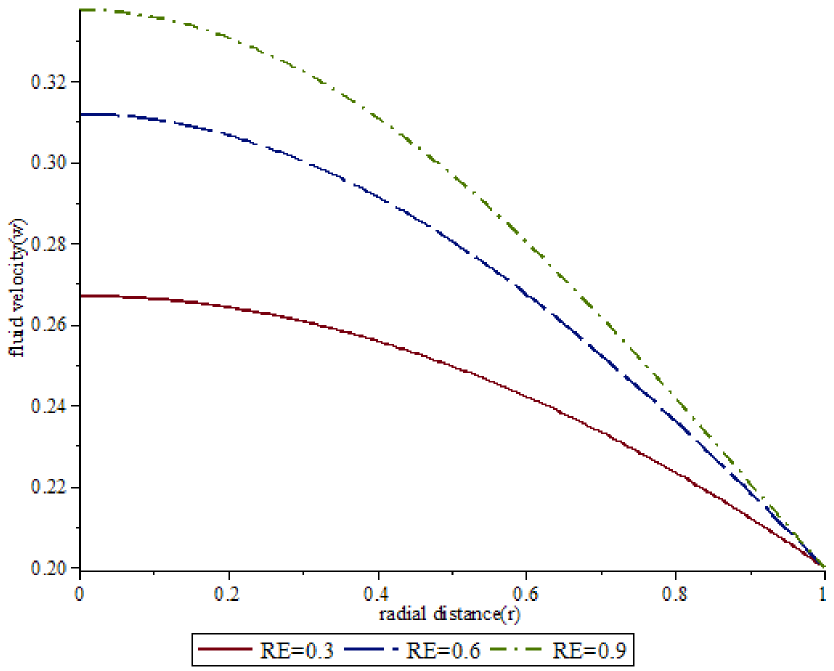

Figure 6a: Variation of Velocity Profile of the Steady Blood Flow Model with increasing values of the Reynold Number in the radial direction.

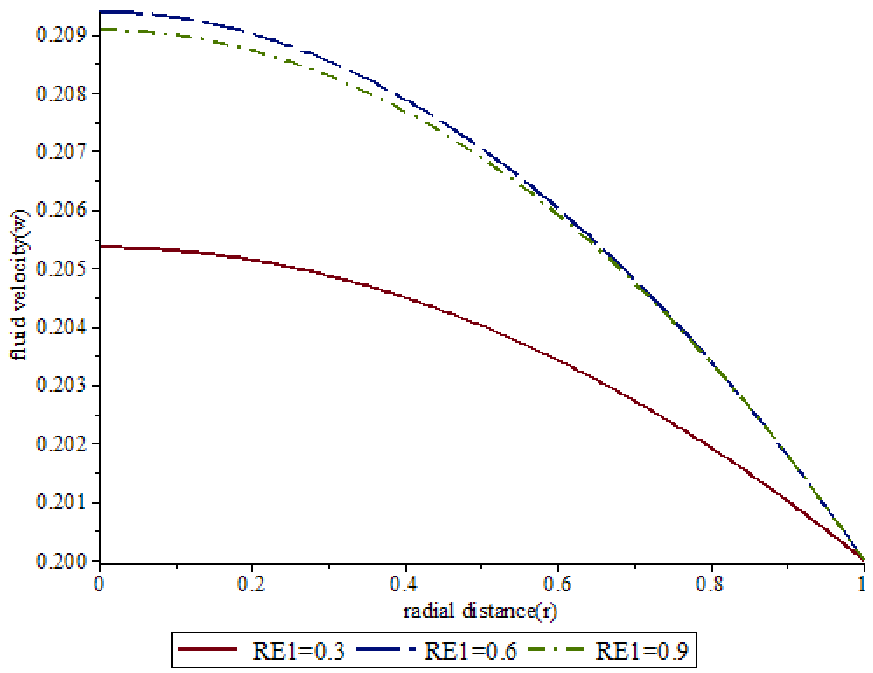

Figure 6b: Variation of Velocity Profile of the Unsteady Blood Flow Model with increasing values of the Reynold Number in the radial direction.

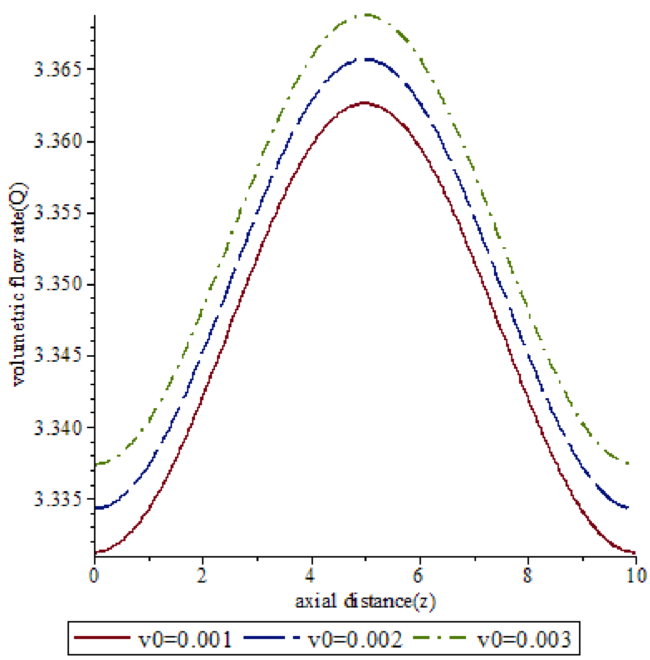

Figure 7a: Variation of Volume Flow Rate of Steady Blood Flow Model with increasing values of the Slip Velocity in the entire stenotic region along the axial direction.

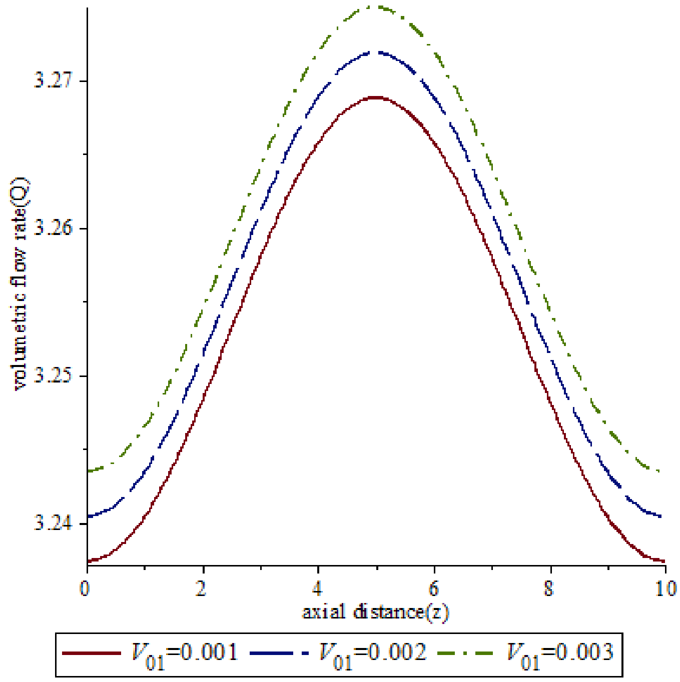

Figure 7b: Variation of Volume Flow Rate of Unsteady Blood Flow Model with increasing values of the Slip Velocity in the entire stenotic region along the axial direction.

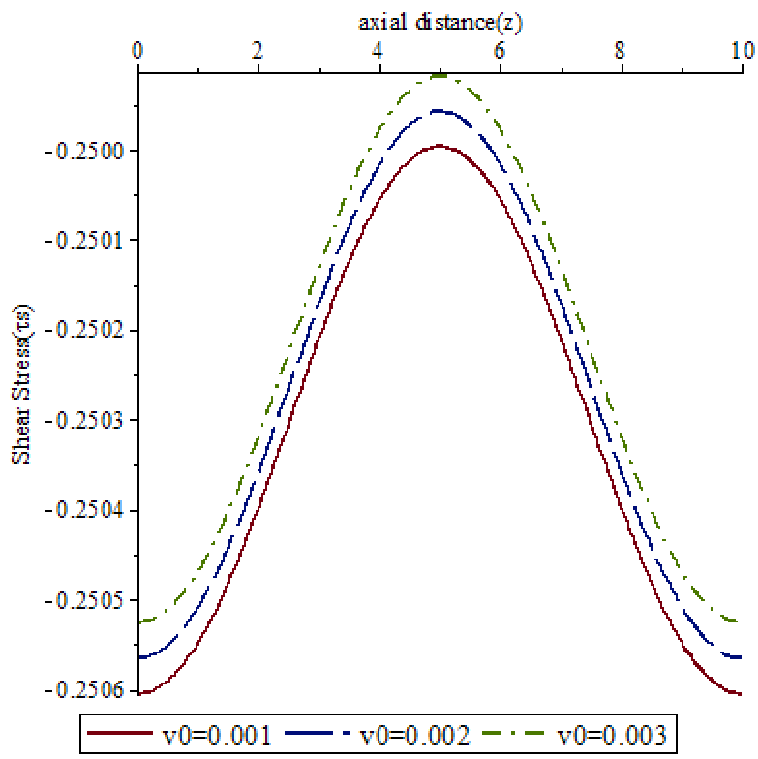

Figure 8a: Variation of Wall Shear Stress of Steady Blood Flow Model with increasing values of the Slip Velocity in the entire stenotic region along the axial direction.

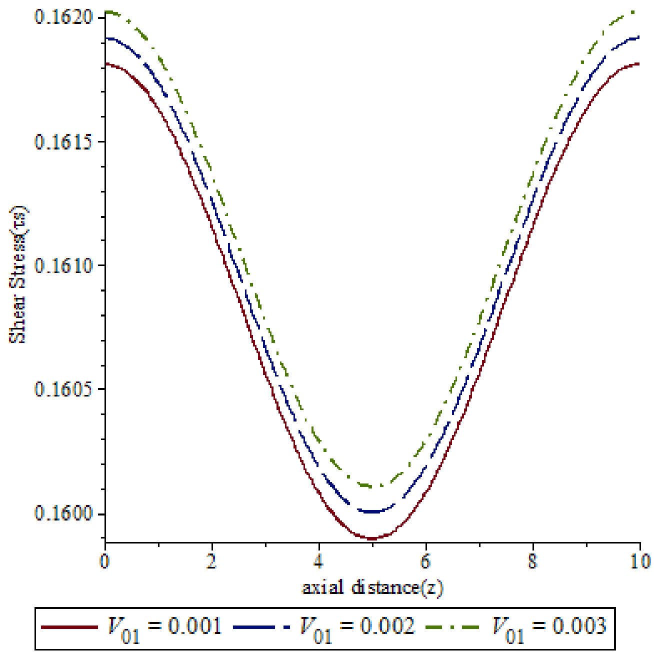

Figure 8b: Variation of Wall Shear Stress of Unsteady Blood Flow Model with increasing values of the Slip Velocity in the entire stenotic region along the axial direction.

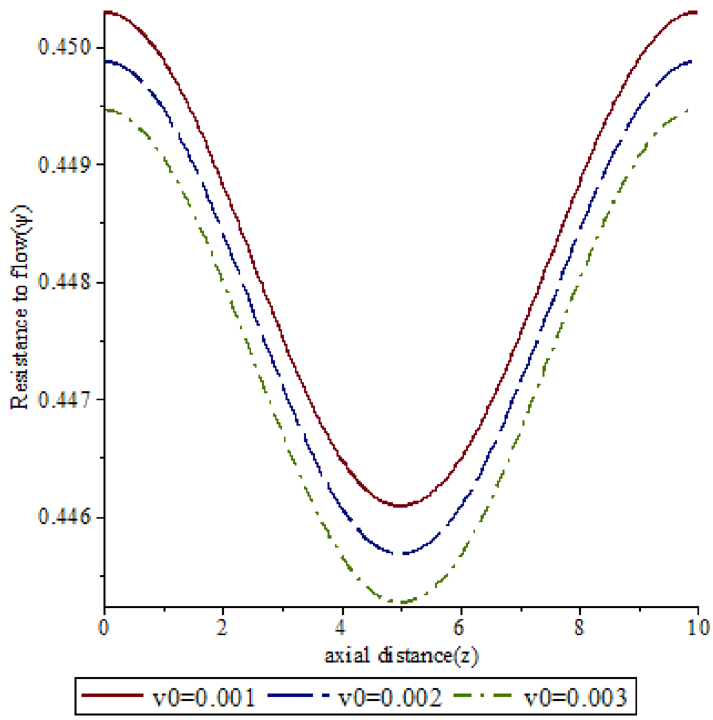

Figure 9a: Variation of Wall Shear Stress of Steady Blood Flow Model with increasing values of the Slip Velocity in the entire stenotic region along the axial direction.

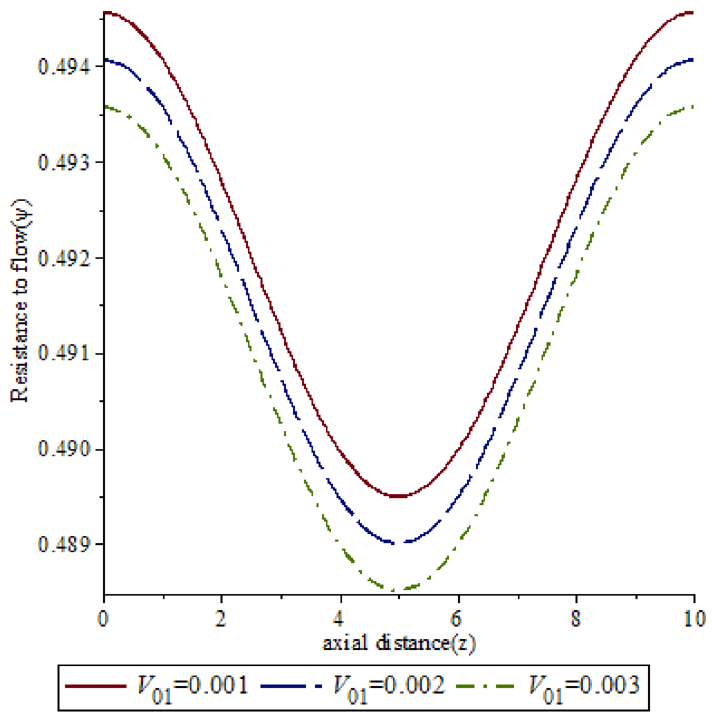

Figure 9b: Variation of Wall Shear Stress of Unsteady Blood Flow Model with increasing values of the Slip Velocity in the entire stenotic region along the axial direction.

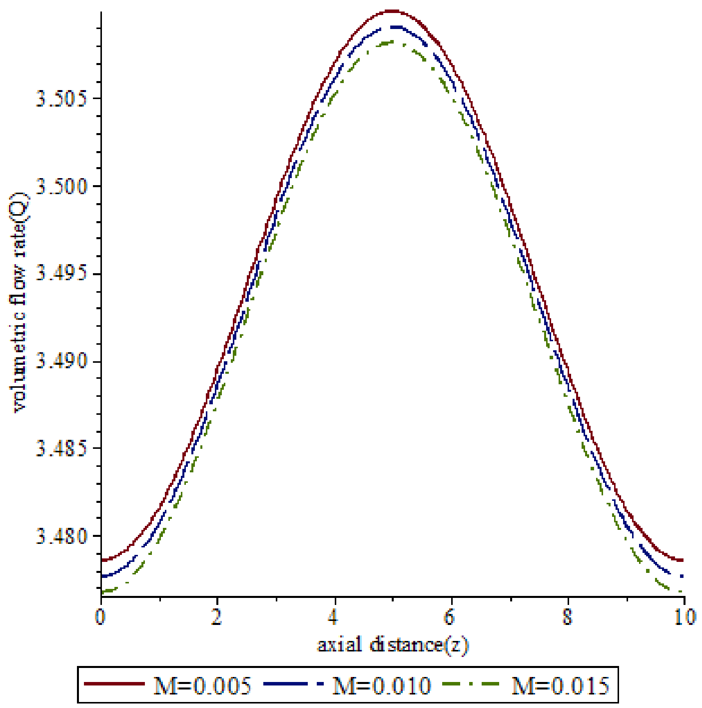

Figure 10a: Variation of Volume Flow Rate of Steady Blood Flow Model with increasing values of the Magnetic Field Parameter in the entire stenotic region along the axial direction.

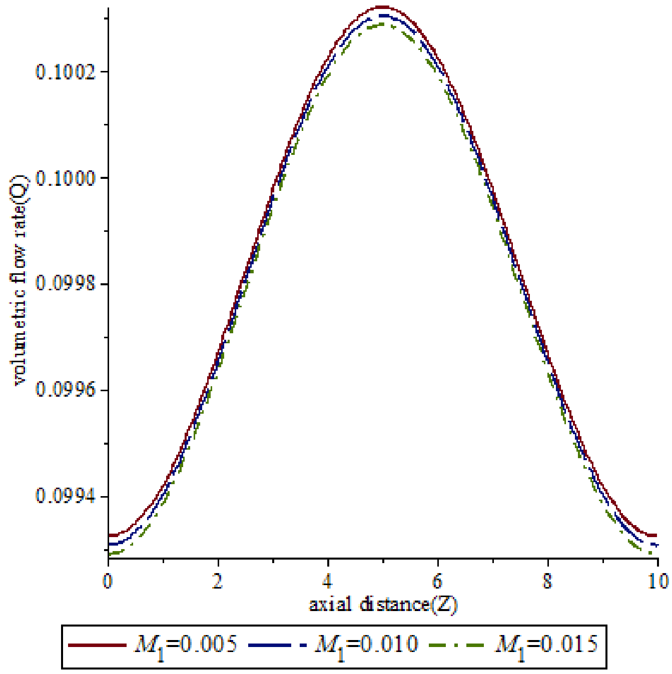

Figure 10b: Variation of Volume Flow Rate of Unsteady Blood Flow Model with increasing values of the Magnetic Field Parameter in the entire stenotic region along the axial direction.

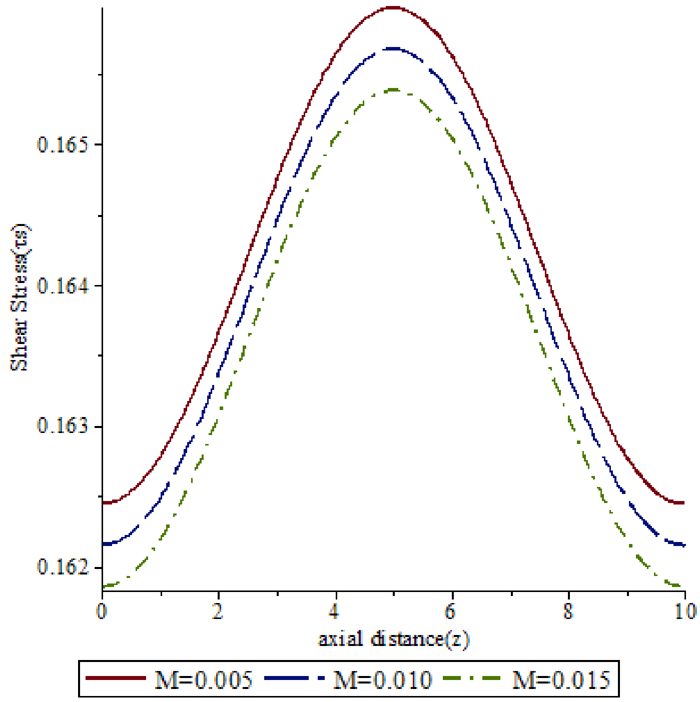

Figure 11a: Variation of Wall Shear Stress of Steady Blood Flow Model with increasing values of the Magnetic Field Parameter in the entire stenotic region along the axial direction.

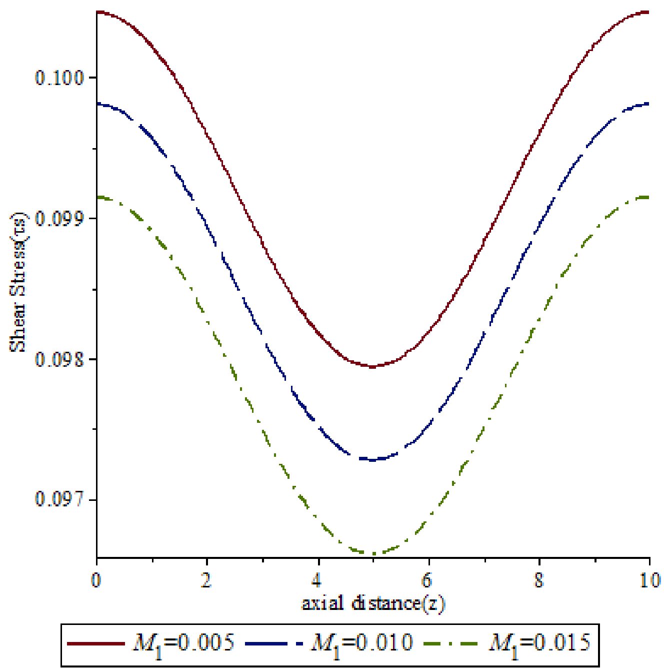

Figure 11b: Variation of Wall Shear Stress of Unsteady Blood Flow Model with increasing values of the Magnetic Field Parameter in the entire stenotic region along the axial direction.

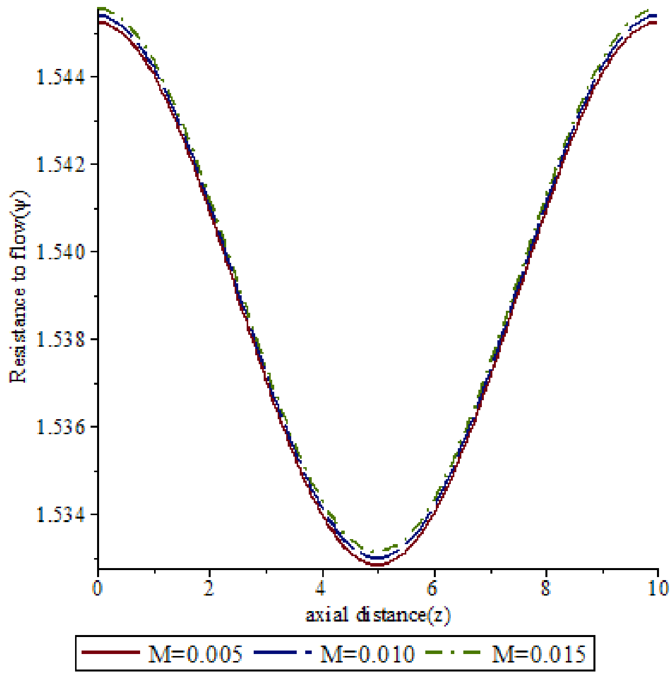

Figure 12a: Variation of Resistance to Steady Blood Flow Model with increasing values of the Magnetic Field Parameter in the entire stenotic region along the axial direction.

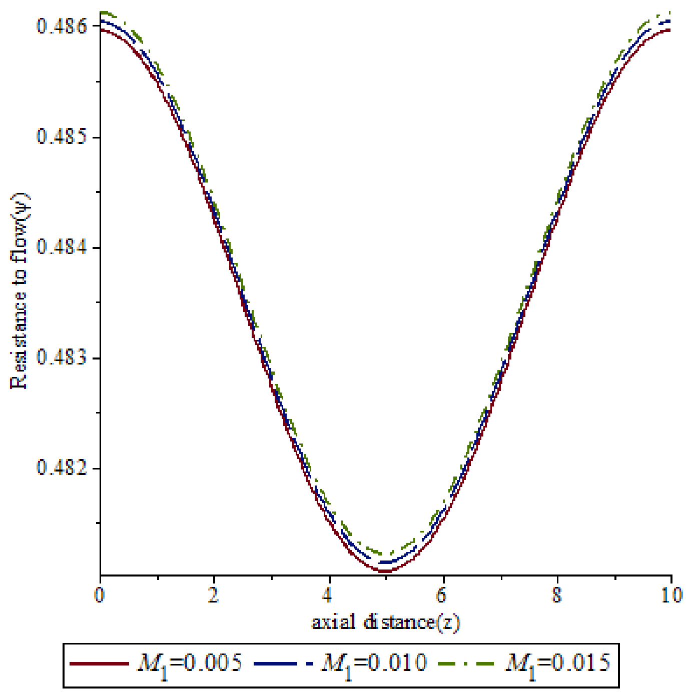

Figure 12b: Variation of Resistance to Unsteady Blood Flow Model with increasing values of the Magnetic Field Parameter in the entire stenotic region along the axial direction.

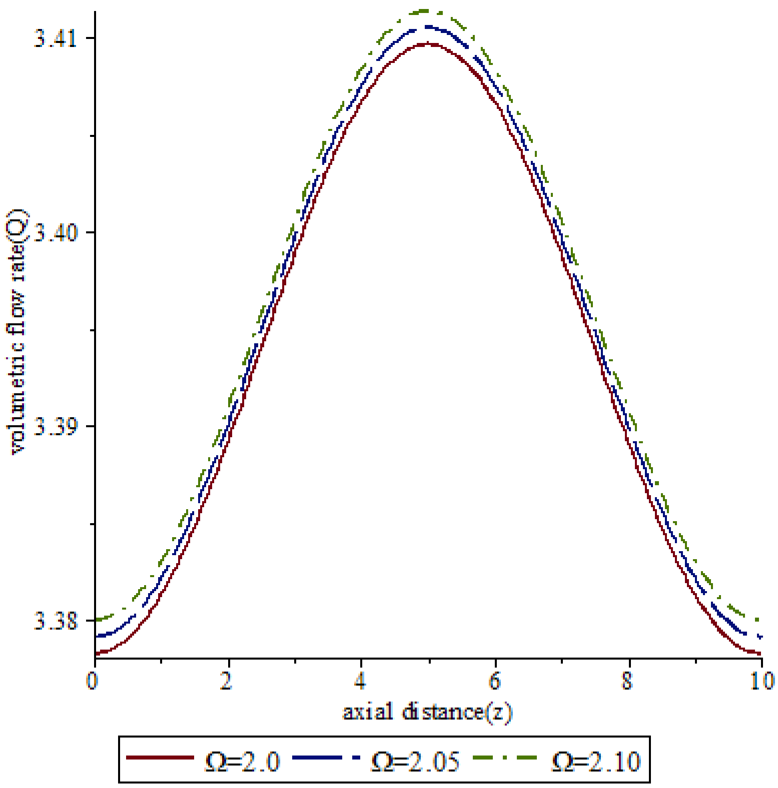

Figure 13a: Variation of Volumetric Flow Rate of Steady Blood Flow Model with increasing values of the Shear thinning in the entire stenotic region along the axial direction.

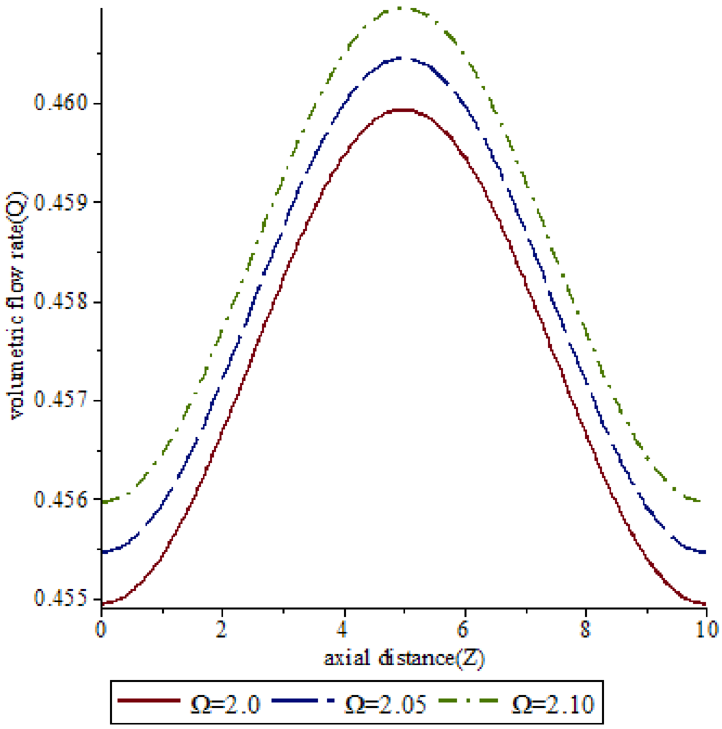

Figure 13b: Variation of Volumetric Flow Rate of Unsteady Blood Flow Model with increasing values of the Shear thinning in the entire stenotic region along the axial direction.

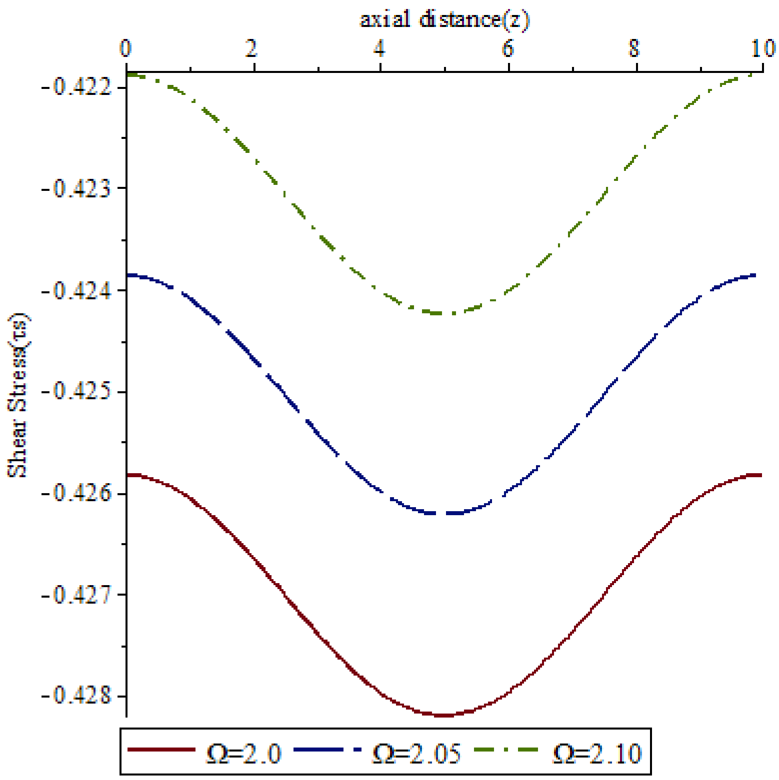

Figure 14a: Variation of Wall Shear Stress of Steady Blood Flow Model with increasing values of the Shear Thinning in the entire stenotic region along the axial direction.

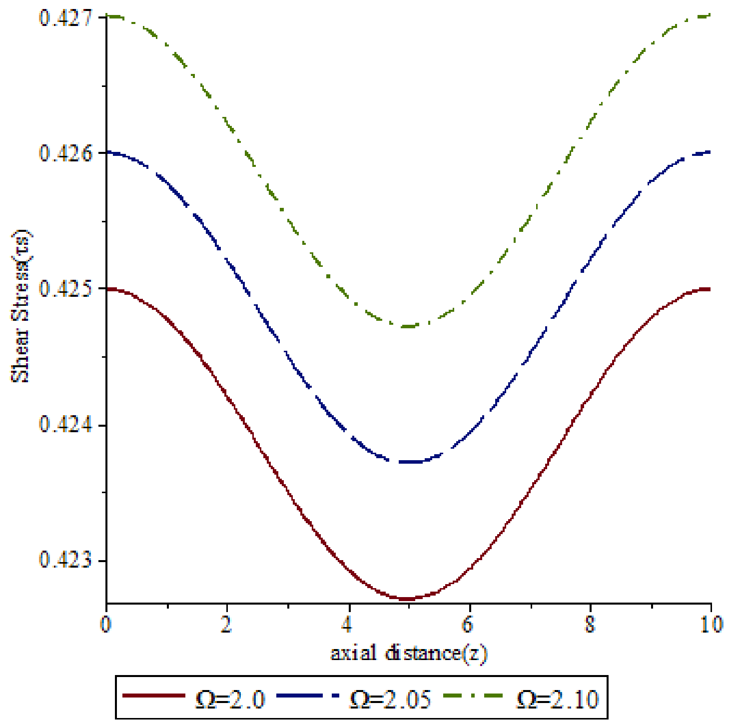

Figure 14b: Variation of Wall Shear Stress of Unsteady Blood Flow Model with increasing values of the Shear Thinning in the entire stenotic region along the axial direction.

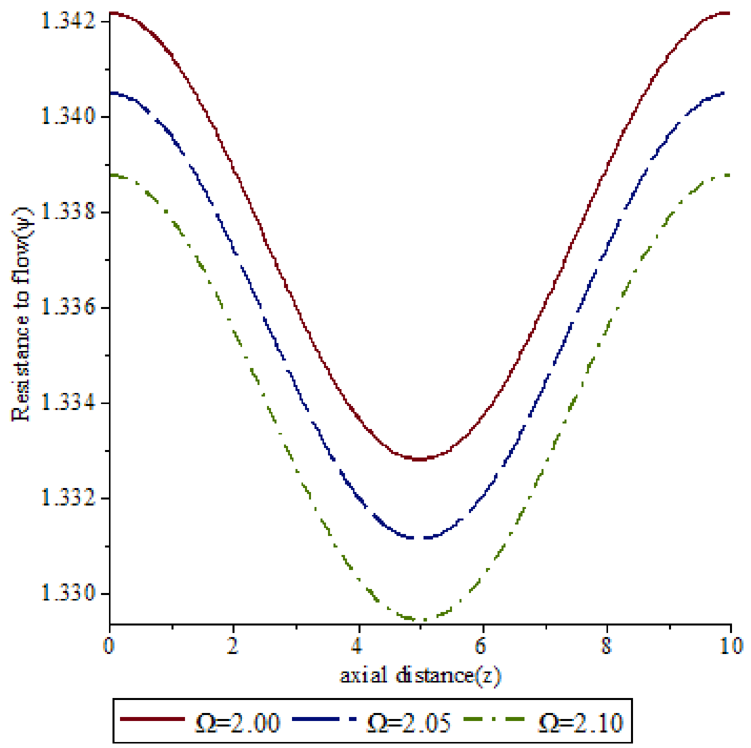

Figure 15a: Variation of Resistance to Steady Blood Flow Model with increasing values of the Shear Thinning in the entire stenotic region along the axial direction.

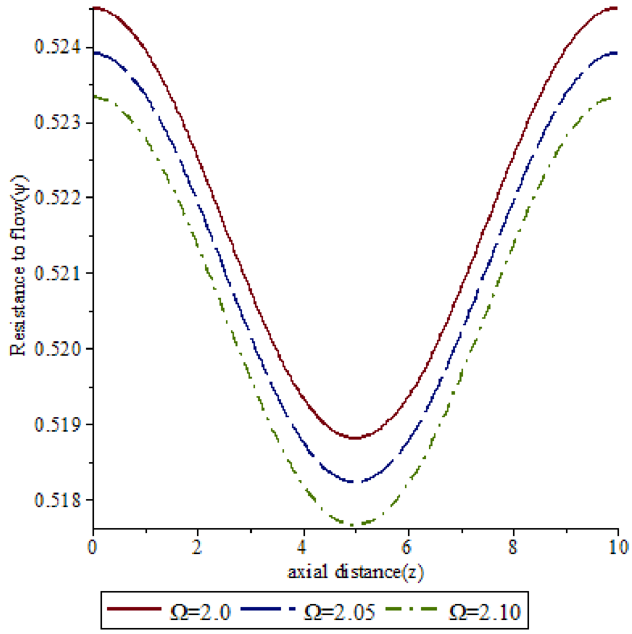

Figure 15b: Variation of Resistance to Unsteady Blood Flow Model with increasing values of the Shear Thinning in the entire stenotic region along the axial direction.

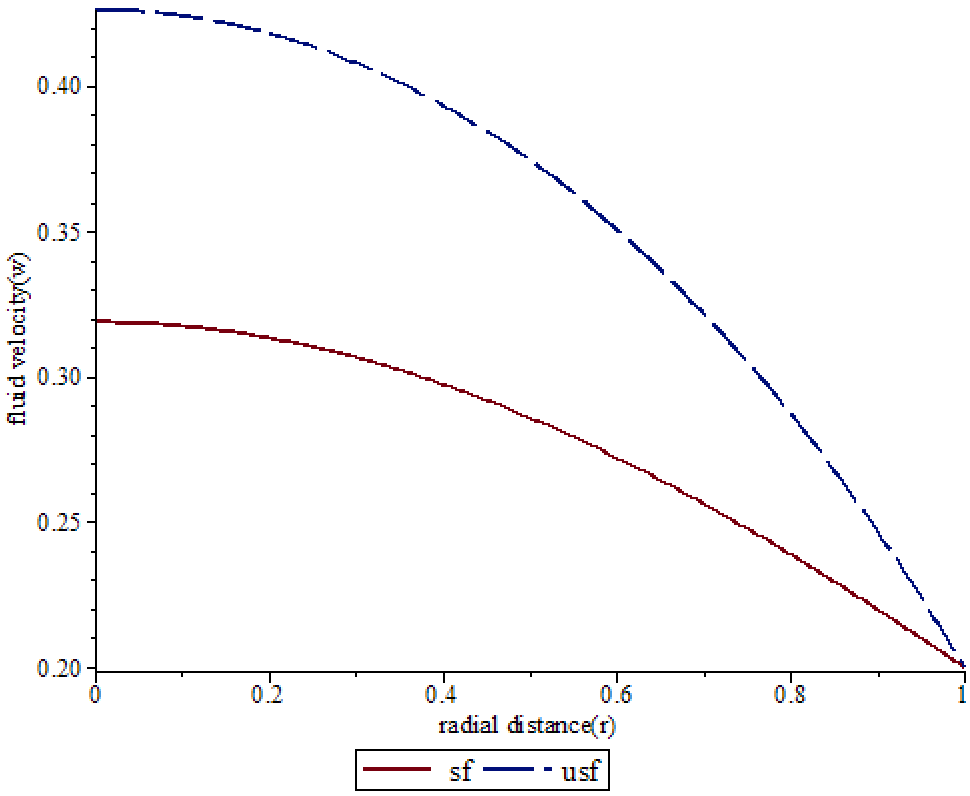

Figure 16: Comparison of the Velocity profiles of Steady and Unsteady Blood Flow Models for constant values of all the parameters in the Radial Direction.

IV. Discussion of results

I presented in this work, effect of unsteadiness on blood flow through a stenosed artery using a third grade fluid models with slip conditions. Externally applied magnetic field is also taken into consideration and after the analysis and numerical simulation carried out, it was reveals that increase in slip velocity significantly lead to an increase in flow velocity, flow rate and shear stress but reduces the resistance to fluid flow for both steady and unsteady blood flow models and these are shown in figures 4a, 4b, 7a, 7b, 8a, 8b, 9a and 9b respectively. Increase in magnetic field parameter lead to decease in flow velocity, flow rate, and shear stress but increases the resistances to fluid flow for both steady and unsteady blood flow models as indicated in figures 3a, 3b, 10a, 10b, 11a, 11b, 12a and 12b respectively. Increase in shear thinning lead to increase in flow velocity, flow rate and shear stress but reduces the flow resistance as shown in figures 5a, 5b, 13a, 13b,14a, 14b, 15a and 15b respectively. Pressure gradient and Reynold number increase with flow velocity as shown in figures 2a, 2b, 6a and 6b respectively. Finally, the flow velocity of unsteady blood flow model is higher than that of steady flow model as indicated in figure 16.

V. Conclusion

The present analysis focused on effect of unsteadiness on third grade blood flow through a stenosed artery with slip conditions and incorporation of externally applied magnetics in the models. The mathematical problems are solved analytically and the significant findings are summarized belows;

(i) Velocity profile, volumetric flow rate, and shear stress increases while the resistance to flow decreases with increasing values of the slip velocity and shear thinning.

(ii) Velocity profile, volumetric flow rate, and shear stress decreases while resistance to flow increase as magnetic field parameter increase.

(iii) Effect of slip velocity, shear thinning and magnetic field parameters on volumetric flow rate and resistance to flow are more noticeable on unsteady blood flow models compare to that of steady blood flow models. Hence, lower values of those parameters are required for more noticeable effect on flow rate for the unsteady blood flow model when compare to that of steady blood flow model.

(iv) Effect of slip velocity, shear thinning and magnetic parameters on shear stress are more noticeable on steady blood flow model compare to that of unsteady blood flow model. Hence, lower values of those parameters are required for more noticeable effect on shear stress for the steady blood flow model when compare to that of unsteady blood flow model.

(v) Finally, the velocity profile of unsteady blood flow model is higher than that of the steady blood flow model.

The research analysis incorporating externally applied magnetic field useful for the reduction of blood flow during surgery and magnetic resonance imaging (MRI). Also, the incorporating slip velocity in the constricted artery can help to reduce blood viscosity because high blood viscosity is a risk factor in the cardiovascular disorder Cirillo *et al* [21]. It is clear that the combined effects of slip velocity and magnetic field are strong parameters influencing fluid flow. Hence, this could save humanity from deaths associated with various cardiovascular related diseases.

REFERENCE

[1] Shanthi, M., Pekka, P. and Norrving, B. (2011). Global Atlas on Cardiovascular Disease Prevention and Control. World Health Organization in Collaboration with the World Heart Federation and The World Stoke Organization. Pp. 3-18.

[2] Young, D. F and Tsai, F. Y (1973).Flow Characteristics in Model of Arterial Stenosis Steady Flow. *Journal of Biomechanics, 6, 395-410.*

[3] Miller, J, D. (2013). Cardiovascular Calcification: Orbicular Origins. *Nature Material 12, 476-478.*

[4] Biswas, D and Chakraborty. U. S (2010). Pulsatile Blood Flow through a catheterized Artery with an Axially Nonsymmetrical Stenosis. *AppliedMathematical Sciences. 4(58), 2865-2880.*

[5] Narendra, K. V, Shailesh, M, Shafi, U. S. and Ram, S. G. (2011). Effect of Slip Velocity on Blood Flow through a Catheterized Artery. *Applied Mathematics, 2, 746-700.*

[6] Devajyoti, B and Uday, S.C (2010). Two Layered Blood Flow in Stenosed Artery with a Body Acceleration and Slip at Wall. *International journal of Application and Applied Mathematics, 5(2): 313-320.*

[7] Devajyoti, B, and Razial, B. L (2011). Steady Flow of Blood through a Stenosed Artery: A Non- Newtonian Fluid Model. *Assan University Journal of Science and Technology. Vol. 7, No 11, Pp. 144-153.*

[8] Mallik, B. B., Nanda, S., Das B., Saha, D., Das, D. S. and Paul, K. (2013). A Non-Newtonian Fluid Model for Blood Flow using Power Law through Atherosclerotic Arterial Segment having Slip Velocity. *International Journal of Pharmaceutical, Chemical and Biological Sciences*, 3(3): 752-760.

[9] Tian, F. B., Zhu, L, Fok, P. U., and Lu, X. Y (2013). Simulation of a Pulsatile Non- Newtonian Blood Flow past a Stenosed 2D Artery with Artherosclerosis, *Computer in Biological and Medicine, 43, Pp. 1098-1113.*

[10] Misra, J. C. and Shit, G. C. (2007). Role of Slip Velocity in Blood Flow through Stenosed Arteries: A Non-Newtonian Model. *Journal of Mechanical in Medicine and Biology*, 7: 337-353.

[11] Venkateswarlu, K., and Rao, J. A. (2004). Numerical Solution of Unsteady Blood Flow through an Indented Tube with Atherosclerosis. *Indian Journal of Biochemistry and Biophysics, Vol. 41, Pp. 241-245.*

[12] Geeta, A. and Siddique, S. U. (2016). Analysis of Unsteady Blood Flow through a Stenosed Artery with Slip Effect. *International Journal of Bio-Science and Bio-Technology*, 8(5): 43-54.

[13] Srikanth, D. S., Ramana, R. S. and Jain, A. K (2015). Unsteady Polar Fluid Model of Blood Flow through Tapered X-Shape Stenosed Artery. Effect of Catheter and Velocity Slip. *Ain Shams Engineering Journal*, 6: 1093 – 1104.

[14] Ikbal, M. A, Chakravarty, S, Wong, K. K and Mandal, P. K (2009). Unsteady Response of Non-Newtonian Blood Flow through a Stenosed Artery in Magnetic Field.*Journal of Computational and Applied Mathematics. 230: 243-259.*

[15] Islam, M. E. (2012). Mathematical Analysis of Unsteady MHD Blood Flow through Parallel Plate Channel with Heat Source. *World Journal of Mechanics, 2(3), 2012.*

[16] Mandal, P. K. (2005). An Unsteady Analysis of Non-Newtonian Blood Flow through Tapered Arteries with a Stenosis. *International Journal of Non-Linear Mechanics, 40: 151-164.*

[17] Jimoh, A, Okedayo, G.T, and Ikpakyegh, L. N. (2019). Computational Analysis of Unsteady Flow of Blood and Heat Transfer throug a Stenosed Artery in a third Grade Fluid Model with Slip Conditions. *International OF Applied Mathematics and Statistical Sciences. Vol. 8, Issue 5, Pp. 11-36.*

[18] Mohammed, A. A. (2011). Analytical Solution for MHD Unsteady Flow of a Third Grade Fluid with Constant Viscosity. M.Sc. Thesis, Department of Mathematics, university of Baghdad. Pp. 1-104.

[19] Young, D. F. (1979). Effect of Time Dependent Stenosis on Flow through a Tube. *Journal of Engineering*, 90: 248-254.

[20] Biswas, D. (2000). Blood Flow Model: *A Comparative Study*. New Delhi:Mittal Publication. P. 15.

[21] Cirillo, M, Lourenzi, M, Trevisern, M and Stamler, L. (1992). Hematocrit, Blood Pressure and Hypertensions: *The Gubbio Population Study, Hypertension. 20, 319-326.*

Nomenclatures

w - Fluid velocity $\overline{w}$ - Dimensionless fluid velocity

t - Time component $\bar{t}$ - Dimensionless time component

r - Radial distance y - Dimensionless radial distance

z - Axial distance w_s - Slip velocity

V_{01} - Dimensionless slip velocity for the unsteady blood flow

V_0 - Dimensionless slip velocity for the steady blood flow

R_0 - Radius of the normal artery β_0 - Magnetic Field Strength

R(z) - Radius of the artery in a stenotic regionτ_s - Wall Shear Stress

ψ - Resistance to flow Ƹ - Maximum height of the stenosis

Q - Volumetric flow rate L - Length of the stenosis

W = Fluid velocity G_1-Pressure gradient for the unsteady flow

M_1-Magnetic field parameter for the unsteady flow Ω-Shear thinning for the unsteady flow

M-Magnetic field parameter for the steady flow$G$-Pressure gradient for the unsteady flow