Fuzzy Group And Their Types

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Abstract - *In this paper we introduce the concept of fuzzy group and their some related properties are investigated and we also define the algebraic structure of fuzzy group and their types. The purpose of this study is to implement of fuzzy set theory and group theory in fuzzy group.*

Keywords - *Fuzzy set, Multi -fuzzy set, Fuzzy group, Multi -fuzzy group, Fuzzy subgroup, Fuzzy Abelian group, Fuzzy normal subgroup, Left and Right Coset of Fuzzy group, Multi Anti Fuzzy subgroup.*

I. INTRODUCTION

The concept of fuzzy set was introduced by Lotti A Zadeh in 1965. The theory of multi-fuzzy set is an extension of theories of fuzzy set. And the theory of multi fuzzy sets is proposed by the S. Sabu and T.V. Ramakrishnan.

The investigated the theory of multi-dimensional membership function and some properties of multi- level fuzziness. R. Muthuraj and S. Balamurgan proposed multi-fuzzy group and its level subgroups .The concept of anti-fuzzy subgroup was introduced by BISWAS. In this paper we define a new algebraic structure of multi-fuzzy group and their subgroup and also study some of their related properties

II. RESEARCH OBJECTIVES

Fuzzy mathematics is a branch of mathematics that is related to fuzzy logic. It started in 1965 after the publication of Lotfi Asker Zadeh's seminal work Fuzzy set.

Before knowing Fuzzy group we should know the Fuzzy set. Fuzzy group is an extension of theories of fuzzy sets. In this paper we define a new algebraic structure of fuzzy group and fuzzy subgroups and study some of their related properties.

III. HYPOTHESIS

A hypothesis is an unproven statement which is supported by all the available data and by much weaker result. It is a sequential process of research. It is an indicator of the work to be done for research. It describes the theoretical information in relation to how the research study will work. The hypothesis is a formal suggestion that express the outline of the study's proposal. Since Zadeh introduced the concept of a fuzzy set in 1965, various algebraic structures have been fuzzified. As a result the theory of fuzzy group was developed. In 1971 Rosenfield introduced the notion of a fuzzy subgroup and thus initiated study of fuzzy group. In this paper we preset the further investigation into properties of fuzzy groups and their types.

IV. RESEARCH METHOD

A research design is the set of methods and procedures used in collecting and analyzing measures of variables specified in the problem research. A research design is a frame work that has been created to find answers to research questions. There are numerous types of research design that are appropriate for the different types of research projects. The choice of which design to apply depends on the problems posed by the research aim.
Each type of research design has a range of research methods that are commonly used to collect and analyses the type of data, which is generated by the investigations. Here is a list of some of more common research design, which is used to find expected outcome of my proposed work.

1. *Historical –* This aim at systematic and objective evaluation and synthesis of evidence in order to establish facts and draw conclusion about past events. It uses primary historical data, such as archaeological remains as well as documentary sources of the past, it is usually necessary to carry out tests in order to check the authenticity of these sources.

2. ***Descriptive –*** This design relies on observation as a means of collecting data. It attempts to examine situations in order to establish. Observation can take many forms depending on the type of information sought, people can be interviewed questionnaires distributed.
3. ***Correlation –*** This design is used to examine a relationship between two concepts. There are two broad classifications of relational statements and association between two concepts.
4. ***Comparative –*** This design is used to compare past and present or different parallel situations. It can look at situation at different scales, macro (international, national) or micro (community, individual).

V. ANALYSIS

Fuzzy mathematics is a branch of mathematics that is related to fuzzy logic. It started in 1965 after the publication of Lotfi Asker Zadeh's seminal work Fuzzy set before knowing Fuzzy group we should know the Fuzzy set.

1. *Fuzzy set-*

In this section we shall start to introduce the concept about fuzzy set on the basis definition with some example-

Definition- Let A is a non-empty set and the function M: A→ [01] is called fuzzy set in A

Example- Let X be real number R, then the function M: R→ [01] such that

$$M(X) = \begin{bmatrix} 1 - 1/x & if\ x > 1 \\ 0 & if\ x \leq 1 \end{bmatrix}$$

Is fuzzy set

2. *Multi-fuzzy set*

Let X be a non-empty set. A multi –fuzzy set A in X is defined as a set of ordered sequence.

$$A = \{(x, \mu_1(x), \mu_2(x)\dots\mu_i(x)\ \dots..):\ x \in X\}$$

Where $\mu_i : X \rightarrow [0, 1]$ for all i

Remark –

(i) If the sequences of the membership functions have only K-terms (finite number of terms) Then K is called the dimension of A.
(ii) The set of all multi fuzzy sets in X of dimension K is denoted by $M^k FS(X)$.

Properties of Multi-Fuzzy Sets-

Let A and B be any two multi-fuzzy sets of a non-empty set X then—

(i) $A \cup B = A$, $A \cup A = A$

(ii) $A \subseteq A \cup B$, $B \subseteq A \cup B$, $A \cap B \subseteq A$ AND $A \cup B \subseteq B$

(iii) $A \subseteq B$ iff $A \cup B = B$

(iv) $A \subseteq B$ iff $A \cap B = A$

3. *Fuzzy Group -*

Definition – Let G be any group . A mapping $\mu : G \rightarrow [0\ 1]$ is a fuzzy group if---

(F G1) μ (x y) ≥ min { μ (x) , μ(y) }

(F G2) μ (x^{-1}) = μ (x)   For all x , y ∈ G

4. *Fuzzy Abelian Group –*

The Fuzzy Group (x,) is called a Fuzzy Abelian Group if X is abelian . Moreover in this case the binary operation is denoted by" + ", ". " Thus (x,) is a fuzzy abelian group if..

(1) X is an abelian group.
(2) $\mu (x + y) \geq$ min { μ (x) , μ (y) }

(3) $\mu(-x) = \mu(x)$

For all $x \in X$

5. *Fuzzy Subgroup -*

Let G be a Group and A is a fuzzy subset of G then A is called a fuzzy subgroup of G if –

(i) $A(x,y) \geq \{A(x), A(y)\}$
(ii) $A(x^{-1}) = A(x)$

6. *Normal Fuzzy subgroup –*

Let G be a Group and A is fuzzy subgroup of G, so A is also satisfied the above fuzzy subgroup conditions then A is called normal fuzzy subgroup of G if $A(x,y) = A(yx)$ for all x and y in G

7. *Anti -Fuzzy Subgroup -*

Let A be a fuzzy set on a group G, then A is called an anti-fuzzy subgroup of G, if for all x, y $\in$ G.

(i) $A(xy) \leq \max\{A(x), A(y)\}$
(ii) $A(x^{-1}) = A(x)$

8. *Multi fuzzy Subgroup -*

A multi fuzzy set A of a group G is called a Multi- fuzzy subgroup of G if for all x, y $\in$ G,

(i) $A(xy) \geq \min\{A(x), A(y)\}$
(ii) $A(x^{-1}) = A(x)$.

9. *Multi –Anti Fuzzy Subgroup -*

A multi- anti fuzzy set A of a group G is called multi-anti-fuzzy subgroup of G if for all x,y $\in$ G.

(i) $A(xy) \leq \max\{A(x), A(y)\}$.
(ii) $A(x^{-1}) = A(x)$.

VI. CONCLUSION

In this paper we conclude that the concept of fuzzy group and proved some properties of this new concept. We give the basic properties of fuzzy group, fuzzy subgroups and their types. We present a further investigation into properties of fuzzy subgroup, fuzzy normal subgroup and fuzzy left and right coset. Certainly some other topics of fuzzy subgroups such as the characterization of fuzzy subgroup and operations of fuzzy subgroups are still open for the research in the further.

REFERENCES

[1] P. S. DAS, Fuzzy groups and level subgroups, J. Math. Anal. Appl. 84 (1981), 264-269.
[2] D. DUBOIS AND H. PRADE, "Fuzzy Sets and Systems: Theory and Applications,"Academic Press, New York, 1980.
[3] C. NEGOITA,L . A. ZADEH,A ND H. J. ZIMMERMAN(E ds.), Fuzzy Sets and Systems (1977 ff).
[4] A. ROSENFELD Fuzzy croups, J. Math. Anal. Appl. 35 (1971) 512-517.
[5] ROSENFELD How many are a few? Fuzzy sets, fuzzy numbers and fuzzy mathematics, Math. Inteliigencer 4 (1982), 139-143
[6] M. Asaad, Groups and fuzzy subgroups, Fuzzy Sets and Systems 39 (1991) 323{328.
[7] S. Abdullah, M. Aslam and T. Ahmed Khan, A new generalization of fuzzy subgroup and level subsets, Ann. Fuzzy Math. Inform. 9 (6) (2015) 991{1018.
[8] R. Biswas, Fuzzy subgroups and anti-fuzzy subgroups, Fuzzy Sets and Systems 35 (1990) 121{124.
[9] W. B. VasanthaKandasamy and D. Meiyappan, Pseudo fuzzy cosets of fuzzy subsets, fuzzy subgroups and their generalizations, Vikram Mathematical J. 17 (1997) 33{44.
[10] L. A. Zadeh , Fuzzy Sets, Information and Control 8 (1965) 338{353.
[11] Rosenfeld A (1971) Fuzzy groups. J. Math. Anal. Appl. 35: 512-517
[12] Wu W (1981) Normal fuzzy subgroups. Fuzzy Math. 1: 21-30