Maximal Fuzzy Assignment Problem Involving Dodecagonal Fuzzy Number

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Abstract : *The decision of placement of a right person for a right job is difficult because of uncertainty and imprecise information. However fuzzy assignment problem can certainly solve this purpose. In this paper, we solve Maximal fuzzy assignment prob including dodecagonal fuzzy number using Hungarian method and also we made comparative study for Trapezoidal, Octogonal and dodecagonal fuzzy number.*

Keywords: *Trapezoidal fuzzy number, Octogonal fuzzy number, and dodecagonal fuzzy number.*

I. INTRODUCTION

Our human system and our activites are not certain. They are not clear but they are certain upto a degree and they may be studied and analysed by representing in a system witn varying degree. The first attempt was made by Prof. L. A. Zadeh. He formulated a system in which sets are represented by a membership in the year 1965. Such systems are called Fuzzy sets and system.

A Special type of linear programming known as assignment problem in which our objective is to assign n-machine to n-person at minimum cost or maximaum profit. Assignment problem is of two types Conventional assignment problem and fuzzy assignment problem. Fuzzy assignment problem is more realistic than conventional because in conventional the cost of the problem is certain.

II. PRELIMINARIES

Definition 1A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in X\}$,where $\mu_{\tilde{A}}(x)$ is called membership functin of x in A which maps $X \rightarrow [0, 1]$.

Definition 2 (Fuzzy Number):A fuzzy set $\tilde{A}$ on $\mathbb{R}$ is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0, 1]$ possess the following conditions.

- $\mu_{\tilde{A}}$ is normal. It means that there exists an $X \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$
- $\mu_{\tilde{A}}$ is convex. It means that for every $x_1, x_2 \in \mathbb{R}$, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, $\lambda \in [0, 1]$
- $\mu_{\tilde{A}}$ is upper semi-continuous.

Supp($\mu_{\tilde{A}}$) is bounded in $\mathbb{R}$.

Definition3(Trapezoidal Fuzzy Number):): Atrapezoidal fuzzy number denoted by $\tilde{A}$ is defined as (a_1, a_2, a_3, a_4) where the membership function is given by**,**

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

Definition 4(OCTAGONAL FUZZYNUMBER).A fuzzy number Ã is said to beoctagonal fuzzy number (a_1,a_2, a_3,a_4, a_5, a_6, a_7, a_8) where a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 are real numbers and its membership function .

$$\mu_{\tilde{A}}(x) = \begin{cases} k\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ k, & a_2 \le x \le a_3 \\ k+(1-k)\dfrac{x-a_3}{a_4-a_3}, & a_3 \le x \le a_4 \\ 1, & a_4 \le x \le a_5 \\ k+(1-k)\dfrac{a_6-x}{a_6-a_5}, & a_5 \le x \le a_6 \\ k, & a_6 \le x \le a_7 \\ k\left(\dfrac{a_8-x}{a_8-a_7}\right), & a_7 \le x \le a_8 \\ 0, & otherwise \end{cases}$$

Defintion 5(Dodecagonal Fuzzy Number): The membership function of dodecagonal fuzzy number Ã = (a_1,a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9,a_{10}, a_{11}, a_{12}) wherea_1,a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9,a_{10}, a_{11}, a_{12} are real numbers and is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le a_1 \\ k_1\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ k_1, & a_2 \le x \le a_3 \\ k_1 + (1-k_1)\dfrac{x-a_3}{a_4-a_3}, & a_3 \le x \le a_4 \\ k_2, & a_4 \le x \le a_5 \\ k_2 + (1-k_2)\dfrac{x-a_5}{a_6-a_5}, & a_5 \le x \le a_6 \\ 1, & a_6 \le x \le a_7 \\ k_2+(1-k_2)\dfrac{a_8-x}{a_8-a_7}, & a_7 \le x \le a_8 \\ k_2, & a_8 \le x \le a_9 \\ k_1+(1-k_1)\dfrac{a_{10}-x}{a_{10}-a_9}, & a_9 \le x \le a_{10} \\ k_1, & a_{10} \le x \le a_{11} \\ k_1\left(\dfrac{a_{12}-x}{a_{12}-a_{11}}\right), & a_{11} \le x \le a_{12} \\ 0, & a_{12} \le x \end{cases}$$

Where $0 < k_1 < k_2 < 1$

Definition6 (Measure of dodecagonal Fuzzy Number): Let Ã be a normal dodecagonal fuzzy number. The measure of Ã is given by

$$M_0^{DDEC}(\tilde{A}) = \frac{1}{12}\{(a_1 + a_{12}) + k_1(a_2 + a_3 + a_{10} + a_{11}) + k_2(a_4 + a_5 + a_8 + a_9) + (a_6 + a_7)\}$$

Where $0 < k_1 < k_2 < 1$

Remark: If $0 < k_1 = k_2 < 1$, the dodecagonal fuzzy number reduces to the octagonal fuzzy number (a_1,a_2, a_5, a_6, a_7, a_8, a_{11}, a_{12}) and if $k_1 = k_2 = 1$, it reduces to the trapezoidal fuzzy number (a_1,a_2, a_{11}, a_{12})

III. Assignment Problem

A. Mathematical Formulation

Mathematically, the assignment problem is stated as, Minimize the total cost:

$$z = \sum_{i=1}^{n} \sum_{j=1}^{1} c_{ij}\, x_{ij}, i = 1,2,\dots,n; j = 1,2,\dots,n$$

Subject to condition:

$$x_{ij} = \begin{cases} 1, & if\ the\ ith\ person\ is\ assigned\ jth\ job \\ 0, & if\ not \end{cases}$$

$$\sum_{j=1}^{n} x_{ij} = 1(one\ job\ is\ done\ by\ ith\ person\ i = 1,2,\dots,n)$$

$$\sum_{j=1}^{n} x_{ij} = 1(one\ job\ is\ done\ by\ ith\ person\ i = 1,2,\dots,n)$$

Where, x_{ij} denotes that jth job is to be assigned to the ith person

c_{ij} represents the cost of assignment of a person i to the job j.

B. PROCEDURE FOR SOLVING MAXIMAL FUZZYASSIGNMENT PROBLEM USING HUNGARIAN METHOD

Step 1: As we are considering maximal assignment problem, we first convert it into minimization problem by subtracting all the elements from the highest element.
Step 2: Subtract minimum element of each row from all the elements of that row.
Step 3: Next subtract the smallest element of each column from every element of that column.
Step 4: Now we check whether zero assignment is possible or not. Starting with row 1 examine one by one row containing exactly one zero and mark that cell as zero. Cross all other zeros in the column in which the assignment has been made. When all rows are examined follow the identical procedure for the columns. When the assignment is made in column, cross all zeros in the row in which the assignment is made. Continue this until all zeros are either assigned or crossed out.

IV. NUMERICAL EXAMPLE

A. Solving Maximal Fuzzy Assignment Problem of Dodecagonal Fuzzy Numbers:

	J_1	J_2	J_3	J_4
P_1	10,13,16,19,22,25, 28,31,34,37,40,43	7,10,13,16,19,22, 25,28,31,34,37,40	8,11,14,17,20,23, 26,29,32,35,38,41	7,10,13,16,19,22, 25,28,31,34,37,40
P_2	9,12,15,18,21,24, 27,30,33,36,39,42	10,13,16,19,22,25, 28,31,34,37,40,43	8,11,14,17,20,23, 26,29,32,35,38,41	9,12,15,18,21,24, 27,30,33,36,39,42
P_3	5,8,11,14,17,20, 23,26,29,32,35,38	7,10,13,16,19,22, 25,28,31,34,37,40	7,10,13,16,19,22, 25,28,31,34,37,40	5,8,11,14,17,20, 23,26,29,32,35,38
P_4	8,11,14,17,20,23, 26,29,32,35,38,41	6,9,12,15,18,21, 24,27,30,33,36,39	8,11,14,17,20,23, 26,29,32,35,38,41	5,8,11,14,17,20, 23,26,29,32,35,38

Now, the measure of $\tilde{A}$ is calculated as disscussed in section 1. This problem is done by taking the values of $k_1 = 0.3$ and $k_2 = 0.7$

$M_0(10,13,16,19,22,25,28,31,34,37,40,43) = 17.67$
$M_0(7,10,13,16,19,22,25,28,31,34,37,40) = 15.67$
$M_0(8,11,14,17,20,23,26,29,32,35,38,41) = 16.33$
$M_0(9,12,15,18,21,24,27,30,33,36,39,42) = 17$
$M_0(5,8,11,14,17,20,23,26,29,32,35,38) = 14.33$
$M_0(6,9,12,15,18,21,24,27,30,33,36,39) = 15$

Step 1: Converting into crisp values.

	J_1	J_2	J_3	J_4
P_1	17.67	15.67	16.33	15.67
P_2	17	17.67	16.33	17
P_3	14.33	15.67	15.67	14.33
P_4	16.33	15	16.33	14.33

Step 2: Converting into minimization problem.

	J_1	J_2	J_3	J_4
P_1	0	2	1.34	2
P_2	0.67	0	1.34	0.67
P_3	3.34	2	2	3.34
P_4	1.34	2.67	1.34	3.34

Step 3*:* Hungarian Method.

	J_1	J_2	J_3	J_4
P_1	(0)	2	1.34	1.33
P_2	0.67	0	1.34	(0)
P_3	1.34	(0)	0	0.67
P_4	0	1.4	(0)	1.33

Fuzzy Optimal Assignment is $E_1 \rightarrow A_1$, $E_2 \rightarrow A_4$, $E_3 \rightarrow A_2$, $E_4 \rightarrow A_3$

Fuzzy Maximal Profit = (10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43)
+ (9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42)
+ (7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40)
+ (8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41)
= (34, 46, 58, 70, 82, 94, 106, 118, 130, 142, 154, 166)
= 66.67

B. Solving Maximal Assignment Problem of Octagonal Fuzzy Numbers

If $k_1 = k_2$, then the above dodecagonal fuzzy number is reduces to octagonal fuzzy number and we get

10,13,22,25,28,31,40,43	7,10,19,22,25,28,37,40	8,11,20,23,26,29,38,41	7,10,19,22,25,28,37,40
9,12,21,24,27,30,39,42	10,13,22,25,28,31,40,43	8,11,20,23,26,29,38,41	9,12,21,24,27,30,39,42
5,8,17,20,23,26,35,38	7,10,19,22,25,28,37,40	7,10,19,22,25,28,37,40	5,8,17,20,23,26,35,38
8,11,20,23,26,29,38,41	6,9,18,21,24,27,36,39	8,17,20,23,26,29,38,41	5,8,17,20,23,26,35,38

When this problem is solved as in [3] by taking $k_1 = k_2 = 0.7$ we would get

Fuzzy Maximal Profit = (10, 13, 22, 25, 28, 31, 40, 43)
+ (9, 12, 21, 24, 27, 30, 39, 42)
+ (7, 10, 19, 22, 25, 28, 37, 40)
+ (8, 17, 20, 23, 26, 29, 38, 41)
= (34, 46, 82, 94, 106, 118, 154, 166)
= 56.5

C. Solving Maximal Assignment Problem of Trapezoidal Fuzzy Numbers

If $k_1 = k_2 = 1$, then the above dodecagonal fuzzy number is reduced to trapezoidal fuzzy number and we get.

10,13,40,43	7,10,37,40	8,11,38,41	7,10,37,40
9,12,39,42	10,13,40,43	8,11,38,41	9,12,39,42
5,8,35,38	7,10,37,40	7,10,37,40	5,8,35,38
8,11,38,41	6,9,36,39	8,11,38,41	5,8,35,38

When this problem is solved as in [5] we would get

Fuzzy Maximal Profit = (10, 13, 40, 43) + (9, 12, 39, 42)

+ (7, 10, 37, 40) + (8, 11, 38, 41)
= (34, 46, 154, 166)
= 33.33.

V. CONCLUSION

In this paper, we have shown that the maximal profit obtained using Dodecagonal fuzzy number gives more profit in comparison with Octagonal and Trapezoidal fuzzy numbers.

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