Onset of Magnetoconvection in a Rotating Darcy-Brinkman Porous Layer Heated from Below with Temperature Dependent Heat Source

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Abstract: *The onset of stationary and oscillatory magnetoconvection in a rotating Darcy-Brinkman infinitely horizontal porous layer filled with electrically conducting Newtonian fluid heated from below with temperature – dependent heat source using linear stability analysis using free – free boundaries are investigated.The criteriafor the onset of convection in the system are derived analytically. The effects of heat source, γ, magnetic field, Ha, rotation and ratio of viscosities, Λparameters on the onset of convection are presented graphically and analyzed in detail. The effects of increasing magnetic field, rotation and ratio of viscosities delayed the onset of stationary and oscillatory convection, thereby stabilizing the system. However, increment in the heat source parameter accelerates the onset of convection and the system is more unstable, while Prandtl number slowed the onset of oscillatory convection.*

Keywords*:Magnetoconvection; rotating Darcy-Brinkman porous layer; temperature–dependent heat source; ratio of viscosities; free- free boundaries.*

I. INTRODUCTION

Several investigations has been conducted on the problem of viscous incompressible fluid in a rotating medium. We need mention the studies in [1], [2], [3],[4], [5], [6] which were done under various configurations and conditions. Investigation of the effect of the Cariolis force in thermal convection when the Darcy model is extended by including the acceleration term was carried out by [7]. The influence of gravity and centrifugal forces on the onset of convection in a rotating porous medium was taken into account in [8]. High Rayleigh number steady state thermal convection in a rotating porous half spacewas studied in [9]. This study showed that rotation led to rise in downward flow in contrast to the upward thermal convection. Experimental investigation in [10] of free convection in a horizontal circular cylinder embedded in a porous medium reveals that deviations from the Darcy's law occur when the Reynolds number based pore diameter exceeds 1-10. This suggests that a different flow scenero will prevail for sufficiently high Reynolds number.

It was stressed in [11] that in high permeability porous medium flow, the momentum equation for porous medium need reduce to the viscous flow limit and therefore advocated that classical frictional term be added in the Darcy's law. This was a basis for the Brinkman model which of course with the addition of the convective terms was looked at in [12] in their numerical study of free convection about an isothermal vertical plate in a porous medium. The Brinkman model to perform a detailed study on mixed convection boundary flow past a horizontal circular cylinder in a porous medium in [13]. The solution depends on the non dimensional Darcy-Brinkman parameter,Γ and the mixed convection parameter, λ.

We recognize that many convective instability problems of practical importance involve electrically conducting fluids. The linear theory of Rayleigh-Bernard magnetoconvection, in particular, how the onset of instability is affected by vertically imposed magnetic field was studied in detail in [1]. Studies of the effects of externally imposed magnetic field and rotation on couette flow in a porous medium was considered in [14]. A number ofsituations allow for the simultaneous occurrence of magnetic field and rotation in the same direction to the gravitational field, for example, [1] showed the dependence of the critical Rayleigh number, Ra on the magnetic field, Q and rotation, Ta for free – free boundaries when the medium adjoining the fluid is electrically non- conducting. The problem of the onset of thermal convection of Boussinesq fluid with simultaneous effects of rotation and magnetic field for rigid-rigid surfaces and non-conducting boundaries wasstudied in [15]. The study considered magnetic field and rotation in different directions relative to gravity.

Situations in which the onset of convection is induced by internal heating abound in nature such as in geophysics and engineering, for example, internal heat cores, nuclear waste disposal, crystal growth within porous media. Researchers have touched on them such as [16], [17], [18],[19]. Darcy and Brinkman limits in the problem of thermal instability with combined effects of centrifugal acceleration and anisotropy was considered in [20]. The effect of rotation on the onset of Darcy – Brinkman convection in a porous layer irradiated by purely internal heating with free-free, rigid-rigid and lower-rigid and upper-free boundaries was carried out in [21].

There are sufficient literature on thermal convection in a porous medium but,to the best of our knowledge, none has been devoted to study the combined effects of internal heating in a rotating Darcy-Brinkman layer with free-free boundaries. This is the subject of this present investigation.

II. MATHEMATICAL FORMULATION

Consider an infinite electrically conducting horizontal fluid saturated porous layer confined between two parallel boundaries located at $z^* = 0$ and $z^* = d$, and are maintained at temperatures T_h^*, and $T_c^*(T_h^* > T_c^*)$, respectively. The porous layer is rotating uniformly about the vertical axis at a constant angular velocity $\boldsymbol{\omega}^* = \omega\boldsymbol{k}$. A uniform magnetic field $\boldsymbol{B}^* = B_0^*\boldsymbol{k}$is applied across the fluid layer in the vertically upward direction, where the induced magnetic field is neglected on the account of small magnetic Reynolds number. A Cartesian coordinate system $(x^*,\ y^*, z^*)$ is chosen such that the origin is at the bottom of the porous layer and the gravity force acting in the negative $z^* -$ direction. The schematic diagram of the system considered is shown in Fig. 1.

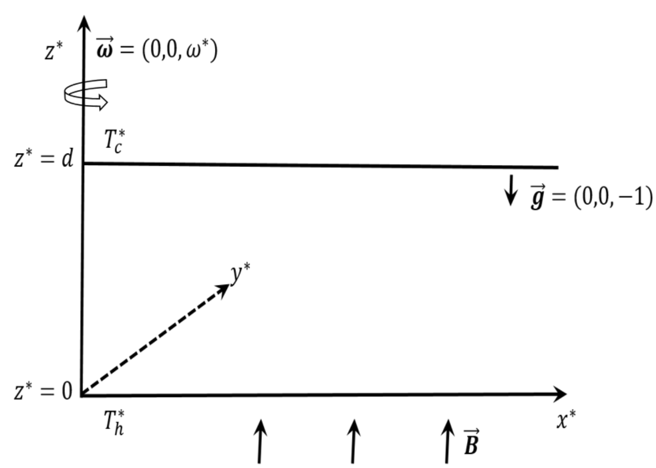

Fig. 1: Physical model and coordinate system.

Taking into consideration temperature dependent heat source, the Coriolis force and acceleration coefficient terms, the governing equations of the fluid motion in a homogeneous and isotropic medium follow Darcy-Brinkman model under Bousssinesq approximation together with Lorentz force are ([1], [22], [7],[19], [23])

$$\nabla^* \cdot \boldsymbol{V} = 0 \quad (1)$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \boldsymbol{V}}{\partial t} + \nabla^* P^* + \rho_f g \boldsymbol{k} + \frac{\mu_f}{K}\boldsymbol{V} - \mu_e \nabla^{*2} + 2\frac{\rho_0}{\varepsilon}\boldsymbol{\omega}^* \times \boldsymbol{V} - \boldsymbol{J}^* \times \boldsymbol{B}^* = 0 \quad (2)$$

$$A\frac{\partial T^*}{\partial t^*} + \boldsymbol{V}^*.\nabla^* T^* = \alpha_m \nabla^{*2} T^* + Q(T^* - T_0) \quad (3)$$

$$\rho_f = \rho_0[1 - \beta_T(T^* - T_0)] \quad (4)$$

$$\boldsymbol{J}^* = \sigma_c(\boldsymbol{E}^* + \boldsymbol{V}^* \times \boldsymbol{B}^*), \nabla^*.\boldsymbol{J}^* = 0 \quad (5)$$

Where $\rho_0$is the reference density, g is the acceleration due to gravity, $\boldsymbol{k}$ is the unit vector in the vertical direction,$\boldsymbol{V}^* = (u^*, v^*, w^*)$is the velocity, $\mu_e$is the effective viscosity, $\mu_f$is the fluid viscosity, P^* is the pressure, K is the permeability of the porous medium, μ is the fluid viscosity, $\alpha_m = \frac{\kappa}{(\rho c_p)_f}$ is the thermal diffusivity, where $(\rho c_p)_f$ is the volumetric heat capacity of the fluid, $A = \frac{(\rho c_p)_m}{(\rho c_p)_f}$ is the ratio of heat capacities, $(\rho c_p)_m = (1-\varepsilon)(\rho c_p)_s + \varepsilon(\rho c_p)_f$ is volumetric heat capacity of the porous medium, $c_p$is the specific heat capacity, ε is the porosity parameter where the subscripts *f*, *s* and *m* denotes the properties of the fluid, solid and porous matrix, respectively.β_T is the cofficient of thermal expansion, Further,$Q = \frac{Q_0}{(\rho c_p)_f}$ is constant of proportionality,$\boldsymbol{J}^*$is the current density, σ_c is the electrical conductivity and$\boldsymbol{E}^*$ is the electric field.

The electric field given in Eq. (5) can be written in terms of the electrostatic potential, ϕas $\boldsymbol{E}^* = -\nabla^*\phi$. We assume that the boundaries are electrically insulated for which ϕ is a constant hence the current density, $\boldsymbol{J}^*$ reduces to

$$\boldsymbol{J}^* = \sigma_c(\boldsymbol{V}^* \times \boldsymbol{B}^*) \quad (6)$$

and consequently, the Lorentz force takes the form

$$\boldsymbol{J}^* \times \boldsymbol{B}^* = \sigma_c(\boldsymbol{V}^* \times \boldsymbol{B}^*) \times \boldsymbol{B}^* = -\sigma_c B_0^2(U^*, V^*, 0) \quad (7)$$

Due to the geometry of the flow region, the following boundary condition are assumed for the velocity field

$$\boldsymbol{V}.\boldsymbol{k} = 0 \text{ on } z^* = 0, d \quad (8)$$

while for the temperature field

$$T^* = T_0 + \Delta T \qquad \text{on } z^* = 0 \quad (9)$$

$$T^* = T_0 \qquad \text{on} z^* = d \quad (10)$$

We now introduce the following non-dimensional quantities

$$(x, y, z) = \frac{1}{d}(x^*, y^*, z^*), t = \frac{\alpha_m}{Ad^2}t^*, \boldsymbol{V} = \frac{d}{\alpha_m}\boldsymbol{V}^*, T = \frac{T^* - T_0}{\Delta T^*}, P = \frac{K}{\alpha_m \mu_f}P^*$$

into the governing equations together with Eqs. (4) and (7), to obtain

$$\nabla.\boldsymbol{V} = 0 \quad (11)$$

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} + \frac{1}{Da}\right)\boldsymbol{V} + \nabla P - \mathrm{Ra}\, T\, \boldsymbol{k} - \Lambda \nabla^2 \boldsymbol{V} + \sqrt{T_D}\boldsymbol{k} \times \boldsymbol{V} + Ha^2(u, v, 0) = 0 \quad (12)$$

$$\frac{\partial T}{\partial t} + \boldsymbol{V}.\nabla T = (\nabla^2 + \gamma)T \quad (13)$$

The parameters that have entered the flow are:

$Pr = \frac{\mu_f \varepsilon A}{\rho_0 \alpha_m}$ is the Prandtl number, $Ra = \frac{\rho_0 g \beta_T d^3 \Delta T}{\alpha_m \mu_f}$ is the Raleigh number, $T_D = \left(\frac{2\rho_0 \omega^* d^2}{\mu_f \varepsilon}\right)^2$ is the Darcy-Taylor number, $Da = \frac{K}{d^2}$ is the Darcy number, $Ha = B_0\sqrt{\frac{\sigma_c d^2}{\mu_f}}$ is the Hartman number (magnetic parameter), $\Lambda = \frac{\mu_e}{\mu_f}$is the ratio of viscosities and $\gamma = \frac{Q_0 d^2}{\alpha_m}$ is the heat source parameter.

Now, the boundary conditionsfor the velocity and temperature fields become

$$w = 0, T = 1 \quad \text{on } z = 0 \quad (14)$$

$$w = 0, T = 0 \quad \text{on } z = 1 \tag{15}$$

III. METHOD OF SOLUTION

3.1. Basic state

We assume the basic state to be independent of time (quiescent) and vary only in the $z-$direction, and hence described by

$$\boldsymbol{V} = 0, T = T_b(z), p = p_b(z) \tag{16}$$

where the subscript "b" denotes the basic state. Substitution Eq. (16) into Eqs. (11) – (13) and the boundary conditions (14) and (15) yields the basic state equations as:

$$\frac{dp_b}{dz} = Ra\, T_b \tag{17}$$

$$\frac{d^2T_b}{dz^2} + \gamma T_b = 0 \tag{18}$$

The boundary conditions for T_b are

$$T_b = 1 \quad \text{on } z = 0 \tag{19}$$

$$T_b = 0 \quad \text{on } z = 1 \tag{20}$$

The solutions of Eqs. (17) – (18) subject to conditions (19) and (20) yield the basic state pressure and temperature profiles as

$$p_b(z) = Ra\, \frac{Cos[(1-z)\sqrt{\gamma}]}{\sqrt{\gamma}\sin[\sqrt{\gamma}]}, \quad T_b(z) = \frac{sin[(1-z)\sqrt{\gamma}]}{\sin[\sqrt{\gamma}]}, \tag{21}$$

3.2 Perturbation equations

To study the stability of the basic state, we superimpose perturbations on the basic state in the form

$$\boldsymbol{V} = \boldsymbol{v}, \quad P = p_b + p, \quad T = T_b(z) + \theta \tag{22}$$

where $p \ll p_b$ and $\theta \ll T_b$, are the perturbed quantities over their equilibrium counterparts and are assumed small. Substituting Eq. (22) into Eqs. (11) – (15), and linearizing by neglecting products of perturbed quantities, we obtain the following equations

$$\boldsymbol{\nabla}.\boldsymbol{v} = 0 \tag{23}$$

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} + k_D - \Lambda\nabla^2\right)\boldsymbol{v} + Ha^2(u, v, 0) + \sqrt{T_{\boldsymbol{D}}}\,\boldsymbol{k} \times \boldsymbol{v} + \nabla p - Ra\,\theta\,\boldsymbol{k} = 0 \tag{24}$$

$$\frac{\partial\theta}{\partial t} - (\nabla^2 + \gamma)\theta + f(z)\, w = 0 \tag{25}$$

where $f(z) = \frac{\partial T_b}{\partial z} = -\frac{\sqrt{\gamma}}{\sin[\sqrt{\gamma}]}\cos[\sqrt{\gamma}(1-z)]$ is the basic temperature gradient distribution and $k_{Ds} = \frac{1}{Da}$.

The boundary conditions are now

$$w = \theta = 0 \quad \text{on } z = 0, 1 \tag{26}$$

3.3 Linear stability analysis

Next, we eliminate the pressure by performing the operations $curl$ and $curl\, curl$ on Eq.(24), using the continuity equation (Eq. (23)), the identity $curl\, curl\, \boldsymbol{A} = grad\, div\boldsymbol{A} - \boldsymbol{\nabla}^2\boldsymbol{A}$ and keeping only the $z-$components yield

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} + k_D + Ha^2 - \Lambda\nabla^2\right)\xi - \sqrt{T_D}\, Dw = 0 \tag{27}$$

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} + k_D + Ha^2 - \Lambda\nabla^2\right)\nabla^2 w + Ha^2 D^2 w + \sqrt{T_D}\, D\xi - Ra\,\nabla_h^2\theta = 0 \tag{28}$$

where $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z – component of the vorticity, $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian in the horizontal plane and $D = \frac{\partial}{\partial z}$.

We assume that the conductive motion exhibit horizontal periodicity, hence we seek a time-dependent periodic disturbance of the form:

$$\begin{pmatrix} W \\ \xi \\ \theta \end{pmatrix} = \begin{pmatrix} W(z) \\ Z(z) \\ \Theta(z) \end{pmatrix} f(x,y) e^{st} \tag{29}$$

where $W(z), Z(z)$ and $\Theta(z)$ represent the vertical velocity, vorticity and temperature, respectively. Also, $s(=\omega_r + i\omega_i,\ \omega_r, \omega_i$ real) is the growth rate of disturbances; and $f(x,y)$ is a function in the $xy-$plane, such that $\nabla_h^2 f(x,y) + a^2 f(x,y) = 0$ and a is the wave number. The substitution of Eq. (29) into Eqs. (25), (27) and (28) yield

$$(D^2 - a^2 + \gamma - s)\Theta + f(z)W = 0 \tag{30}$$

$$\left(\frac{s}{Pr} + k_D + Ha^2 - \Lambda(\mathrm{D}^2 - \mathrm{a}^2)\right) Z - \sqrt{T_D}\, DW = 0 \tag{31}$$

$$\left(\frac{s}{Pr} + k_D - \Lambda(\mathrm{D}^2 - \mathrm{a}^2)\right)(D^2 - a^2)W + Ha^2 D^2 W + a^2 Ra\Theta + \sqrt{T_D}\, DZ = 0 \tag{32}$$

The boundary conditions are now

$$W = D^2 W = \Theta = Z = 0 \qquad \text{on} \qquad z = 0,1 \tag{33}$$

for free-free boundaries.

Equations (30) – (32) together with conditions (33) constitute a linear eigenvalue problem of the system whose solution could be obtained by assuming:

$$W = W_n Cos(n\pi z), \Theta = \Theta_n Cos(n\pi z), Z = Z_n Sin(n\pi z) \tag{34}$$

for arbitrary W_n, Θ_n, Z_n and n an integer. From now on, we restrict our analysis to the lowest mode (idealized mode) $n = 1$, which corresponds to the most dangerous mode. On using Eq. (34) the eigenvalue problemEqs. (30) – (32) in matrix form become

$$B\bar{X} = 0 \tag{35}$$

where

$$B = \begin{pmatrix} -2F & 0 & \delta^2 - \gamma + s \\ \pi\sqrt{T_D} & \frac{s}{Pr_D} + k_D + Ha^2 + \Lambda\delta^2 & 0 \\ \left(\frac{s}{Pr_D} + k_D + \Lambda\delta^2\right)\delta^2 + \pi^2 Ha^2 & -\pi\sqrt{T_D} & -a^2 Ra_D \end{pmatrix}, \quad \bar{X} = (W_1, \Theta_1, Z_1) \text{ and}$$

$$F = \frac{\sqrt{\gamma}}{Sin[\sqrt{\gamma}]} \int_0^1 Cos[\sqrt{\gamma}(1-z)] Sin^2(\pi z)\, dz = \frac{2\pi^2}{4\pi^2 - \gamma},\ \delta^2 = a^2 + \pi^2.$$

The solvability of the system given by Eq. (35) requires that the determinant of B vanish, that is $|B| = 0$.This condition yields the expression for the Rayleigh number, Ra for the determination of the onset of convection in the system as

$$Ra = (\delta^2 - \gamma + s)\left[\left(\frac{Pr(\pi^2 Ha^2 + k_D\delta^2 + \Lambda\delta^4) + \delta^2 s}{2\ a^2 F\ Pr}\right) + \frac{Pr^2 \pi^2 T_D}{2\ a^2 F Pr(Pr(Ha^2 + \Lambda\delta^2 + k_D) + s)}\right] \tag{36}$$

where $\delta^2 = \pi^2 + a^2$.

Since the growth rate $s(=\omega_r + i\omega_i,\ \omega_r,\ \omega_i)$ is in general a complex quantity. The system with $\omega_r < 0$ is always stable , while for $\omega_r > 0$, it will be unstable. For neutral (marginal) stability $s = 0$.

3.4 Onset of stationary convection

For the onset of stationary convection in which the principle of exchange of stabilities is valid, we set $s = 0$ ($\omega_r = \omega_i = 0$) ,and $Ra = Ra^{st}$in Eq. (36). Simplifyingyields the Rayleigh number, Ra^{st}as

$$Ra^{st} = (\delta^2 - \gamma)\left[\frac{k_D\delta^2 + \Lambda\delta^4}{2\,a^2F} + \frac{\pi^2 Ha^2}{2\,a^2F} + \frac{\pi^2 T_D}{2\,a^2F(\Lambda\delta^2 + k_D + Ha^2)}\right] \tag{37}$$

Next, we compute the critical (minimum) values of the critical wave number, a_c and the corresponding critical Rayleigh number, $Ra_c^{(st)}$ for the onset of convection. By setting $a = a_c$ and $Ra^{(st)} = Ra_c^{(st)}$ in Eq. (37) and minimizing according to [1]

$$\frac{\partial Ra_c^{(st)}}{\partial a_c^2} = 0 \tag{38}$$

This minimization condition above yields the following tenth order polynomial in $a_c$given by

$$b_5p^5 + b_4p^4 + b_3p^3 + b_2p^2 - b_1p - b_0 = 0, \tag{39}$$

where

$p = a_c^2,\, b_5 = 2\pi^2,$

$b_4 = \Lambda^2\big((7\pi^2 - \gamma)\Lambda + (4\,Ha^2 + 5k_D)\big),$

$b_3 = 2\Lambda Ha^2(Ha^2 + (5\pi^2 - \gamma)\Lambda) + 2\pi^2\Lambda^3(4\pi^2 - \gamma) + 2(3Ha^2 + (6\pi^2 - \gamma)\Lambda)\Lambda k_D + 4\Lambda k_D^2,$

$b_2 = Ha^4\Lambda(3\pi^2 - \gamma) + Ha^2\pi^2\Lambda^2(5\pi^2 - \gamma) + 2\pi^6\Lambda^3 - \pi^2T_D\Lambda + (2Ha^2 + (5\pi^2 - \gamma)\Lambda k_D^2) + \big(Ha^4 + 2Ha^2\Lambda(4\pi^2 - \gamma) + \pi^2\Lambda^2(6\pi^2 - \gamma)\big)k_D,$

$b_1 = (2Ha^2\pi^2\Lambda + 4Ha^2\pi^4\Lambda^2 + 2\pi^6\Lambda^3 + (4Ha^2\pi^2\Lambda + 4\pi\Lambda^2)k_D + 2\pi^2\Lambda(k_D^2 + T_D))(\pi^2 - \gamma),$

$b_0 = (Ha^6\pi^2 + 3Ha^4\pi^4\Lambda + 3Ha^2\pi^6\Lambda^2 + \pi^8\Lambda^3 + (3Ha^2\pi^2 + 3\pi^4\Lambda)k_D^2 + \pi^2k_D^2 + Ha^2\pi^2T_D$
$\pi^4T_D\Lambda + (3Ha^4 + 6Ha^2\pi^4\Lambda + 3\pi^6\Lambda^2 + \pi^2T_D)k_D)(\pi^2 - \gamma)$

Equation (39) is solved numerically for the critical wave number, a_c for various values of magnetic field, Ha, rotation, T_D, Darcy number, $Da = \frac{1}{k_D}$, and heat source, γ. Using this value of a_c, the critical Rayleigh number, Ra_c^{st} for the determination of the criterion for the onset of stationary convection is obtained.
To validate our results with those in literature, we set the $\gamma = 0, Ha = 0$, and $T_D = 0$ in Eq. (39), and obtain the critical wave number, a_c as

$$a_c = \sqrt{\frac{\left(-(\Lambda\pi^2 + k_D)k + \sqrt{(\Lambda\pi^2 + k_D)(9\pi^2\Lambda + k_D)}\right)}{4\Lambda}} \tag{40}$$

Further as $k_D \to 0$, i.e. as $Da \to \infty$, the critical wave number reduces to$a_c = \frac{\pi}{\sqrt{2}}$and the corresponding critical Rayleigh number $Ra_c^{st} = \frac{27}{4}\pi^4$. This is the exact results obtained for viscous fluids in [1].

3-5 Onset of oscillatory convection

To analyze the onset of oscillatory convection, we set $s = i\omega_i$ and $Ra = Ra^{os}$ in Eq. (36) and obtain the expression for the oscillatory Rayleigh number, Ra^{os}as

$$Ra^{os} = \Delta_1 + i\omega_i\Delta_2 \tag{41}$$

where

$$\Delta_1 = \frac{1}{2FPr\,a^2}\left(b_1b_2 - \delta^2\omega_i^2 + \frac{\pi^2Pr^2T_Db_0}{b_0^2 + \omega_i^2}\right),\ \Delta_2 = \frac{1}{2F\,Pr\,a^2}\left(\delta^2b_1 + b_2 - \frac{\pi^2Pr^2T_D}{b_0^2 + \omega_i^2}\right),$$

$$b_0 = Pr(\Lambda\delta^2 + k_D + Ha^2),\ b_1 = \delta^2 - \gamma,\ b_2 = Pr(\Lambda\delta^4 + k_D\delta^2 + \pi^2Ha^2)$$

Now, since the oscillatory Rayleigh number, Ra^{os} is a physical quantity, it must be real. This requires that the imaginary part of Eq. (41) be zero. That is

$$\delta^2 b_1 + b_2 - \frac{\pi^2 Pr^2 T_D}{b_0^2 + \omega_i^2} = 0 \tag{42}$$

From Eq. (42), it follows that the frequency, ω_i^2 for the occurrence of oscillatory convection is

$$\omega_i^2 = \frac{\pi^2 Pr^2 T_D - b_0^2(b_2 + b_1\delta^2)}{b_2 + b_1\delta^2} \tag{43}$$

Hence, from Eq. (43) oscillatory convection can occur for a particular wave number, a_c if the following inequality is satisfied

$$T_D > \left(\frac{b_0^2(b_2 + b_1\delta^2)}{(\pi^2 Pr^2}\right) \tag{44}$$

Hence, the corresponding oscillatory Rayleigh number, Ra^{os} is given by the expression

$$Ra^{os} = \frac{1}{2F\,Pra^2}\Big((\delta^2 - \gamma)(\Lambda\delta^4 + k_D + \pi^2 Ha^2 - Pr\delta^2\omega_i^2) + Pr\pi^2 T_D\left(\frac{\Lambda\delta^2 + k_D + Ha^2}{(\Lambda\delta^2 + k_D + Ha^2)^2 + \omega_i^2}\right)\Big) \tag{45}$$

IV. RESULTS AND DISCUSSION

The criteria for onset of magnetoconvectionin a rotating Darcy-Brinkman porous layer saturated with electrically conducting fluid with temperature dependent heat source taking into consideration the ratio of viscosities has been studied analytically. In this section, we discuss the effects of various parameters in the governing equations on the onset of both stationary and oscillatory convections numerically and graphically. The results of effects of different parameters $Ha, Ta, Da, \gamma, \Lambda$and Pr are presented in Tables 1 – 4 for the stationary critical wave number, a_c and critical Rayleigh number, Ra_c^{st}, while the stability curves in $Ra - a$ plane for the onset of both stationary and oscillatory convection for various values of magnetic field, rotation, heat source, viscosity ratio and Prandtl number are shown in Figs. (2) – (6), respectively.

Table 1: Critical Values of Wave Number, a_c and Rayleigh Number, Ra_c^{st} Against Heat Source, γ for $Ha = 2, Da = 100, \Lambda = 10$, for $Ta = 0, 10, 20, 50$.

	$Ta = 0$		$Ta = 10$		$Ta = 20$		$Ta = 50$	
γ	a_c	Ra_c^{st}	a_c	Ra_c^{st}	a_c	Ra_c^{st}	a_c	Ra_c^{st}
0.0	2.2412	6693.27	2.2416	6695.18	2.2421	6697.10	2.2435	6702.83
0.5	2.2217	6385.97	2.2222	6387.83	2.2227	6389.69	2.2241	6395.25
1.0	2.2012	6083.01	2.2016	6084.82	2.2021	6086.62	2.2035	6092.01
1.5	2.1794	5784.31	2.1798	5786.00	2.1803	5787.80	2.1817	5793.03
2.0	2.1562	5489.77	2.1566	5491.46	2.1571	5493.15	2.1585	5498.21
2.5	2.1314	5199.26	2.1319	5200.90	2.1324	5202.54	2.1338	5207.44
3.0	2.1049	4912.66	2.1054	4914.24	2.1059	4915.82	2.1072	4920.56
3.5	2.0764	4629.79	2.0769	4631.32	2.0774	4632.85	2.0787	4637.81
4.0	2.0456	4350.45	2.0461	4351.92	2.0465	4353.39	2.0479	4357.81
4.5	2.0121	4074.37	2.0126	4075.79	2.0130	4077.21	2.0144	4081.47
5.0	1.9754	3801.23	1.9759	3802.60	1.9763	3803.97	1.9777	3808.06

Table 2: Critical Values of Wave Number, a_c and Rayleigh Number, Ra_c^{st} Against Heat Source, γ for$Ta = 10, Da = 100, \Lambda = 10$, for $Ha = 0, 2, 5, 10$.

	$Ha = 0$		$Ha = 2$		$Ha = 5$		$Ha = 10$	
γ	a_c	Ra_c^{st}	a_c	Ra_c^{st}	a_c	Ra_c^{st}	a_c	Ra_c^{st}
0.0	2.2220	6577.53	2.2416	6695.18	2.3356	7292.53	2.5900	9232.00
0.5	2.2027	6274.29	2.2222	6387.83	2.3154	6964.52	2.5675	8839.14
1.0	2.1823	5975.36	2.2016	6084.82	2.2939	6641.03	2.5438	8451.42
1.5	2.1607	5680.64	2.1798	5786.06	2.2712	6321.96	2.5187	8068.58

2.0	2.1377	5390.06	2.1566	5491.46	2.2471	6007.21	2.4920	7690.54
2.5	2.1132	5103.48	2.1319	5200.90	2.2214	5696.64	2.4636	7317.11
3.0	2.0869	4820.78	2.1054	4914.24	2.1938	5390.11	2.4331	6948.11
3.5	2.0586	4541.79	2.0769	4631.32	2.1642	5087.42	2.4004	6583.29
4.0	2.0281	4266.30	2.0461	4351.92	2.1321	4788.35	2.3651	6222.37
4.5	1.9949	3994.07	2.0126	4075.79	2.0973	4492.62	2.3268	5864.97
5.0	1.9585	3724.78	1.9759	3802.60	2.0592	4199.62	2.2849	5510.64

Table 3: Critical Values of Wave Number, a_c and Rayleigh Number, Ra_c^{st} Against Viscosity Ratio, Λ for $Ha = 2, Da = 100, \gamma = 0.5$, for $Ta = 0, 10, 20, 50$.

Λ	$Ta = 0$		$Ta = 10$		$Ta = 20$		$Ta = 50$	
	a_c	Ra_c^{st}	a_c	Ra_c^{st}	a_c	Ra_c^{st}	a_c	Ra_c^{st}
1	2.3742	736.19	2.4018	749.35	2.4283	762.23	2.5021	799.45
2	2.2942	1366.00	2.3033	1374.13	2.3123	1381.69	2.3386	1404.80
3	2.2650	1994.16	2.2695	1999.77	2.2740	2005.36	2.2872	2022.00
4	2.2499	2621.86	2.2526	2626.22	2.2552	2630.56	2.2632	2643.53
5	2.2407	3249.37	2.2424	3252.92	2.2442	3256.48	2.2494	3267.10
6	2.2344	3876.77	2.2357	3879.78	2.2369	3882.78	2.2407	3891.77
7	2.2299	4504.12	2.2308	4506.72	2.2318	4509.32	2.2346	4517.11
8	2.2265	5131.42	2.2272	5133.72	2.2280	5136.02	2.2301	5142.89
9	2.2239	5758.71	2.2244	5760.76	2.2250	5762.81	2.2267	5768.96
10	2.2217	6385.97	2.2222	6387.83	2.2223	6389.69	2.2241	6395.25

Table 4: Critical Values of Wave Number, a_c and Rayleigh Number, Ra_c^{st} Against Viscosity Ratio, Λ for $Ta = 10, Da = 100, \gamma = 0.5$, for $Ha = 0, 2, 5, 10$.

Λ	$Ha = 0$		$Ha = 2$		$Ha = 5$		$Ha = 10$	
	a_c	Ra_c^{st}	a_c	Ra_c^{st}	a_c	Ra_c^{st}	a_c	Ra_c^{st}
1	2.2510	646.59	2.4018	749.35	2.9010	1217.63	3.6701	2565.97
2	2.2148	1264.46	2.3033	1374.13	2.6370	1888.08	3.2423	3396.15
3	2.2079	1888.46	2.2695	1999.77	2.5211	2535.75	3.0295	4142.62
4	2.2054	2514.04	2.2526	2626.22	2.4548	3175.05	2.8958	4851.53
5	2.2043	3140.27	2.2424	3252.92	2.4115	3810.29	2.8023	5539.20
6	2.2036	3766.82	2.2357	3879.78	2.3810	4443.22	2.7324	6123.36
7	2.2033	4393.55	2.2308	4506.72	2.3583	5074.71	2.6779	6878.26
8	2.2030	5020.39	2.2272	5133.72	2.3408	5705.26	2.6341	7536.51
9	2.2028	5647.32	2.2244	5760.76	2.3268	6335.13	2.5979	8189.77
10	2.2027	6274.29	2.2222	6387.83	2.3154	6964.52	2.5675	8839.19

The effect of the heat source parameter, γ on the stability of the system is depicted in Figs. 2a – 2b. The thermal Rayleigh numbers for onset stationary and oscillatory convections decreased with increases in the heat source parameter. This shows, that the heat source parameter accelerates the onset of convection in the system.

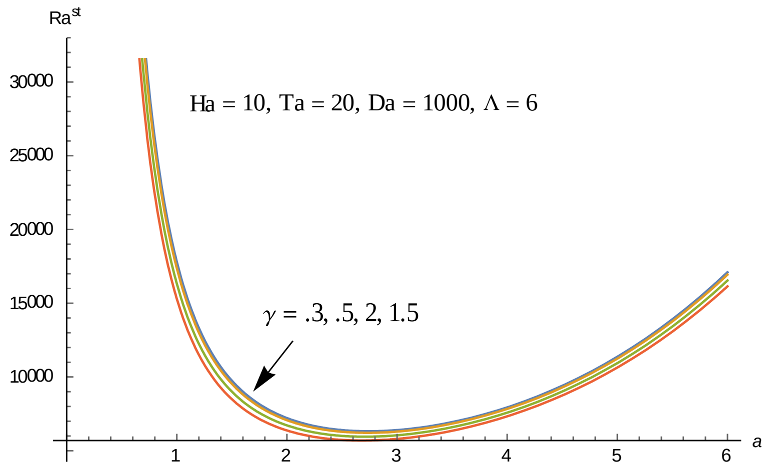

Fig. 2a: Effect of heat source, γ on the thermal Rayleigh number, Ra^{st} with respect to wave number, *a* for stationary convection.

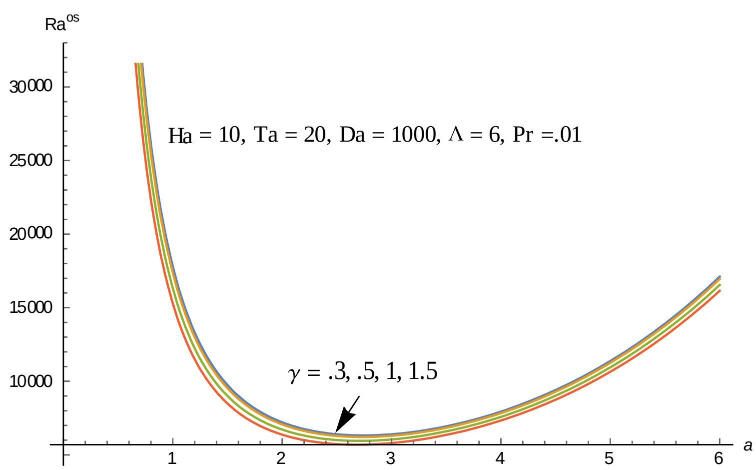

Fig. 2b: Effect of heat source, γ on the thermal Rayleigh number, Ra^{os} with respect to wave number, *a* for oscillatory convection.

Figures 3 - 6show the effects of magnetic field, Ha, rotation, Ta, ratio of viscosities, Λ, and Prandtl number, Pron the onset of stationary and oscillatory convections of the system. It is observed from these figures that increases in the parameters result in increase in the critical Rayleigh numbers. This shows that these parameters delayed the onset of convection and hence have stabilizing factor to make the system more stable.

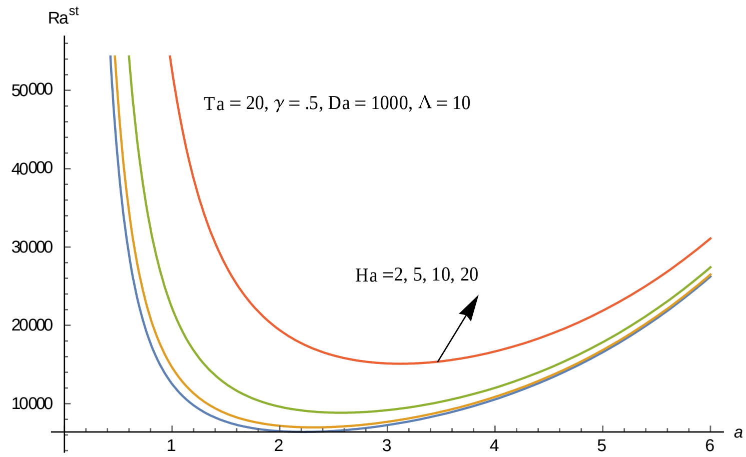

Fig. 3a: Effect of magnetic field, Ha on the thermal Rayleigh number, Ra^{st} with respect towave number, a for stationary convection.

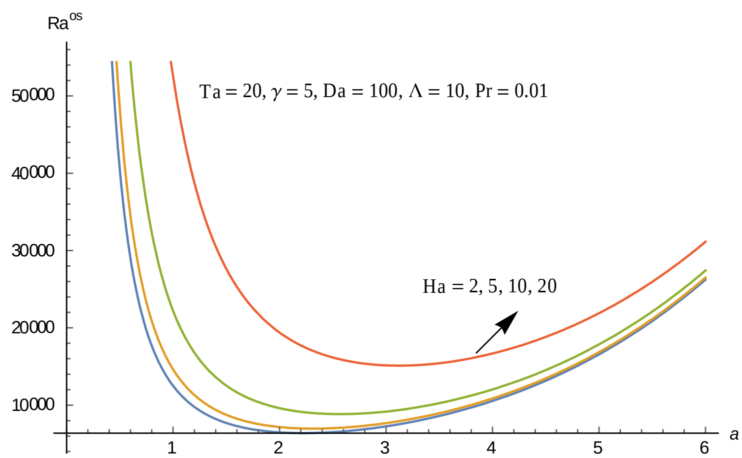

Fig. 3b: Effect of magnetic field, Ha on the thermal Rayleigh number, Ra^{os} with respect to wave number, a for oscillatory convection.

respectively. It is observed that for the thermal Rayleigh numbers for both stationary and oscillatory increase as the magnetic field parameter increases, for $Ta = 20, \gamma = 0.5, Da = 100, Pr = 0.01$ and $\Lambda = 10$. Hence, magnetic field parameter, Ha delays the onset of convection in the electrically saturated – rotating porous layer. This is because, the Lorentz force suppresses the vertical motion and hence convection, by restricting the motion in the horizontal plane.

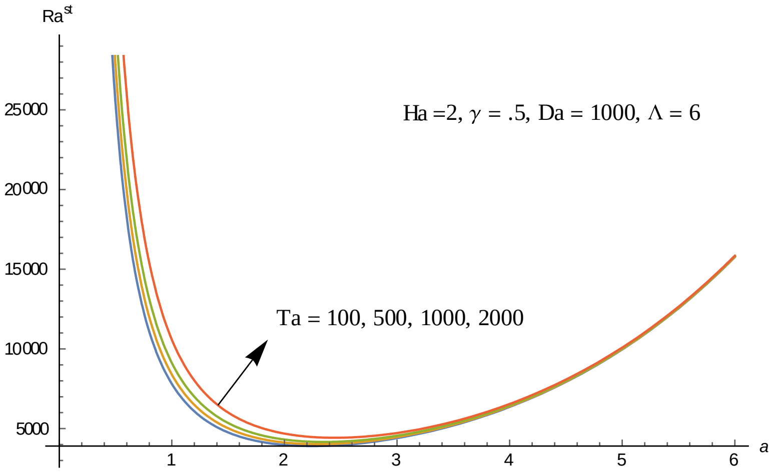

Fig. 4a:Effect of rotation, *Ta* on the thermal Rayleigh number, Ra^{st} with respect to wave number, *a* for stationary convection

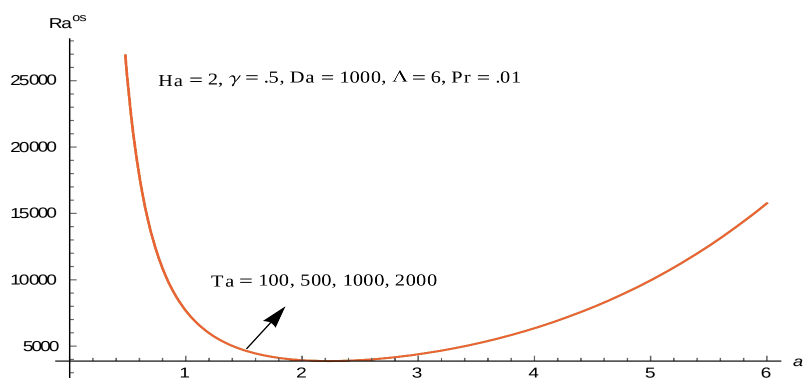

Fig. 4b: Effect of rotation, *Ta* on the thermal Rayleigh number, Ra^{os} with respect to wave number, *a* for stationary convection.

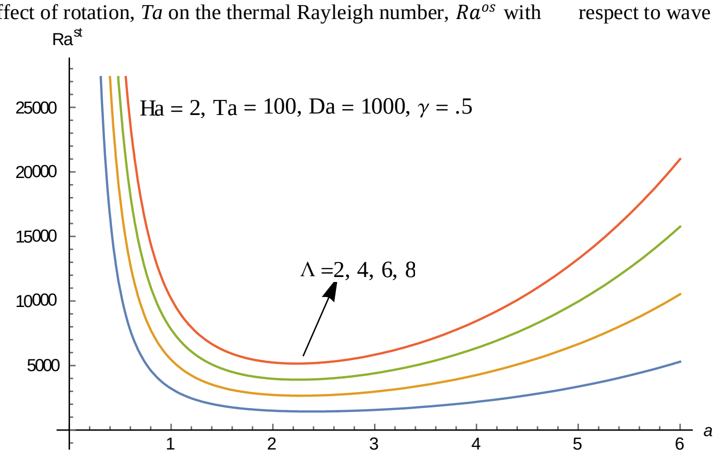

Fig. 5a: Effect of viscosity ratio, Λ on the thermal Rayleigh number, Ra^{st} with respect to wave number, *a* for stationary convection.

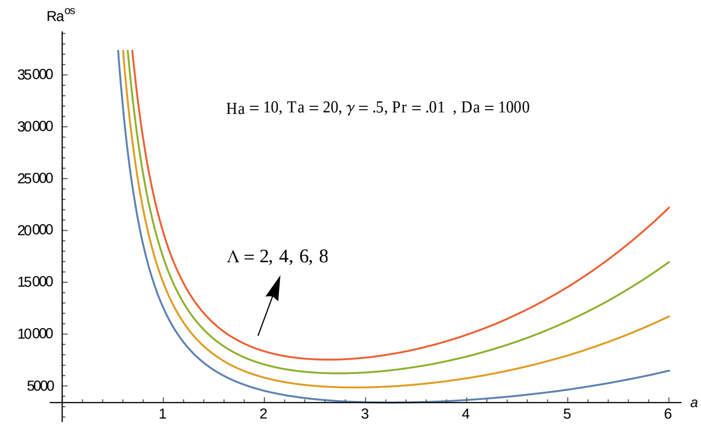

Fig. 5b: Effect of viscosity ratio, Λ on the thermal Rayleigh number, Ra^{os} with respect to wave number, *a* for oscillatory convection.

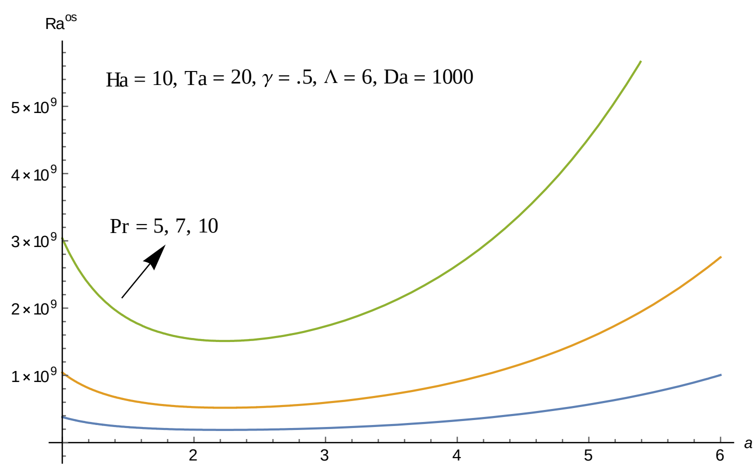

Fig. 6: Effect of Prandtl number, , Pr on the thermal Rayleigh number, Ra^{os} with respect to wave number, *a* for oscillatory convection.

V. CONCLUSION

The criterion for the onset of magnetoconvection in a rotating Darcy layer filled with electrically conducting fluid with temperature – dependent heat source, which is heated from below for free – free boundaries has been investigated. The linear stability analysis is used for establishing the criteria for onset of both stationary and oscillatory convections in the system. The effects of physical parameters in the governing equations, such as heat source parameter, γ, magnetic field parameter, Ha, rotation parameter, T_D, and the Prandtl number, Pr are shown graphically. The effects of increasing magnetic field and rotation parameters slow down the onset of both stationary and oscillatory convection, while the Prandtl number delays the onset of oscillatory convection. This means that magnetic field parameter, Ha, rotation parameter, T_D, and the Prandtl number, Pr are stabilizing factors. On the other hand, the heat source parameter, γ accelerates the onset of convection.

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