Choquet Integral Compared With Weighted Mean In Decision Making

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Abstract:
Weighted mean is most popular method. The Choquet integral is too tough to apply in practice. In this paper, we investigate the decision making problem using the Choquet integral compare with weighted mean. Also, we discuss the accuracy of the proposed method.

Keywords: *Fuzzy Set, Fuzzy Measure, Choquet Integral.*

I. INTRODUCTION

Fuzzy theory was initiated by Lotfi A. Zadeh[6] in 1965 with his seminal paper "Fuzzy Sets" as early as 1962, he wrote that to handle biological system "We need a radically different kind of mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions". Later he formalized the ideas into the paper "Fuzzy Sets". Since its birth, fuzzy theory has been speaking. Richard Bellman endorsed the idea and began to work in this new field. In the late 1960's many few fuzzy methods like fuzzy algorithms, fuzzy decision making, etc., were proposed.

Measure theory was developed in successive stages during the late 19th and early 20th centuries by Emile Borel, Henri Lebesgue, Johann Radon, and Maurice Frechet among others. Probability theory considers measures that assign to the whole set the size 1, and considers measurable subsets to be events whose probability is given by measure.

Fuzzy measure theory considers generalized measures in which the additive property is replaced by the weaker property of monotonicity. The central concept of fuzzy measure theory is the fuzzy measure which was introduced by Choquet[2]in1953 and independently defined by Sugeno[4] in 1974 in the context of fuzzy integrals. There exists a numbers of different classes of fuzzy measures. Sugeno[4] proposed the concept of non-additive fuzzy measure and fuzzy integral. Sergey Sakulin and Alexander Alfimtsev [3]were analyzed the practical applications of fuzzy measure and Choquet integral.

Some of the most commonly used aggregation operators are: Family of quasi arithmetic means operators(such as simple Arithmetic mean, Geometric mean, Harmonic mean, etc.,) Median (taking into account not the values themselves but only their ordering), Weighted minimum, Weighted maximum, Ordered weighted averaging operators (OWA).

All these operators are idempotent, continuous, and monotonically non decreasing. Their main common characteristic is that they all are averaging operators.

All these operators have some drawbacks: Some do not posses all the desirable properties (e.g. Quasi-arithmetic mean are not stable under positive linear transformation), and some seem to be too restrictive (arithmetic sums, OWA, etc.,).The main point here is that no one is able to model interaction between criteria in some understandable way. For interacting criteria decision making, the Choquet integrals[2] represents a suitable aggregation operator. M.E.Zuanon[7]initiated a characterization of the existence of a probability distortion about Choquet integral.

II. Preliminaries

In this section, we have presented the basic definitions.

Definition 2.1:[2]
The weighted mean is a type of mean that is calculated by multiplying the weight (or probability) associated with a particular event or outcome with its associated quantitative outcome and then summing all the products together.

The **Weighted mean** for given set of non-negative data $x_1,x_2,x_3,\ldots.x_n$ with non-negative weights $w_1,w_2,w_3,\ldots.w_n$ can be derived from the formula.

$$\text{Weighted mean} = \frac{\sum_{i=1}^{n} x_i w(i)}{\sum_{i=1}^{n} w(i)}$$

Definition 2.2:[6]

Let X be a non empty set, then a **fuzzy set** μ over X is a function from X into I = [0,1].That is μ : X → I.

Definition 2.3:[2]

A function μ on (X ,2^X) is a **fuzzy measure** if it satisfies the following axioms :

(i) μ(∅) = 0 , μ(X) = 1. (Boundary conditions)

(ii)A⊆B implies μ(A)≤μ(B) (Monotonicity)

Definition 2.4:[2]

A fuzzy measure (or the **Choquet capacity**) on C = {C_1 ,……C_m } is a monotonic set function μ : P(C) → [0,1], where P(C) is the power set of the set C, with μ(∅) = 0 and μ(C) = 1. Monotonicity means that μ(S) ≤ μ(T), whenever S ⊆T ⊆ C. An interpretation of μ(S) can be that it is the weight related to the subset S of criteria.

Definition 2.5:[2]

Given μ, the **Choquet integral** of x ∈ $(R^+)^n$ with respect to μ is defined by

$$Ch_\mu(x) = \sum_{i=1}^{n}(x_{(i)}-x_{(i-1)})\mu(\{(i),\ldots(n)\}) \qquad \ldots\ldots (1)$$

In (1) means a permutation of the elements of C such that

$x_1 \leq x_2 \ldots \leq x_n$ and $x_0 = 0$.

III. THE CHOQUET INTEGRAL FOR INTERACTING CRITERIA MODELING

In this section, we compared the results of weighted mean and Choquet integral.

Let x_i, i=1,2,…N. be vectors of objects properties, which are considered in decision making process; C_j, j=1,2,…..m, are decision making criteria; φ_{ij} (x_i),i=1,2,….N, j=1,2,…m, are scores, thatis degrees in which an object x_i satisfies the criteriaC_j. D_i, i=1,2,….N, are decisions of an object x_i with respect to all the criteria C_j. Decisions D_i are obtained by aggregation information φ_{ij} (x_i), using suitable aggregation operation.

The decisionD^*, on object $x_i$that best satisfies all criteria C_j, j=1,2,…..m, is obtained by aggregation of decisions. D_i, using suitable aggregation operation, appropriate for the considered problem.

In fuzzy multicriteria decision making systems, a score has the following property:

φ_{ij} (x_i) = μ_{ij} (x_i) ∈ [0,1] and is treated as a fuzzy measure. Thus the aggregation process of fuzzy information is an important element of fuzzy decision making system.

3.1 DECISION MAKING

Students are evaluated according to their level in 3 subjects: Mathematics, Physics and Literature. More importance attributed to mathematics and physics, and the two are considered equally important. Coefficient of importance is chosen accordingly: 3 for mathematics, 3 for physics, and 2 for literature.

Computing the average evaluation of the students by using a simple weighted mean, and with marks given on scale from 0 to 20, 3 students are evaluated in Table

Student	Mathematics	Physics	Literature	Global evaluation(Weighted mean)
A	18	16	10	15.25
B	10	12	18	12.75
C	14	15	15	14.62

Weighted mean = $\frac{\sum_{i=1}^{n} x_i \mu(i)}{\sum_{i=1}^{n} \mu(i)}$

Using fuzzy measure:

1. Boundary conditions:
   (always true for the weighted mean)

2. Relative importance of scientific versus literary subjects:
   μ(Mathematics) = μ(Physics)=0.45, μ(Literature)=0.3.

For the Students A, B & C the weighted mean is calculated as follows.

For Student A

Weighted mean = $\frac{\sum_{i=1}^{n} x_i \mu(i)}{\sum_{i=1}^{n} \mu(i)} = \frac{x_1\mu(1)+x_2\mu(2)+x_3\mu(3)}{\mu(1)+\mu(2)+\mu(3)}$

$= \frac{(18\times 0.45)+(16\times 0.45)+(10\times 0.3)}{0.45+0.45+0.3}$

= **15.25**

Similarly we get the weighted mean value for **Student B= 12.75** and for**Student C=14.62.**

The shown weighted mean student ranking is not satisfactory if the school ranking. Student A hassevere weakness is literature, but is still ranked higher than Student C, which has no weak points. This is due to too much importance being given to mathematics and physics, which are in a sense redundant, since usually, students good at are good at physics(and vice versa). This kind of evaluation tends to overestimate (resp. underestimate) students good (resp. bad)at mathematics and / or physics. Through use of the Choquet integral, a more complex decision making process reflecting criteria interaction can be modeled.

For the student ranking example, suppose the decision makers preferences are:

1. Scientific subjects (Mathematics, Physics) are more important.

2. Scientific subjects are more or less similar and students good at Mathematics (resp. Physics) are in general also good at Physics (resp. Mathematics) so that students good at both must not be too favored.

3. Students good at mathematics (or Physics) and Literature are rather uncommon and must be favored.

   These can be directly translated in term of fuzzy measure as:

   (a) Boundary conditions:
   μ (∅) = 0, μ({M, P, L}) = 1
   The importance of the empty set is 0.
   The set consisting of all objects has maximum importance.

   (b) μ ({Mathematics}) = μ ({Physics}) =0.45, μ ({ Literature}) = 0.3
   (relative importance of scientific versus literary subjects)

   (c) μ ({Mathematics, Physics}) =0.5 < μ ({ Mathematics}) + μ ({ Physics})
   (redundancy between Mathematics and Physics)

   (d) μ ({Mathematics, Literature}) =μ ({Physics, Literature})
   =0.9 > 0.45 +0.3

(Support between Literature and scientific subjects)

The idea is that superadditivity of the fuzzy measure implies synergy between criteria, and subadditivity implies redundancy. Note that it is up to expert to scale these values to the extent that he feels expresses the importance and interaction.

Applying the Choquet integral with the above fuzzy measure leads to the

following new global evaluation shown in Table:

Here Students are properly ranked in accordance to the preference relation.

Calculation:

$Ch_{\mu}(x) = \sum_{i=1}^{n}(x_{(i)}-x_{(i-1)})$ µ({(i),…..(n)})

For the students A, B and C, the Choquet integral is calculated as follows:

For Student A

$x_1 \leq x_2 \leq x_3$, 10 ≤ 16≤ 18, L ≤ P ≤ M

$Ch_{\mu}(A) = \sum_{i=1}^{3}(x_{(i)}-x_{(i-1)})$µ({(i),…..(3)})

$Ch_{\mu}(A) = (x_{(1)}-x_{(0)})$µ{(1),(2),(3)} +$(x_{(2)}-x_{(1)})$µ{(2),(3)}+$(x_{(3)}- x_{(2)})$µ{(3)}

= (10-0) µ{L,P,M}+(16-10)µ{P,M}+(18-16)µ{M}

= (10×1)+(6×0.5)+(2×0.45)

=10+3+0.9

= 13.9

Similarly we have the answer for **StudentB = 13.6**and for**Student C=14.9**

Student	Mathematics	Physics	Literature	Global evaluation(The Choquet integral)
A	18	16	10	13.9
B	10	12	18	13.6
C	14	15	15	14.9

Hence the students are ranked based on **Choquet integral** is as follows

I rank : Student C
II rank : Student A
III rank : Student B

CONCLUSIONS

In this paper we discussed about the results of weighted mean and Choquet integral. Also we investigated the students ranking accuracy using Choquet method.

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